ECT 307 CONTROL SYSTEMS
UNIVERSITY QUESTION PAPER WITH SOLUTION

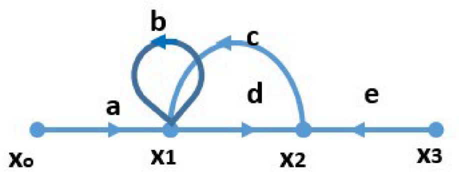
PART A

1. Distinguish between open loop and closed loop system.

Open loop	Closed loop
Inaccurate and unreliable	Accurate and reliable
Simple and economical	Complex and costlier
The changes in output due to external disturbances are not corrected automatically	The changes in output due to external disturbances are corrected automatically
They are generally stable	Great efforts are needed to design a stable system.

Text : Modern Control Engineering, Ogata – Page - 7

2. Draw the signal flow graph for the following set of algebraic equations:
$x_1=ax_0+bx_1+cx_2$, $x_2=dx_1+ex_3$



Text : Control Systems Engineering, Nagarath M Gopal – Page - 64

3. Calculate the rise time of a second order system having damping ratio 0.5 and natural frequency of oscillation 10 rad/sec.

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}}{\omega_n\sqrt{1-\xi^2}}$$

$$t_r = \frac{\pi - \tan^{-1}\frac{\sqrt{1-0.5^2}}{0.5}}{\omega_n\sqrt{1-0.5^2}}$$

Tr=0.214sec
Text : Modern Control Engineering, Ogata – Page – 156

4. Derive the expression for unit step response of a first order unity negative feedback system.

The closed loop transfer function of first order system, $\frac{C(s)}{R(s)} = \frac{1}{1+Ts}$

If the input is unit step then, $r(t) = 1$ and $R(s) = \frac{1}{s}$

$\therefore$ The response in s-domain, $C(s) = R(s)\frac{1}{(1+Ts)} = \frac{1}{s}.\frac{1}{(1+Ts)}$

$$= \frac{1}{sT(\frac{1}{T}+s)} = \frac{\frac{1}{T}}{s(s+\frac{1}{T})}$$

By partial fraction expansion,

$$C(s) = \frac{\frac{1}{T}}{s(s+\frac{1}{T})} = \frac{A}{s} + \frac{B}{(s+\frac{1}{T})}$$

A is obtained by multiplying C(s) by s and letting $s = 0$

$$A = C(s)s = \frac{\frac{1}{T}}{s(s+\frac{1}{T})}s = \left.\frac{\frac{1}{T}}{s+\frac{1}{T}}\right|_{s=0} = \frac{\frac{1}{T}}{\frac{1}{T}} = 1$$

B is obtained by multiplying C(s) by $(s+1/T)$ and letting $s = -1/T$,

$$B = C(s)\,(s+\frac{1}{T}) = \frac{\frac{1}{T}}{s(s+\frac{1}{T})}\,(s+\frac{1}{T}) = \left.\frac{\frac{1}{T}}{s}\right|_{s=-\frac{1}{T}} = \frac{\frac{1}{T}}{\frac{-1}{T}} = -1$$

$$\therefore C(s) = \frac{1}{s} - \frac{1}{s+\frac{1}{T}}$$

The response in time domain is given by,

$$c(t) = L^{-1}[C(s)] = L^{-1}\left[\frac{1}{s} - \frac{1}{s+\frac{1}{T}}\right] = 1 - e^{-\frac{t}{T}}$$

Text : Control Systems Engineering, Nagarath M Gopal – Page - 197

5. Distinguish between absolute stability and marginal stability? Indicate the pole locations in S-plane for absolute stability and marginal stability.

Pole Locations in the S-Plane

The stability of a linear time-invariant (LTI) control system is determined by the locations of its poles in the S-plane. The S-plane is a complex plane where the horizontal axis represents the real part (σ) and the vertical axis represents the imaginary part (jω) of the system's poles.

Absolute Stability Pole Locations

For a system to be absolutely stable, all of its poles must lie in the **left half of the S-plane**. This means that the real part (σ) of every pole must be negative ($\sigma<0$).

- **Real poles:** Poles on the negative real axis correspond to exponentially decaying responses.
- **Complex conjugate poles:** Pairs of complex conjugate poles in the left half of the S-plane correspond to exponentially decaying sinusoidal responses.

In both cases, the transient response of the system will eventually diminish to zero, and the system will reach a stable equilibrium.

Marginal Stability Pole Locations

For a system to be marginally stable, all of its poles must have a non-positive real part ($\sigma \le 0$), with at least one pair of simple (non-repeated) poles located on the **imaginary axis** of the S-plane.

- **Poles on the imaginary axis:** A simple pair of complex conjugate poles on the imaginary axis (at $s=\pm j\omega$) corresponds to a sustained, constant-amplitude sinusoidal oscillation in the system's response.
- **A pole at the origin:** A simple pole at the origin (at $s=0$) corresponds to a constant, non-decaying response.

