

Simple and Tighter Derivation of Achievability for Classical Communication Over Quantum Channels

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Achievability in information theory refers to demonstrating a coding strategy that accomplishes a prescribed performance benchmark for the underlying task. In quantum information theory, the crafted Hayashi-Nagaoka operator inequality is an essential technique in proving a wealth of one-shot achievability bounds since it effectively resembles a union bound in various problems. In this work, we show that the so-called pretty-good measurement naturally plays a role as the union bound as well. A judicious application of it considerably simplifies the derivation of one-shot achievability for classical-quantum channel coding via an elegant three-line proof. The proposed analysis enjoys the following favorable features. (i) The established one-shot bound admits a closed-form expression as in the celebrated Holevo-Helstrom theorem. Namely, the average error probability of sending M messages through a classical-quantum channel is upper bounded by the minimum error of distinguishing the joint channel input-output state against $(M-1)$ decoupled product states. (ii) Our bound directly yields asymptotic achievability results in the large deviation, small deviation, and moderate deviation regimes in a unified manner. (iii) The coefficients incurred in applying the Hayashi-Nagaoka operator inequality or the quantum union bound are no longer needed. Hence, the derived one-shot bound sharpens existing results relying on the Hayashi-Nagaoka operator inequality. In particular, we obtain the tightest achievable ε -one-shot capacity for classical communication over quantum channels heretofore, improving the third-order coding rate in the asymptotic scenario. (iv) Our result holds for infinite-dimensional Hilbert space. (v) The proposed method applies to deriving one-shot achievability for classical data compression with quantum side information, entanglement-assisted classical communication over quantum channels, and various quantum network information-processing protocols.

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