Updated, June 2024

OVERVIEW:

This full-year Algebra 2 course is designed to provide students with the opportunity to strengthen their skills in the areas of linear, quadratic, and exponential functions, while extending their content base and knowledge to include higher-degree polynomial functions (including now solving these over the set of complex numbers), rational exponents and radical functions, solving exponential and logarithmic equations, rational functions and trigonometric ratios and functions. In addition to these topics, students will refine their abilities to model with mathematics, reason abstractly and quantitatively while attending to precision both in calculations and vocabulary, and to make sense of problem situations as an essential part of the solution process.

Learning Plan

Unit 1 Linear Functions (*3 weeks*)

1.1 Day 1

- Introduce parent functions: constant, linear, absolute value and quadratic. Students should create and graph functions from tables of parent functions. Introduce vertical translations and identify equations given graphs.

1.1 Day 2

- Show the remaining transformations (horizontal translation, vertical stretch, reflection over x-axis)

1.2

- Apply multiple transformations to linear and absolute value functions. Identify the equation of a given graph. Graph a given function.
- Apply transformations on a non-parent function and find a new equation.

Mid Chapter Quiz (review and quiz day)

1.3 Day 1

- Graph linear equations.
- Find equations of lines given the graph.
- Compare linear equations to real life problems

1.3 Day 2

- Plot data to find line of fit using straightedge, calculating the slope and solving for the y-intercept.
- Use technology to find the line of Best fit.
- Interpret correlation coefficient. Use line of fit or best fit to predict.

1.4 2 days

- Discuss intersection(s) of linear systems in three variables.
- Use algebra to solve systems of linear equations in three variables.

Second half Chapter Quiz (review and quiz day)

SGO or PSAT practice in between

Chapter Test (2023, Test Date 10/10)

Unit 2 Graphing Quadratic Functions (*3 weeks*) (Big Ideas Chapter 2.1, 2.2, 2.4)

2.1 Transformations (2 days)

- Students will apply knowledge from previous chapter to quadratic functions, using x^2 as the new parent function
- Transformations will include: horizontal and vertical translations, reflections over both x and y axis, and vertical stretches.
- Students will graph a transformed equation
- Students will write equation, given a graph
- Given a description of transformations, students will name the equation and graph the equation.
- Students will name the vertex as a characteristic of the graph and identify the values for (h, k) in the equation.
- Students will recognize $y = a(x - h)^2 + k$ as the vertex form of a quadratic equation.

2.2 Graphing using 3 Forms of Quadratic Equations (3 days)

- Introduce the Axis of Symmetry focusing on its relationship with the vertex and how points on the parabola are reflected across the Axis of Symmetry
- Graph a quadratic given in vertex form, using Axis of Symmetry to quickly find additional points on the curve.
- Derive Standard Form by expanding Vertex Form.
- Recognize properties of the graph of a quadratic in Standard Form: whether it opens up or down, how to calculate the Axis of Symmetry and the Vertex, finding the y-intercept.
- Graph a Quadratic in Standard Form
- Identify characteristics of the Graph of a Quadratic curve: name the maximum or minimum value, name the increasing and decreasing interval, name the domain and range.
- Discuss Intercept Form, $y = a(x - p)(x - q)$, including the vertex can be found by finding the midpoint between p and q and substituting the value for the x-coordinate of the vertex back into the equation to find the y-coordinate.
- Use quadratic models of real life problems to answer questions about the problem: max height, furthest distance, etc.

2.4 Finding Equations of Quadratic Equations (2 days)

- Given the vertex and a point on the parabola, find the equation of the parabola in vertex form (solve for a); be able to expand to standard form.
- Given the intercepts and a point on the parabola, find the equation of the parabola in intercept form; be able to expand to standard form.
- Find the average rate of change from the vertex to each intercept; interpret the results
- Use technology (graphing calculator, or desmos) to find a regression equation that fits the data. Interpret its correlation coefficient and use the equation to predict.