In a marginally stable system, there are no poles in the right half of the S-plane ($\sigma > 0$), which would cause the output to grow unbounded. However, the presence of non-repeated poles on the imaginary axis means that the system's response will not die out, but will instead oscillate indefinitely or remain at a constant value.

Text : Control Systems Engineering, Nagarath M Gopal – Page – 275, Control Systems, Nagoor Kani -458

6. Define the angle and magnitude criteria on the open-loop transfer function of a system used for constructing root locus plot.

The angle criterion states that $s = s_a$ will be a point on root locus if for that value of s the argument or phase of G(s) H(s) is equal to an odd multiple of 180° [i.e., $\angle G(s)H(s) = \pm 180°(2q+1)$]

Let $G(s)\,H(s) = K\dfrac{(s+z_1)\,(s+z_2)\,(s+z_3)\,.....}{(s+p_1)\,(s+p_2)\,(s+p_3)\,.....}$

for $s = s_a$ be a point on root locus

$$\angle G(s)\,H(s) = \angle(s+z_1) + \angle(s+z_2) + \angle(s+z_3)\,.....-\angle(s+p_1) - \angle(s+p_2)....$$
$$= \pm 180°(2q+1)$$

$$\therefore \begin{pmatrix} \text{Sum of angles} \\ \text{of vectors from zeros} \\ \text{to the point } s = s_a \end{pmatrix} - \begin{pmatrix} \text{Sum of angles} \\ \text{of vectors from poles} \\ \text{to the point } s = s_a \end{pmatrix} = \pm 180°(2q+1)$$

The magnitude condition states that $s = s_a$, will be a point on root locus if for that value of s, magnitude of G(s) H(s) is equal to 1 (i.e. $|G(s)\,H(s)| = 1$)

$$\text{Let } G(s)\,H(s) = \frac{K\,(s+z_1)\,(s+z_2)\,(s+z_3)\ \ldots}{(s+p_1)\,(s+p_2)\,(s+p_3)\ \ldots}$$

$\therefore$ for $s = s_a$ be a point in root locus

$$|G(s)\,H(s)| = \frac{K\,|s_a+z_1|\,|s_a+z_2|\,|s_a+z_3|\ \ldots}{|s_a+p_1|\,|s_a+p_2|\,|s_a+p_3|\ \ldots} = 1$$

$$= K\,\frac{\text{Product of length of vectors from open loop zeros to the point } s_a}{\text{Product of length of vectors from open loop poles to the point } s_a} = 1$$

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7. Explain the need of compensators and list the different types of compensators.
A compensator is a component in the control system and it is used to regulate another system.
When a set of specifications are given for system then a suitable compensator should be designed so that overall system will meet the given specification.
In control system compensators are required,
*when system is unstable, then compensator is needed to stabilise the system.
*when system is stable, compensator is provided to obtain desired performance.
Types of compensator
Lead compensator
Lag compensator
Lag - lead compensator
Lead – lag compensator
Text : Control Systems Engineering, Nagarath M Gopal – Page – 435

8. Draw the s-plane contour used for mapping, for stability analysis, to the plane of open-loop transfer function.

$$G(s)\,H(s) = 2(s+1)/s(s-1)$$

Explain the choice of the contour.

The s-plane contour used for mapping, for stability analysis, to the plane of open-loop transfer function. $G(s)H(s) = \frac{2(s+1)}{s(s-1)}$
The contour is bounded by the semicircle with R → ∞ on the right half of GH plane. The remaining part of the contour is the imaginary axis with an infinitesimally small semicircle that excludes the pole at origin from the s-plane contour. This is by the condition to be met for the principle of argument used for the Nyquist criteria.
Text : Control Systems Engineering, Nagarath M Gopal – Page – 384

9. Obtain the state model of the system whose transfer function is given by

$$\frac{Y(s)}{U(s)} = \frac{10}{s^3+4s^2+2s+1}$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -1 & -2 & -4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 10\end{bmatrix}[u]$$

$$y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$$

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10. List the properties of state transition matrix.

1. Identity at Initial Time	When evaluated at the initial time t_0 (or $t = 0$ for LTI systems), the state transition matrix becomes the identity matrix (I). This signifies that the state remains unchanged at the starting point.	$\Phi(t_0, t_0) = I$ (or $\Phi(0) = I$ for LTI systems)
2. Semi-Group Property	The transition from t_0 to t_2 can be performed in steps: first from t_0 to an intermediate time t_1, then from t_1 to t_2. This allows for breaking down state evolution over composite intervals.	$\Phi(t_2, t_0) = \Phi(t_2, t_1)\Phi(t_1, t_0)$
3. Invertibility	The state transition matrix is always invertible. This implies that if you know the system's state at a certain time, you can uniquely determine its state at any previous or future time.	$\Phi^{-1}(t, t_0) = \Phi(t_0, t)$ (or $\Phi^{-1}(t) = \Phi(-t)$ for LTI systems)

