Sir: ##

Definitions & Theorems

State and prove Cauchy's residue theorem. [4]
Define modulus and principal arguments.
Define the analyticity of a function.
State Cauchy's integral formula. [2]
Explain Laurent's theorem.
Define pole and residue with examples.
Define square matrix, unit matrix, transpose matrix, periodic matrix, Idempotent matrix, Hermitian Matrix with examples. [7]
State Cayley Hamilton Theorem.
Define conjugate complex number.

State the Cauchy's Fundamental theorem and prove it. [6]

Definition of Rank

(iv) The **rank** of a matrix A is the maximum number of linearly independent rows or columns in the matrix, or, equivalently

(v) Let A be an $m \times n$ matrix and let A_R be the row echelon form of A. Then the **rank r** of the matrix A is the number of non-zero rows of A_R.

Harmonic Function

Show that $u(x,y)$ is harmonic. Find the harmonic conjugate function $v(x,y)$ of u and the corresponding analytic function $f(z)$ of Z, where $u = y^3 - 3x^2y$. 5

Prove a Function is Continuous and Analytic

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Prove that the complex function $f(z) = \sin x \cos hy + i \cos x \sin hy$ is continuous as well as analytic at everywhere. 5

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Matrix

Finding Inverse of Matrix

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Find the inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -1 \\ 3 & 3 & 2 \end{bmatrix}$. 5

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Use elementary row operations to find the inverse of the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ (5)

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Given $A = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 0 & -2 \\ 2 & -2 & 0 \end{pmatrix}$. Find the inverse of A using co-factor matrix if possible. [7]

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Finding Rank of Matrix:

Find the rank of the matrix $B = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$. 5

Echelon Matrix

Check whether following is in row echelon form or reduced row echelon from or neither or both. (3)

(i) $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \end{bmatrix}$

Others:

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Find all the eigenvalues and associated eigenvectors of the matrix (5)

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

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State Cayley Hamilton theorem. Using this theorem find out A^3 and A^{-1} for the following matrix (7)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

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Prove that $A = \frac{1}{6}\begin{pmatrix} 3 & 3 & 3 & 3 \\ 3 & -5 & 1 & 1 \\ 3 & 1 & 1 & -5 \\ 3 & 1 & -5 & 1 \end{pmatrix}$ is an orthogonal matrix. $A \cdot A^T = I$ [5]

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a) Define eigenvalues and eigenvectors of a square matrix. Find the eigenvalues of the matrix: [6]

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{pmatrix}$$

b) Solve the system of the following equati...

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b) Solve the system of the following equations using matrix method: [6]

$$3x - 2y = 3$$
$$-x + 4y = 11$$

8.

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$(ii)\ 1 < |z| < 2$

b) Using Cayley-Hamilton theorem find A^{-1}, where A matrix is: [6]

$$\begin{pmatrix} 2 & 3 & 3 \\ 1 & 3 & 3 \\ 1 & 2 & 4 \end{pmatrix}$$

Finding Modulus & Principal Argument

Define modulus and principal arguments. Find the modulus and principal arguments of the following complex numbers (6)

(i) $z = \left(\frac{1+i}{1-i}\right)^2$

(ii) Show that $|2z + 3\bar{z}| \leq 4|Re(z)| + |z|$

Cauchy's Theorem

Using Cauchy's theorem, evaluate the following integral. [6]

$$\oint \frac{e^{zt}}{(z^2+1)^2} dz \ ; t > 0 \text{ and } c: |z| = 3$$

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Let $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$ when $z \neq 0$ & $f(0) = 0$, when $z = 0$. (3)

Show that $f(z)$ satisfies the Cauchy Riemann equations but is not differentiable at z=0.

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Let $f(z) = \frac{(x^3 - 3xy^2) + i(y^3 - 3x^2y)}{x^2+y^2}$ when $z \neq 0$ & $f(0) = 0$, when $z = 0$. (6)

Show that $f(z)$ satisfies the Cauchy Riemann equations but is not differentiable at z=0.

#

Evaluate the following integral by Cauchy's integral formula, (6)

(i) $\oint_c \frac{z}{(9-z^2)(z+i)} dz$, where c is a circle $|z| = 2$

(ii) $\oint_c \frac{e^{3z}}{z - \pi i} dz$, where c is a curve $|z - 1| = 4$

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Laurent's Theorem

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State Laurent's theorem and expand $f(z) = \frac{1}{(z+1)(z+3)}$ in Laurent series, valid for the region $|z| > 3$ (6)

... $f(z)$ be analytic inside and on a simple (6)

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Expand the function $f(z) = \frac{1}{(z-1)(z-2)}$ in a Laurent Series valid for: [6]

$(i)\, 0 < |z| < 1$

$(ii)\, 1 < |z| < 2$

Using Cayley-Hamilton theorem ...

Mixed

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Using the property $|a + b| \le |a| + |b|$ show that, (2)

$$|2z + 3\bar{z}| \le 4|Re(z)| + |z|$$

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Find all values of $(-8i)^{\frac{1}{4}}$. (2)

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Check whether following is in row echelon form or reduced row echelon from or neither or both. (3)

(i) $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

(iii) $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \end{bmatrix}$

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Show that the following system of linear equations given by (4)

$$2x - 3y + 5z = 1$$
$$3x + y - z = -2$$
$$x + 4y - 6z = -3$$

has infinite number of solutions.

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Determine the value of λ and μ such that the following system has (i) no solution, (ii) more than one solution, (iii) a unique solution (5)

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu$$

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Define conjugate complex number. Find all the roots of the equation $sinhz = i$. [6]

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Consider that z is any complex number and show that: [6]

(i) $Im(iz) = Re(z)$ and $Re(iz) = |z|^2 Im(z^{-1})$

(ii) $Re(z) = \frac{z+\bar{z}}{2}$, $Im(z) = \frac{z-\bar{z}}{2i}$

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If $\lim_{z \to z0} f(z)$ exists, prove that it must be unique. [3]

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Show that $\oint_C \frac{e^{tz}}{z^2+1} dz = 2\pi i \sin t$, where C is the circle $|z| = 3$ and $t > 0$. [6]

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Show that $\oint_c \frac{zf'(z)}{f(z)} dz = 4\pi i$, where $f(z) = z^4 - 2z^3 + z^2 - 12z + 20$ and c is the circle [8]

$|z| = 5$. [4]

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$|z| = 5$.

Find the Residue of $\frac{z^2}{(z-1)(z-2)}$ at $z = 1$. [4]

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Sketching Z-Plane, W-Plane

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Determine the equation of the curve in the W-plane into which the straight line $x + y = 1$ is mapped under the transformation $w = \frac{1}{z}$. [6]

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mapped under the transformation $w = \frac{1}{z}$.

b) Define bilinear transformation which transforms points $z = 0, -i, -1$ into $w = i, 1, 0$ respectively. [6]

Exists or Not

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Check whether $\lim_{z \to 0} \frac{Re(z)Im(z)}{Re(z)+Im(Z)}$ exists or not. (3)

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Analytic Function

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Define the analyticity of a function. Show that the function $f(z) = z\bar{z}$ is nowhere analytic. (5)

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Show that the function $f(z) = z|z|$ is not analytic. [3]

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Show that the function $u = 4xy - 3x + 2$ is harmonic. Construct the corresponding analytic function $f(z) = u(x, y) + iv(x, y)$. [6]

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Harmonic Function

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Show that the function $u = 4xy - 3x + 2$ is harmonic. Construct the corresponding analytic function $f(z) = u(x, y) + iv(x, y)$. [6]

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