Frequent assessments, with next-day results, should be given throughout this chapter as there is a lot of new material that should be memorized.
Each section, in and of itself, could have it's own big assessment.
Real life examples could be used to review for a large chapter test. (2023 Test Date 10/25)

Unit 3 should begin during the first marking period, but most likely will not be completed until the second marking period has begun.

Unit 3 Quadratic Equations and Complex Numbers (*3 weeks*) (Chapter 3)
*Before starting with the sections in the textbook, students should review factoring: AC (include Grouping) or Trial and Error, Difference of Squares, GCF (2 days) This need to spend time reviewing factoring from Algebra 1 should lessen over time as foundational skills will improve.
3.1 Solving Quadratic Equations (2 days new material, 1 day practice)

- Make the connection, using graphing from the previous chapter, that the *solution* to a quadratic equation is the x-intercept. Also, use the term *root*, which can be interchanged with solution.
- Discuss three methods for solving quadratic equations: factoring, taking the square root, and graphing. Discuss characteristics of equations that would suggest the most efficient method for solving.
- Discuss the Zero Product Property and why it works when factoring to solve.
- Review simplifying radicals when solving by taking the square root.
- This section may take three days, if you include a pure work day, where students solve equations. These methods are important to the class and students may need more practice, other than homework.

**Quiz on factoring and solving quadratics using the first three methods. Students should recognize the difference between the instructions to “solve” and to “factor”.
3.2 Complex Numbers (2 days)

- Introduce Complex numbers by asking the solutions to a quadratic that doesn’t intersect the x-axis.
- Discuss the classification of numbers, now including Complex Numbers and Pure Imaginary numbers.
- Define $i = \sqrt{-1}$
- Simplify square roots of negative numbers.
- Students should understand that an imaginary number multiplied by another imaginary number gives a real number: $i^2 = -1$
- Operations on complex numbers: adding, subtracting, multiplying and combine operations (distributive property).
- Recognize complex conjugates: $(a + bi)(a - bi) = a^2 - (-1)(b^2)$.
- Omit division of complex numbers, including by a single term and binomial. Omit higher powers of i
- Solve quadratic equations by taking the square root and simplifying the negative square root into two imaginary solutions.

3.3 Solving Quadratic Equations by Completing the Square (1 day)

- Review re-writing a perfect square trinomial as a binomial squared
- Solve a quadratic where a perfect square trinomial is given and students rewrite as a binomial squared. Take the square root of both sides and continue to solve, where the solution could be any form of a complex number.
- Decide the process for creating a perfect square trinomial by adding $(b/2)^2$ to both sides of the equation.
- Solve, when $a \neq 1$, by completing the square.

3.4 Solve Quadratic Equations by using the Quadratic Formula (1 day plus 1 practice day for 3.3 and 3.4)

- Use a mnemonic device (video, song, etc) so students memorize the Quadratic Formula
- Review solving Quadratics using the Quadratic Formula by setting the equation equal and combining like terms.
- Discuss, in detail, how to simplify solutions. Students tend to cancel common factors in only two terms, but must cancel from all three terms.
- Find and interpret the discriminant of a quadratic equation. Students should find the discriminant to decide which method of solving the equation to use. Students should understand that the discriminant is not the solution to the equation.

**Quiz on 3.2, 3.3 and 3.4

Modeling with Quadratic Equations

Use real world problems throughout the chapter to review techniques in preparation for a test on Factoring and 3.1-3.4. Also, projectile problems can be used. 11th grade students will make a connection with problems from Physics class.

** Test on Factoring and 3.1-3.4

The next two sections can be completed as independent units, depending on timing within the school building and calendar.

3.5: Solving Nonlinear systems. (2 days)

- Review graphing circles in standard form, covered in Geometry. (If time permits: Convert from Standard Form to (h,k) form by completing the square for both x and y.)
- Solve nonlinear systems (linear, quadratic, circles) by graphing. Review graphing and transformation techniques as well as technology to find intersection points.
- Solve nonlinear systems by substitution. Students need to be reminded that the solution is an ordered pair, and that they need to substitute their value for x back into either equation.
- Solve nonlinear systems by elimination.
- Students should be aware of limitations of each method and decide which method to use for any given system.