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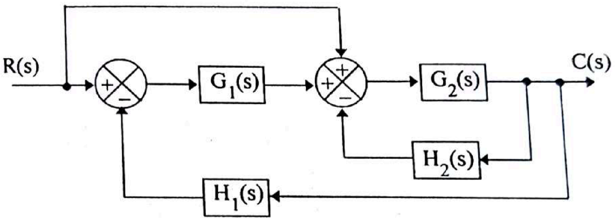
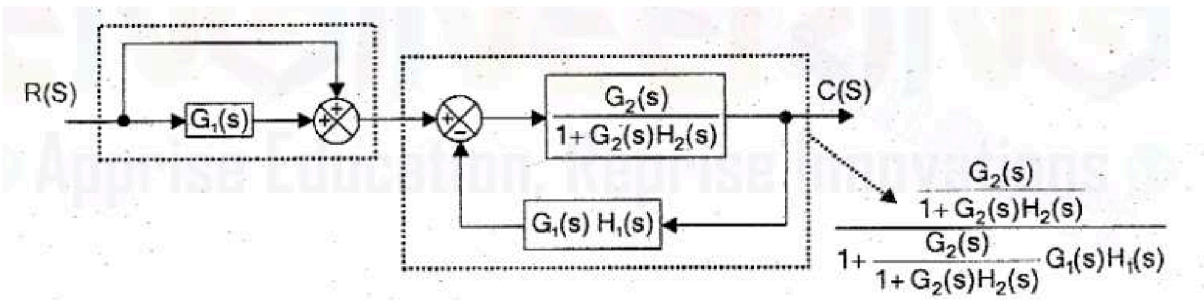
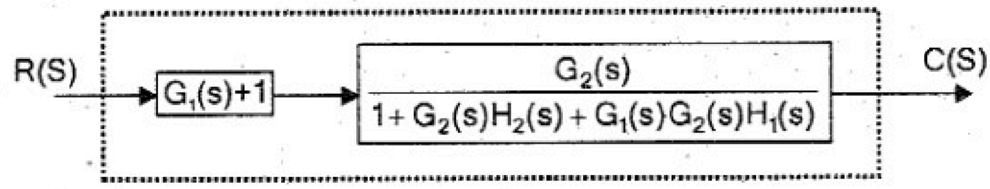
PART B

11. A. Find the transfer function of the following block diagram using block diagram reduction technique. Verify the same using SFG and mason's gain formula



Solution





$$\therefore \frac{C(s)}{R(s)} = \frac{G_2(s)\,[G_1(s)+1]}{1+G_2(s)\,H_2(s)+G_1(s)\,G_2(s)\,H_1(s)}$$

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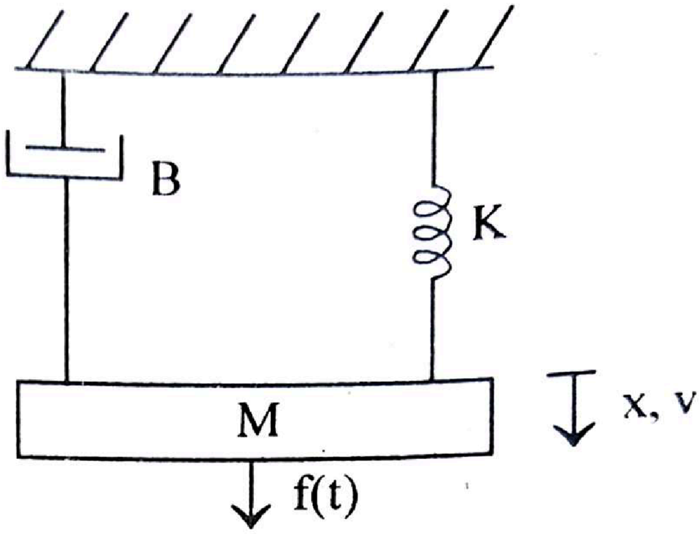
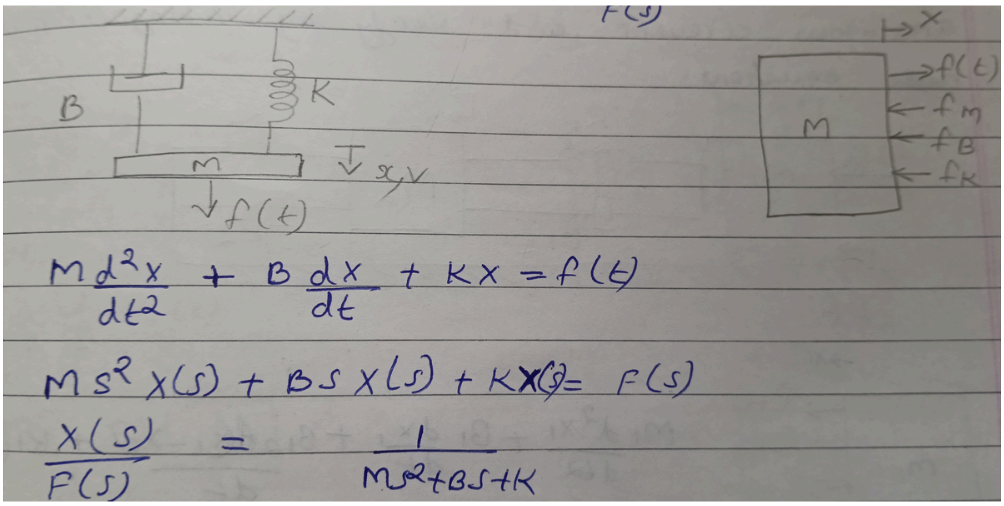
b. For the mechanical system shown in fig, derive the transfer function X(s)/F(s).





