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## Definitions & Theorems

- # State and prove Cauchy's residue theorem. [4]
- # Define modulus and principal arguments.
- # Define the analyticity of a function.
- # State Cauchy's integral formula. [2]
- # Explain Laurent's theorem.
- # Define pole and residue with examples.
- # Define square matrix, unit matrix, transpose matrix, periodic matrix, Idempotent matrix, Hermitian Matrix with examples. [7]
- # State Cayley Hamilton Theorem.
- # Define conjugate complex number.

State the Cauchy's Fundamental theorem and prove it.

[6]

## ## Definition of Rank

(iv) The **rank** of a matrix A is the maximum number of linearly independent rows or columns in the matrix,

or, equivalently

(v) Let A be an  $m \times n$  matrix and let  $A_R$  be the row echelon form of A. Then the **rank r** of the matrix A is the number of non-zero rows of  $A_R$ .

## Harmonic Function

Show that  $u(x,y)$  is harmonic. Find the harmonic conjugate function  $v(x,y)$  of  $u$  and the corresponding analytic function  $f(z)$  of  $Z$ , where  $u = y^3 - 3x^2y$ .

## Prove a Function is Continuous and Analytic

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Prove that the complex function  $f(z) = \sin x \cos hy + i \cos x \sin hy$  is continuous as well as analytic at everywhere.

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## Matrix

### Finding Inverse of Matrix

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Find the inverse of the matrix  $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & -1 \\ 3 & 3 & 2 \end{bmatrix}$ .

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Use elementary row operations to find the inverse of the matrix  $A =$

$$\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$$

(5)

#

Given  $A = \begin{pmatrix} 1 & 1 & -1 \\ -2 & 0 & -2 \\ 2 & -2 & 0 \end{pmatrix}$ . Find the inverse of  $A$  using co-factor matrix if possible.

[7]

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### Finding Rank of Matrix:

Find the rank of the matrix  $B = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$ .

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### Echelon Matrix

Check whether following is in row echelon form or reduced row echelon form or neither or both. (3)

(i)  $\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$

(ii)  $\begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}$

(iii)  $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \end{bmatrix}$

### Others:

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Find all the eigenvalues and associated eigenvectors of the matrix (5)

$$A = \begin{bmatrix} 1 & 0 & C \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

#

State Cayley Hamilton theorem. Using this theorem find out  $A^3$  and  $A^{-1}$  for the following matrix (7)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

#

Prove that  $A = \frac{1}{6} \begin{pmatrix} 3 & 3 & 3 & 3 \\ 3 & -5 & 1 & 1 \\ 3 & 1 & 1 & -5 \\ 3 & 1 & -5 & 1 \end{pmatrix}$  is an orthogonal matrix.

$$A A^T = I$$

[5]

#

Define eigenvalues and eigenvectors of a square matrix. Find the eigenvalues of the matrix: [6]

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{pmatrix}$$

Solve the system of the following equations

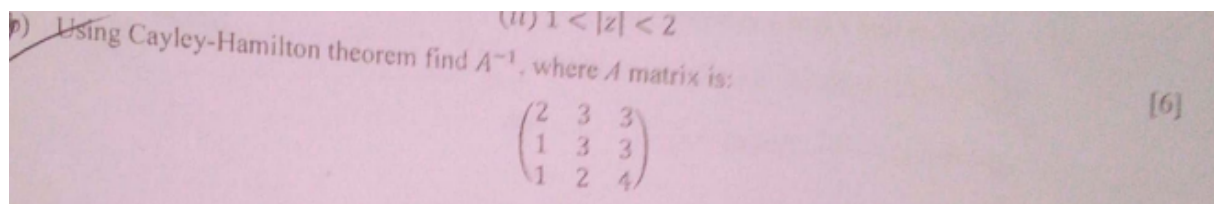
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Solve the system of the following equations using matrix method: [6]

$$3x - 2y = 3$$

$$-x + 4y = 11$$

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## Finding Modulus & Principal Argument

Define modulus and principal arguments. Find the modulus and principal arguments of the following complex numbers (6)

(i)  $z = \left(\frac{1+i}{1-i}\right)^2$   
 (ii) Show that  $|2z + 3\bar{z}| \leq 4|\operatorname{Re}(z)| + |z|$

## Cauchy's Theorem

# Using Cauchy's theorem, evaluate the following integral. [6]

$$\oint \frac{e^{zt}}{(z^2 + 1)^2} dz ; t > 0 \text{ and } c: |z| = 3$$

#

Let  $f(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2 + y^2}$  when  $z \neq 0$  &  $f(0) = 0$ , when  $z = 0$ . (3)  
 Show that  $f(z)$  satisfies the Cauchy Riemann equations but is not differentiable at  $z=0$ .

#

Let  $f(z) = \frac{(x^3 - 3xy^2) + i(y^3 - 3x^2y)}{x^2 + y^2}$  when  $z \neq 0$  &  $f(0) = 0$ , when  $z = 0$ . (6)  
 Show that  $f(z)$  satisfies the Cauchy Riemann equations but is not differentiable at  $z=0$ .