3.6 Quadratic Inequalities (2 days)

- Use quadratic graphing techniques to graph a quadratic inequality in two variables. Use solid or dotted lines to include or exclude border points.
- Graph a system of quadratic inequalities in two variables, shading the final solution set.
- Solve quadratic inequalities in one variable algebraically and graphically.
- Introduce interval notation to represent the solution set as opposed to inequality notation.

Unit 4 Polynomials (4 weeks, overlapping first and second semester)

4.2 Adding, Subtracting, and Multiplying Polynomials (1 day)

- Name Polynomials according to the number of terms and degree (4.1)
- Adding, Subtracting, and Multiplying Polynomial Expressions
- Remind students to distribute the negative sign when subtracting
- Use both traditional distribution and box method to multiply

4.3 Dividing Polynomials (2 days)

- Use Synthetic Division to divide polynomials by binomials of the form (x-k)
- Write remainder as the last term of the quotient over the binomial divisor
- Discuss the Remainder Theorem. Students should evaluate functions f(k) and divide by x-k to see that the remainder is the same as the output when evaluated.

Quiz

4.4 Factoring Polynomials (2 days)

- Open lesson by multiplying polynomials where factoring patterns will be revealed later in lesson (ex: (2x)(3x-5)(3x+5) and (2x+1)(4xx-2x+1))
- Students will review all types of factoring, including GCF, Trinomials, Grouping, Difference of Squares. Students should factor completely where multiple types of factoring are required.
- Students will learn a new factoring pattern: sum and difference of cubes. Introduce, review from warm up, practice problems.
- Show students that the Factor Theorem is a specialized case of the Remainder Theorem; (x-k) is a factor of f(x) if f(k)=0
- Use the Factor Theorem to decide if a binomial is a factor of an unfactorable polynomial. Then use Synthetic Division to divide and continue to factor the polynomial

4.5 Solving Polynomial Equations (2 days).

- Solve polynomials using factoring and the Zero-Product Property. Include repeated solutions and introduce the concept of multiplicities.
- Recognize that the terms zero, solution and root are synonymous. Also, that if k is a solution, then (x-k) is a factor. Students should understand that if they are given a factor of a polynomial, then they also know a solution of the polynomial and an x-intercept.
- Use Rational Root Theorem to name possible rational solutions.
- Use Synthetic Division to test possible roots and find all solutions by factoring quotient.
- Write polynomials in factored form after finding all solutions.
- Discuss implications when the value of the lead coefficient changes from 1 to a different value. The total number of possible rational roots increases. Students can use technology to find rational roots, then use Synthetic Division to reduce the polynomial.
- After using the Quadratic Formula to fully solve a polynomial, decide that irrational solutions are always found in pairs. Discuss the Irrational Conjugates Theorem.

Last big assessment before Midterms

SEMESTER BREAK

4.6 Fundamental Theorem of Algebra (2 days)

- The degree of a polynomial in standard form is easily identified by the highest exponent.
- The degree of a polynomial in factored form can be found by adding the multiplicities of the factors.
- The Fundamental Theorem of Algebra states that the total number of solutions of a polynomial equals the degree, when the total number of solutions includes repeated solutions, and pairs of irrational and complex solutions.

- Find all zeros of a polynomial by using the Rational Root Theorem and Factoring. Students will discover the Complex Conjugates Theorem by solving the remaining quadratic. Complex solutions always come in +/- pairs like irrational solutions.
- Students will write a polynomial in factored form and standard form given the zeros and at least one irrational or complex solution.