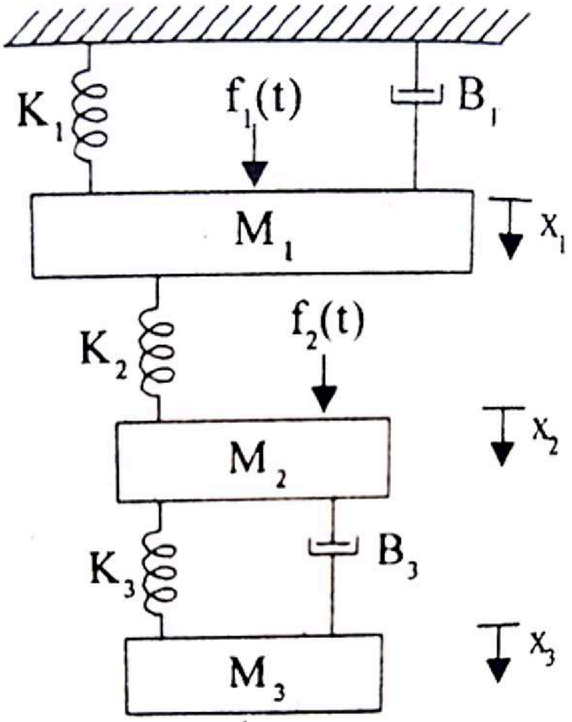
$$M\frac{d^2x}{dt^2} + B\frac{dx}{dt} + Kx = f(t)$$

$$Ms^2X(s) + BsX(s) + KX(s) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{Ms^2+Bs+K}$$

Text : Control Systems, Nagoor Kani -19

12. A. Find the differential equation governing the mechanical system shown in fig. Obtain the corresponding Force-Voltage analogous circuit.



The free body diagram of M_1 is shown in fig 1.9.2. The opposing forces are marked as f_{m1}, f_{b1}, f_{k2} and f_{k1}.

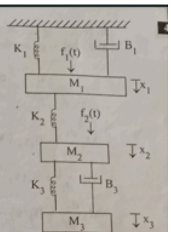


Fig 1.9.1

$$f_{m1} = M_1 \frac{d^2 x_1}{dt^2}$$

$$f_{b1} = B_1 \frac{dx_1}{dt}$$

$$f_{k2} = K_2 (x_1 - x_2)$$

$$f_{k1} = K_1 x_1$$

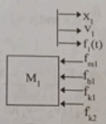


Fig 1.9.2

By Newton's second law, $f_{m1} + f_{b1} + f_{k2} + f_{k1} = f_1(t)$

$$M_1 \frac{d^2 x_1}{dt^2} + B_1 \frac{dx_1}{dt} + K_1 x_1 + K_2 (x_1 - x_2) = f_1(t) \qquad .....(1.9.1)$$

The free body diagram of M_2 is shown in fig 1.9.3. The opposing forces are marked as f_{m2}, f_{b3}, f_{k2}, f_{k3}.

$$f_{m2} = M_2 \frac{d^2 x_2}{dt^2} \quad ; \quad f_{b3} = B_3 \frac{d}{dt}(x_2 - x_3)$$

$$f_{k2} = K_2 (x_2 - x_1) \text{ and } f_{k3} = K_3 (x_2 - x_3)$$

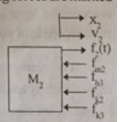


Fig 1.9.3

By Newton's second law, $f_{m2} + f_{b3} + f_{k2} + f_{k3} = f_2(t)$

$$M_2 \frac{d^2 x_2}{dt^2} + B_3 \frac{d}{dt}(x_2 - x_3) + K_2 (x_2 - x_1) + K_3 (x_2 - x_3) = f_2(t) \qquad .....(1.9.2)$$

The free body diagram of M_3 is shown in fig 1.9.4. The opposing forces are marked as f_{m3}, f_{b3}, f_{k3}.

$$f_{m3} = M_3 \frac{d^2 x_3}{dt^2} \; ; \; f_{b3} = B_3 \frac{d}{dt}(x_3 - x_2)$$

$$f_{k3} = K_3 (x_3 - x_2)$$

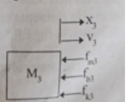


Fig 1.9.4

By Newton's second law, $f_{m3} + f_{b3} + f_{k3} = 0$

$$M_3 \frac{d^2 x_3}{dt^2} + B_3 \frac{d}{dt}(x_3 - x_2) + K_3 (x_3 - x_2) = 0 \qquad .....(1.9.3)$$