#

Evaluate the following integral by Cauchy's integral formula, (6)

(i)  $\oint_c \frac{z}{(9-z^2)(z+i)} dz$ , where  $c$  is a circle  $|z| = 2$   
 (ii)  $\oint_c \frac{e^{3z}}{z-\pi i} dz$ , where  $c$  is a curve  $|z-1| = 4$

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## Laurent's Theorem

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State Laurent's theorem and expand  $f(z) = \frac{1}{(z+1)(z+3)}$  in Laurent series, valid for the region  $|z| > 3$  (6)

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Expand the function  $f(z) = \frac{1}{(z-1)(z-2)}$  in a Laurent Series valid for: (6)

(i)  $0 < |z| < 1$

(ii)  $1 < |z| < 2$

Using Cayley-Hamilton theorem find...

## Mixed

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Using the property  $|a + b| \leq |a| + |b|$  show that, (2)

$$|2z + 3\bar{z}| \leq 4|\operatorname{Re}(z)| + |z|$$

#

Find all values of  $(-8i)^{\frac{1}{2}}$ . (2)

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Check whether following is in row echelon form or reduced row echelon form or neither or both. (3)

(i) 
$$\begin{bmatrix} 1 & 0 & 3 & 1 \\ 0 & 1 & 2 & 4 \end{bmatrix}$$

(ii) 
$$\begin{bmatrix} 1 & -7 & 5 & 5 \\ 0 & 1 & 3 & 2 \end{bmatrix}$$

(iii) 
$$\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 3 \\ 0 & 1 & 0 & 4 \end{bmatrix}$$

#



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Show that the following system of linear equations given by (4)

$$\begin{aligned} 2x - 3y + 5z &= 1 \\ 3x + y - z &= -2 \\ x + 4y - 6z &= -3 \end{aligned}$$

has infinite number of solutions.

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Determine the value of  $\lambda$  and  $\mu$  such that the following system has (i) no solution, (ii) more than one solution, (iii) a unique solution (5)

$$\begin{aligned}x + y + z &= 6 \\x + 2y + 3z &= 10 \\x + 2y + \lambda z &= \mu\end{aligned}$$

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Define conjugate complex number. Find all the roots of the equation  $\sinh z = i$ . [6]

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Consider that  $z$  is any complex number and show that: [6]

(i)  $Im(iz) = Re(z)$  and  $Re(iz) = |z|^2 Im(z^{-1})$

(ii)  $Re(z) = \frac{z+\bar{z}}{2}$ ,  $Im(z) = \frac{z-\bar{z}}{2i}$

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If  $\lim_{z \rightarrow z_0} f(z)$  exists, prove that it must be unique. [3]

#

Show that  $\oint_C \frac{e^{tz}}{z^2+1} dz = 2\pi i \sin t$ , where  $C$  is the circle  $|z| = 3$  and  $t > 0$ . [6]

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Show that  $\oint_C \frac{zf'(z)}{f(z)} dz = 4\pi i$ , where  $f(z) = z^4 - 2z^3 + z^2 - 12z + 20$  and  $c$  is the circle  $|z| = 5$ . [8]

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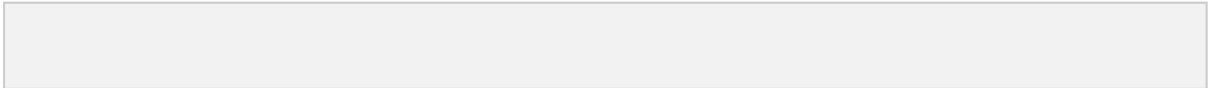
$|z| = 5$ . [4]

Find the Residue of  $\frac{z^2}{(z-1)(z-2)}$  at  $z = 1$ .

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## Sketching Z-Plane, W-Plane

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Determine the equation of the curve in the W-plane into which the straight line  $x + y = 1$  is mapped under the transformation  $w = \frac{1}{z}$ . [6]

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Define bilinear transformation which transforms points  $z = 0, -i, -1$  into  $w = i, 1, 0$  respectively. [6]

## Exists or Not

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Check whether  $\lim_{z \rightarrow 0} \frac{\operatorname{Re}(z)\operatorname{Im}(z)}{\operatorname{Re}(z)+\operatorname{Im}(z)}$  exists or not. (3)

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## Analytic Function



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Define the analyticity of a function. Show that the function  $f(z) = z\bar{z}$  is nowhere analytic. (5)

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Show that the function  $f(z) = z|z|$  is not analytic. [3]

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Show that the function  $u = 4xy - 3x + 2$  is harmonic. Construct the corresponding analytic function  $f(z) = u(x, y) + iv(x, y)$ . [6]

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## Harmonic Function

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Show that the function  $u = 4xy - 3x + 2$  is harmonic. Construct the corresponding analytic function  $f(z) = u(x, y) + iv(x, y)$ . [6]

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