4.8, 4.1: Graphing Polynomials (2 days, 1 day practice)

- Introduce the concept of end behavior and how it can be predicted by the characteristics of the given polynomial. Review parent function x squared and the reflection across the x-axis. Connect degree of quadratic and sign of lead coefficient with end behavior. Also, introduce the function x cubed and negative x cubed. Use these functions to remember end behaviors given degrees and signs of lead coefficients.
- Review multiplicities of factors from 4.5. When the multiplicity is even, the graph will stay of the same side of the x-axis or "bounce". When the multiplicity is odd, the graph will cross through the x-axis; as the multiplicity increases, the shape of the graph at the zero flattens. Use technology (Delta Math) to illustrate this concept. Students are given equations in factored form and should use Delta Math to graph and observe how the change of multiplicity is reflected in the graph.
- Graph polynomials using end behavior, zeros, and multiplicities. Emphasize to students that they should connect tails and zeros, with correct shapes according to multiplicities, moving from left to right.
- Students should evaluate an x value into the given polynomial to confirm the graph is correct.
- Sketch graph given constraints including increasing and decreasing intervals and values for x when $f(x) > 0$ and $f(x) < 0$. Students should also use interval notation.
- Students will identify odd and even functions and realize that functions could also be neither odd or even. They should see the symmetry visually and be able to decide if a polynomial is odd or even algebraically.

**Test on second half of Chapter 4: 4.6, 4.8, 4.1

Unit 5 Rational Functions (Chapter 7 2 weeks)

Depending on timing and schedule, Operations and Solving Rational Equations can be covered and assessed separately from Graphing.

Throughout this unit, students should recognize the values that are excluded; values that make the rational expression undefined. This leads to realizing extraneous solutions and values of discontinuity on a graph.

7.3: Simplifying, Multiplying and Dividing Rational Expressions (1 day)

- Factor and cancel common factors
- Students often struggle with monomial terms.
- When dividing, discuss "factoring and flipping" in the same step

7.4: Adding and Subtracting Rational Expressions (2 day2)

- Begin with rational expressions that have common denominators
- When subtracting, students should recognize that they are subtracting the entire numerator and must distribute the negative sign.
- Find common denominators by first writing each denominator in factored form.
- 2nd Day, additional practice and complex fractions.

7.5: Solving Rational Equations, Extraneous Solutions (1 Day)

- Review solving proportions by cross multiplying
- Solve equations with more than 2 terms by multiplying through by the LCD to cancel all denominators.
- Students often struggle with determining the LCD. They may need to take two steps to cancel all denominators.

Practice Day/Review for Assessment

7.2: Graphing Rational Functions (2-3 days)

- Discuss the parent function 1/x and find points on this graph.
- Discuss discontinuous domains of rational expressions or equations; values excluded from solution sets and why. These values will be areas where the graph is discontinuous. The values are either vertical asymptotes or holes.
- Discuss how to find characteristics of the graphs of rational functions: x-intercept, y-intercept.
- Horizontal Asymptotes have three possibilities. Use a pneumonic device for students to remember them. Plug in a large value for x to see how it affects the graph. Discuss the fact that the graph could go through a horizontal asymptote around the origin; the asymptote is a value as x gets very large or very small.
- Students should create tables to find points on either side of a vertical asymptote.
- At teacher's discretion, students could take formal assessment or complete a project on this topic.

Unit 6 Rational Exponents and Radical Functions (Chapter 5) (3 weeks)

This unit can be split into three mini units, with 3 assessments and a possible comprehensive unit assessment if time allows.

Prior to the start of the unit, rules for exponents should be reviewed. These are standards that are included in Algebra 1 and were included with the summer review. It would be most beneficial if rules of exponents were worked in throughout the year during spiral type review.

5.1: nth roots and Rational Exponents (1 day)

- Discuss nth roots and how many solutions are found when n is even or n is odd.
- Introduce Rational Exponents. Spend time with examples and have students write the same expression in exponential form and radical form. Also stress that in radical form it doesn't matter which is evaluated first, the root or the exponent. Students may be given

a list of perfect squares, cubes, fourths and a few fifths to use as a reference during this unit.
- Solve equations by taking the nth root. Remember when n is even, there could be two or zero solutions, but when n is odd, there is only one solution.