On replacing the displacements by velocity in the differential equations (1.9.1), (1.9.2) and (1.9.3) governing the mechanical system we get,

$$\left(\text{i.e.,} \quad \frac{d^2x}{dt^2} = \frac{dv}{dt} \; ; \quad \frac{dx}{dt} = v \text{ and } x = \int v\,dt \right)$$

$$M_1 \frac{dv_1}{dt} + B_1 v_1 + K_1 \int v_1 dt + K_2 \int (v_1 - v_2) dt = f_1(t) \qquad .....(1.9.4)$$

$$M_2 \frac{dv_2}{dt} + B_3 (v_2 - v_3) + K_2 \int (v_2 - v_1) dt + K_3 \int (v_2 - v_3) dt = f_2(t) \qquad .....(1.9.5)$$

$$M_3 \frac{dv_3}{dt} + B_3 (v_3 - v_2) + K_3 \int (v_3 - v_2) dt = 0 \qquad .....(1.9.6)$$

The electrical analogous elements for the elements of mechanical system are given below

$f_1(t) \rightarrow e_1(t)$	$v_1 \rightarrow i_1$	$M_1 \rightarrow L_1$	$B_1 \rightarrow R_1$	$K_1 \rightarrow 1/C_1$
$f_2(t) \rightarrow e_2(t)$	$v_2 \rightarrow i_2$	$M_2 \rightarrow L_2$	$B_3 \rightarrow R_3$	$K_2 \rightarrow 1/C_2$
	$v_3 \rightarrow i_3$	$M_3 \rightarrow L_3$		$K_3 \rightarrow 1/C_3$

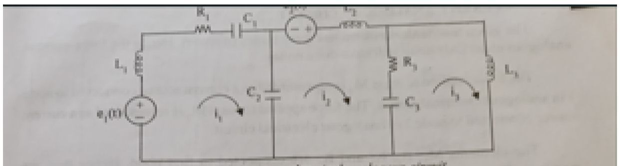


Fig 1.9.5 : Force-voltage electrical analogous circuit

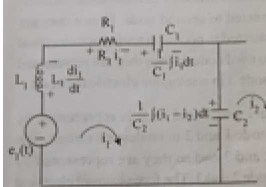


Fig 1.9.6

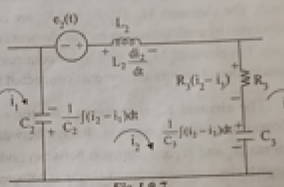


Fig 1.9.7

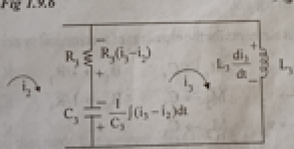


Fig 1.9.8

The mesh basis equations using Kirchoff's voltage law for the circuit shown in fig 1.9.5 are given below [Refer figures (1.9.6), (1.9.7) and (1.9.8).]

$$L_1 \frac{di_1}{dt} + R_1 i_1 + \frac{1}{C_1} \int i_1 dt + \frac{1}{C_2} \int (i_1 - i_2) dt = e_1(t) \qquad .....(1.9.7)$$

$$L_2 \frac{di_2}{dt} + R_3 (i_2 - i_3) + \frac{1}{C_3} \int (i_2 - i_3) dt + \frac{1}{C_2} \int (i_2 - i_1) dt = e_2(t) \qquad .....(1.9.8)$$

$$L_3 \frac{di_3}{dt} + R_3 (i_3 - i_2) + \frac{1}{C_3} \int (i_3 - i_2) dt = 0 \qquad .....(1.9.9)$$

It is observed that the mesh equations (1.9.7), (1.9.8) and (1.9.9) are similar to the differential equations (1.9.4), (1.9.5) and (1.9.6) governing the mechanical system.

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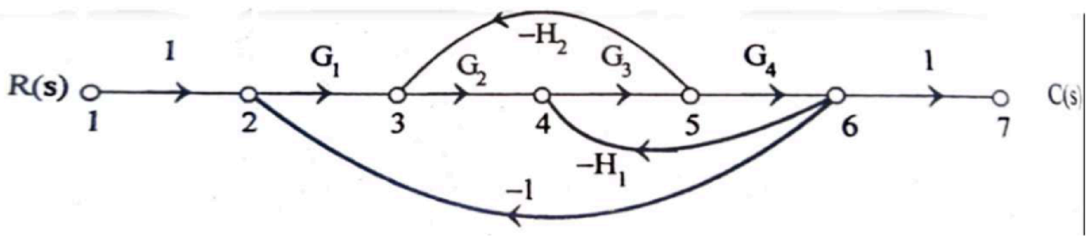
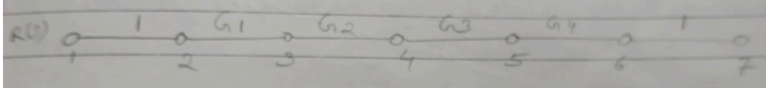
12.b. Find the overall gain C(s)/R(s) for the signal flow graph shown in fig.

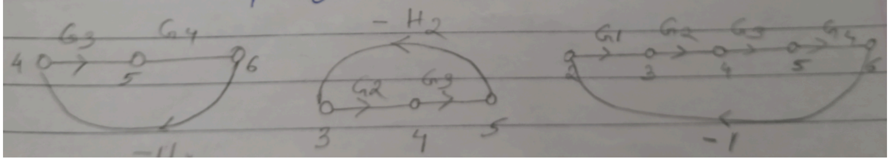


Forward path gains



$P_1 = G_1 G_2 G_3 G_4$

Individual loop gain



Loop gain of individual loop-1 , $P_{11} = -G_3G_4H_1$
" -2 , $P_{21} = -G_2G_3H_2$
-3 , $P_{31} = -G_1G_2G_3G_4$

Gain products of non touching loops

There are no possible combinations of two non touching loops, three non touching loops etc.

Calculation of Δ and Δ_K

$\Delta = 1 - (P_{11} + P_{21} + P_{31})$
$= 1 + G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4$

Since no part of the graph is non touching with forward path -1, $\Delta_1 = 1$

$T = \frac{C(S)}{R(S)} = \frac{1}{\Delta} \sum_K P_K \Delta_K$

$= \frac{1}{\Delta}(P_1 \Delta_1)$

$= \frac{G_1G_2G_3G_4}{1 + G_3G_4H_1 + G_2G_3H_2 + G_1G_2G_3G_4}$

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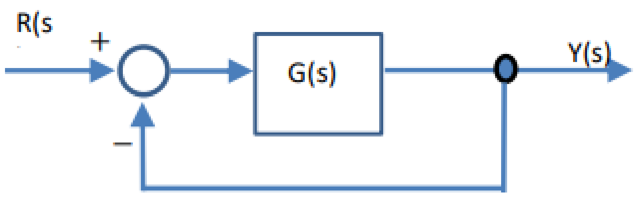
13. A. For the system in the block diagram, G(s) = 81/s(s+7.2). Find the output of the system for unit step input.



Transfer Function $\frac{C(s)}{R(s)} = \frac{81}{s^2+7.2s+81}$ – – – – –

Output

$c(t) = 1 - 1.0911e^{-3.6t}\sin(8.25t + 1.1593)$

Text : Control Systems Engineering, Nagarath M Gopal – Page – 197, Control Systems, Nagoor Kani - 284

B. Evaluate the static error constants of a negative unity feedback system whose open loop transfer function is(s) = 10/s(0.1s + 1) . Also find the steady state error when subjected to an input given by polynomial r(t) $=a_0 + a_1t + a_2t^2$

To find static error constant

For unity feedback system, $H(s) = 1$.

$\therefore$ Loop transfer function, $G(s)H(s) = G(s)$

The static error constants are K_p, K_v and K_a.

$$\text{Position error constant, } K_p = \underset{s\to 0}{Lt}\ G(s) = \underset{s\to 0}{Lt} \frac{10}{s(0.1s+1)} = \infty$$

$$\text{Velocity error constant, } K_v = \underset{s\to 0}{Lt}\ sG(s) = \underset{s\to 0}{Lt}\ s\frac{10}{s(0.1s+1)} = 10$$

$$\text{Acceleration error constant, } K_a = \underset{s\to 0}{Lt}\ s^2G(s) = \underset{s\to 0}{Lt}\ s^2\frac{10}{s(0.1s+1)} = 0$$

The error signal in s - domain, $E(s) = \dfrac{R(s)}{1+G(s)H(s)}$

Given that, $r(t) = a_o + a_1 t + \dfrac{a_2}{2}t^2$; $\quad G(s) = \dfrac{10}{s(0.1s+1)}$; $\quad H(s) = 1$

On taking Laplace transform of r(t) we get R(s),

$$\therefore R(s) = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{2}\frac{2!}{s^3} = \frac{a_0}{s} + \frac{a_1}{s^2} + \frac{a_2}{s^3}$$

$$\therefore E(s) = \frac{R(s)}{1+G(s)H(s)} = \frac{\dfrac{a_o}{s} + \dfrac{a_1}{s^2} + \dfrac{a_2}{s^3}}{1 + \dfrac{10}{s(0.1s+1)}} = \frac{\dfrac{a_o}{s} + \dfrac{a_1}{s^2} + \dfrac{a_2}{s^3}}{\dfrac{s(0.1s+1)+10}{s(0.1s+1)}}$$

$$= \frac{a_0}{s}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right] + \frac{a_1}{s^2}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right] + \frac{a_2}{s^3}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right]$$

The steady state error e_{ss} can be obtained from final value theorem.

$$\text{Steady state error, } e_{ss} = \underset{t\to\infty}{Lt}\ e(t) = \underset{s\to 0}{Lt}\ sE(s)$$

$$\therefore e_{ss} = \underset{s\to 0}{Lt}\ s\left\{\frac{a_o}{s}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right] + \frac{a_1}{s^2}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right] + \frac{a_2}{s^3}\left[\frac{s(0.1s+1)}{s(0.1s+1)+10}\right]\right\}$$

$$= \underset{s\to 0}{Lt}\left\{\frac{a_o s(0.1s+1)}{s(0.1s+1)+10} + \frac{a_1(0.1s+1)}{s(0.1s+1)+10} + \frac{a_2(0.1s+1)}{s[s(0.1s+1)+10]}\right\} = 0 + \frac{a_1}{10} + \infty = \infty$$

Text : Control Systems Engineering, Nagarath M Gopal – Page – 321, Control Systems, Nagoor Kani -458

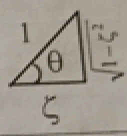
14. Derive the expression for peak time of a second order underdamped system with negative feedback when subjected to unit step input.