5.2: Properties of Rational Exponents and Radicals (3 days instruction, 1 additional day practice)
- Day 1–Properties of Rational Exponents
    - Review properties of exponents using rational exponents. Students should have ample time to practice in class. Give students the list of the properties of exponents and have them refer to it to decide which to use for the given problem.
- Day 2–Simplifying Radical Expressions
    - Product and Quotient Properites
    - Discuss when a radical expression is fully simplified
    - Rationalize a denominator when the root is something other than 2
    - Use conjugates to rationalize when a binomial is in the denominator.
    - Adding and Subtracting “like” Radicals
- Day 3–Simplifying Variable Expressions
    - Apply previous properties and skills when variables are involved
    - Students will simplify radicals with variables when the variable has an exponent that is higher than the root.
    - If the root is even, remember to use absolute value (from 5.1) when needed.

Homework should be assigned each day. It is important that students check their homework; find their mistakes. Answers should be supplied. Only trouble HW problems should be reviewed as the class time will be needed for new material and practice.

Review day, then assessment on 5.1 and 5.2

5.3: Graphing Radical Functions (2 days)
- Graphing square and cube roots. Define parent function and its characteristics. Use xy-table to find parent functions. Discuss restrictions on domain.
- Graph transformations of radical functions. Students should remember rules of transformations.
- Reflection over y-axis for cube root is the same as reflection over x-axis. Reflection over y-axis is different than the reflection over the x-axis for square root. Using technology and tables (Desmos) can be efficient with this topic.

5.4: Solving Radical Equations. (2 days)
- Solving radical equations. Solve algebraically and graphically. Using the graphing method can help reinforce why extraneous solutions exist.
- Review factoring, squaring a binomial requires FOIL
- Check for extraneous solutions
- Show example where squaring both sides is required twice.
- Solve equations with Rational Exponents using reciprocals. When the denominator is even, an even root will be taken. This will result in two solutions.

Assessment on 5.3 and 5.4

5.5 Function Operations (1 day)

- Review definition of a function. Discuss what is and is not a function (this will be useful when learning about inverses).
- Review Adding, Subtracting, Multiplying and Dividing Functions.
- Find the domain of resulting function.
- Evaluate the resulting function.
- Function operations on graphs of f(x) and g(x). Students should see that an equation is not always needed to complete function operations.

5.6: Composition of Functions (1 day)

- Define the new operation, composition.
- Perform composition examples, including "both directions" and composition onto itself–f(f(x)).
- Evaluate composed functions.
- Use graphs of f(x) and g(x) to find compositions.

5.7 Inverse Functions (2 days)

- Discuss definition of inverse functions. Use algebra and graphs to illustrate inverse functions.
- Discuss restricting domain of f(x) so that its inverse will be a function. Introduce the Horizontal Line Test.
- Find inverse of linear, quadratic, cubic and radical functions algebraically.
- Find inverse of given points in a table.
- Find inverse of a given graph by simply reversing x and y and sketching new points.
- Use composition to determine if two functions are inverses of each other.

Function assessment from 5.5, 5.6 and 5.7

If time allows, large assessment on entire unit.

Unit 7 Trigonometry (Chapter 10 3-4 weeks, depending on background right triangle knowledge):

10.1: Right Triangle Trig (3-4 days)

- Review Right Triangle Trig with all 6 Trig functions. Students should be familiar with Sine, Cosine and Tangent. Introduce Cosecant, Secant and Cotangent as reciprocal functions.
- Students should build on previous geometry knowledge and incorporate Algebra 2 concepts of functions, by defining the 6 trig functions where the independent variable/input is the acute angle of a right triangle and the dependent variable/output is the value of the trig ratio of that angle.
- Evaluate 6 Trig functions of an acute angle given a right triangle. Use Pythagorean Thm to find missing length. Review simiplyfiying radicals and rationalizing denominator.