Peak time (t_p)

To find the expression for peak time, t_p, differentiate c(t) with respect to t and equate to 0.

i.e., $\left.\dfrac{d}{dt}c(t)\right|_{t=t_p} = 0$

The unit step response of second order system is given by

> ***Note :** On constructing right angle triangle with ζ and $\sqrt{1-\zeta^2}$, we get*
>
> $\sin\theta = \sqrt{1-\zeta^2}$
>
> $\cos\theta = \zeta$
>
> 
>

$$c(t) = 1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \theta)$$

Differentiating c(t) with respect to **t**

$$\frac{d}{dt}c(t) = \frac{-e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}(-\zeta\omega_n)\sin(\omega_d t+\theta) + \left(\frac{-e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\right)\cos(\omega_d t+\theta)\omega_d$$

Put, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\therefore \frac{d}{dt}c(t) = \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}(\zeta\omega_n)\sin(\omega_d t+\theta) - \frac{\omega_n\sqrt{1-\zeta^2}}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\cos(\omega_d t+$$

$$= \frac{\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\left[\zeta\sin(\omega_d t+\theta) - \sqrt{1-\zeta^2}\cos(\omega_d t+\theta)\right]$$

$$= \frac{\omega_n}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}\left[\cos\theta\sin(\omega_d t+\theta) - \sin\theta\cos(\omega_d t+\theta)\right]$$

(refer note)

$$= \frac{\omega_n}{\sqrt{1-\zeta^2}}\ e^{-\zeta\omega_n t}\left[\sin(\omega_d t+\theta)\cos\theta - \cos(\omega_d t+\theta)\sin\theta\right]$$

$$= \frac{\omega_n}{\sqrt{1-\zeta^2}}\ e^{-\zeta\omega_n t}\left[\sin((\omega_d t+\theta)-\theta)\right]$$

$$= \frac{\omega_n}{\sqrt{1-\zeta^2}}\ e^{-\zeta\omega_n t}\ \sin(\omega_d t)$$

at $t = t_p$, $\frac{d}{dt}c(t) = 0$

$$\therefore \frac{d}{dt}c(t)\Big|_{t=t_p} = \frac{\omega_n}{\sqrt{1-\zeta^2}}\ e^{-\zeta\omega_n t_p}\ \sin(\omega_d t_p) = 0$$

$$\therefore \frac{\omega_n}{\sqrt{1-\zeta^2}}\ e^{-\zeta\omega_n t_p}\ \sin(\omega_d t_p) = 0$$

Since, $e^{-\zeta\omega_n t_p} \neq 0$, the term, $\sin(\omega_d t_p) = 0$

$\sin\phi = 0$, when $\phi = 0, \pi, 2\pi, 3\pi$

$\therefore \omega_d t_p = \pi$

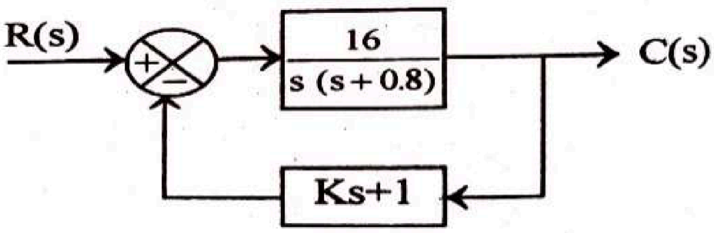
$$\therefore \text{Peak time, } t_p = \frac{\pi}{\omega_d}$$

The damped frequency of oscillation, $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$$\therefore \text{Peak time, } t_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

Text : Control Systems Engineering, Nagarath M Gopal – Page – 277, Control Systems, Nagoor Kani -1111111

B. Determine the unit step response c(t) for a positional control system with velocity feedback as shown in the Fig. Given that ζ = 0.5. Also calculate the rise time, peak time, maximum overshoot and settling time.



$$\therefore \frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{16}{s^2 + (0.8 + 16K)\,s + 16}$$

On comparing we get.