- Use proportions to solve for missing lengths given one side of a right triangle and one acute angle.
- Use inverse functions to find angle of right triangle given two lengths
- Review special right triangles, solving for sides of 30-60-90 and 45-45-90 triangles.
- Find Trig values for special angles. Students should realize that since the triangles are similar, the ratios of their sides will be equal. They should begin to memorize, or be able to find it relatively quickly. There is a lot of material for students to remember, but making sure that they have a strong foundation is paramount.
- Solve a Right Triangle using technology.
- Use trig functions to solve real world problems.

possible assessment on Right Triangle Trig with 6 Trig Functions

10.2 Angles and Radian Measure (2-3 days)

- Students will expand on their use of trig functions from acute angles to any angle.
- Introduce radian measure. Use technology or videos to visually explain the concept of radian measure. Discuss how calculators can change modes between radian to degree.
- Convert from radians to degrees and degrees to radians
- Write special angles, multiples of 30 and 45 in radian measure.
- Draw angles (radian and degree) in standard position.
- Find coterminal angles (radian and degree measure).
- Sector area and arc length
- Real world problems using angles.

possible assessment on first two sections

10.3 Trig Functions of any Angle (3 days)

- Define unit circle and name 6 Trig functions using x and y when radius/hypotenuse is 1.
- Evaluate 6 Trig functions given a point on the unit circle. Students will need to rationalize denominators.
- Define Quadrantal Angles; find trig functions of quadrantal angles.
- Define and find reference angles for each quadrant. This is one area where students should understand conceptually the idea of reference angle instead of memorizing the rule for each Quadrant. Students should find reference angles for positive and negative angles given in degree and radian measure.
- Reference angles allow a student to find a trig function for any angle. Students should also recognize that the sign of each trig function depends on the quadrant where the terminal side of the angle lies. (use mnemonic device, ASTC, All Students Think Clearly, to support this conceptual idea).
- Finally, use concepts from this section and prior knowledge to find the trig function of a given angle (multiple of 30 or 45) given in degree or radian measure by:
    - Sketching the angle
    - Finding the reference angle
    - Evaluating the trig function of the angle
    - Determining the sign of the value depending on the quadrant

    - Students should complete these steps without the use of a calculator as they are finding exact values.

Assessment

If time allows, Units 8, 9 and 10 can be completed. Discuss progress with other Algebra 2/Intro to Trig CP teachers as well as Pre-Calc and Trig CP teachers.

Unit 8 Exponential and Logarithmic Functions (Chapter 6, 2 weeks)

- Graphing exponential growth and decay functions. Discuss characteristics, domain, range, y-intercept. Discuss the meaning of exponential growth and decay using tables. (6.1)
- Solve real world problems using exponential growth. (6.1)
- Solve exponential equations by making like bases, using Property of Equality for Exponential Equations. (6.6)
- Introduce the concept of logs by solving exponential equations where like bases can not be found. Write expressions in log form and exponential form. Evaluate logs without calculator and with a calculator using change of base formula. (6.3)
- Introduce concept of e and continuously compounding interest formula. (6.2)
- Discuss properties of logs. Expand and condense log expressions. (6.5)
- Use properties of logs to solve logarithmic and exponential equations. (6.6)

Unit 9 Series and Sequences (Chapter 11, 2 weeks)

11.1 Defining and Using Sequences and Series
- Use rules to write terms of sequences.
- Introduce and discuss series, summation notation, and sigma notation.
- Write rules for sequences.
- Write and Find sums of series.

11.2 Analyzing Arithmetic Sequences and Series
- Identify arithmetic sequences.
- Write rules for arithmetic sequences.
- Find sums of finite arithmetic series.

11.3 Analyzing Geometric Sequences and Series
- Identify geometric sequences.
- Write rules for geometric sequences.
- Find sums of finite geometric series.

11.4 Finding Sums of Infinite Geometric Series
- Find partial sums of infinite geometric series.
- Find sums of infinite geometric series.

Unit 10 Probability and Statistics (Chapter 8 and 9, 3 weeks)

- Basic theoretical/experimental probability (1 day)
- Independent/dependent rules (1 day)
- Mutually exclusive and overlapping rules (1 day)
- Deciding if a specified model is consistent with the results from a given data-generating process (½ day)
- Recognizing the purposes of and differences among sample surveys, experiments, and observational studies (½ day)
- Using the mean and standard deviation of a data set to fit to a normal distribution (1 day)
- Estimating population percentages (½ day)
- Recognizing when data sets are not appropriate for the use of a normal distribution (½ day)
- Using calculators spreadsheets (1 day)
- Tables to estimate areas under the normal curve (1 day)