$\omega_n^2 = 16$	$0.8 + 16K = 2\zeta\omega_n$
$\therefore\ \omega_n = 4$ rad / sec	$\therefore\ K = \frac{2\zeta\omega_n - 0.8}{16} = \frac{2\times 0.5\times 4 - 0.8}{16} = 0.2$

$$\therefore \frac{C(s)}{R(s)} = \frac{16}{s^2 + (0.8 + 16\times 0.2)\,s + 16} = \frac{16}{s^2 + 4s + 16}$$

Transfer function

$$\therefore\ C(s)=\frac{1}{s}+\frac{-s-4}{s^2+4s+16}=\frac{1}{s}-\frac{s+4}{s^2+4s+4+12}$$

$$=\frac{1}{s}-\frac{s+2+2}{(s+2)^2+12}=\frac{1}{s}-\frac{s+2}{(s+2)^2+12}-\frac{2}{\sqrt{12}}\ \frac{\sqrt{12}}{(s+2)^2+12}$$

Partial fraction output in S domain

$$c(t)=\mathcal{L}^{-1}\{C(s)\}=\mathcal{L}^{-1}\left\{\frac{1}{s}-\frac{s+2}{(s+2)^2+12}-\frac{2}{\sqrt{12}}\ \frac{\sqrt{12}}{(s+2)^2+12}\right\}$$

$$=1-e^{-2t}\cos\sqrt{12}\,t-\frac{2}{2\sqrt{3}}\,e^{-2t}\sin\sqrt{12}\,t$$

$$=1-e^{-2t}\left[\frac{1}{\sqrt{3}}\sin(\sqrt{12}\,t)+\cos(\sqrt{12}\,t)\right]$$

Text : Control Systems Engineering, Nagarath M Gopal – Page – 252, Control Systems, Nagoor Kani -292

15. A. Sketch the root locus of the system whose open loop transfer function is G(S) = K/(s(s+2)(s+4))

Find the value of K so that the damping ratio of the closed loop system is 0.5.

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equation, $s(s+2)(s+4)=0$.

$\therefore$ The poles are, $s = 0, -2, -4$.

The poles are marked by X(cross) as shown in fig 5.23.1.

Step 2 : To find the root locus on real axis

There are three poles on the real axis. Choose a test point on real axis between $s=0$ and $s=-2$. To the right of this point the total number of real poles and zeros is one, which is an odd number. Hence the real axis between $s=0$ and $s=-2$ will be a part of root locus.

Choose a test point on real axis between s = –2 and s = –4. To the right of this point, the total number of real poles and zeros is two which is an even number. Hence the real axis between s = –2 and s = –4 will not be a part of root locus.

Choose a test point on real axis to the left of s = –4. To the right of this point, the total number of real poles and zeros is three, which is an odd number. Hence the entire negative real axis from s = –4 to –∞ will be a part of root locus.

Step 3 : To find asymptotes and centroid

Since there are three poles the number of root locus branches are three. There is no finite zero. Hence all the three root locus branches ends at zeros at infinity. The number of asymptotes required are three.

$$\text{Angles of asymptotes} = \frac{\pm 180\,(2q+1)}{n-m}$$

Here n = 3 and m = 0.

$$\text{If } q=0,\ \text{Angles} = \pm\frac{180}{3} = \pm 60^\circ$$

$$\text{If } q=1,\ \text{Angles} = \pm\frac{180\times 3}{3} = \pm 180^\circ$$

$$\text{If } q=2,\ \text{Angles} = \pm\frac{180\times 5}{3} = \pm 300^\circ = \mp 60^\circ$$

Note : It is enough if you calculate the required number of angles. Here it is given by first three values of angles. The remaining values will be repetitions of the previous values.

$$\text{Centroid} = \frac{\text{Sum of poles} - \text{Sum of zeros}}{n-m}$$

$$= \frac{0-2-4-0}{3} = -2$$

The centroid is marked on real axis and from the centroid the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown in fig 5.23.1.

Step 4 : To find the breakaway and breakin points

$$\left.\begin{array}{l}\text{The closed loop}\\ \text{transfer function}\end{array}\right\} \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)}$$

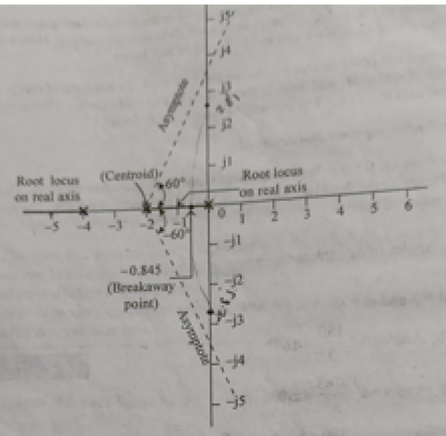


Fig 5.23.1: Figure showing the asymptotes, root locus on real axis and location of poles, centroid and breakaway points.

$$\therefore \frac{C(s)}{R(s)} = \frac{\dfrac{K}{s\,(s+2)\,(s+4)}}{1+\dfrac{K}{s\,(s+2)\,(s+4)}} = \frac{K}{s\,(s+2)\,(s+4) + K}$$

The characteristic equation is given by

$$s\,(s+2)\,(s+4) + K = 0$$

$$s\,(s^2+6s+8) + K = 0$$

$$s^3 + 6s^2 + 8s + K = 0 \qquad \therefore K = -s^3 - 6s^2 - 8s$$

On differentiating the equation of K with respect to s we get,

$$\frac{dK}{ds} = -(3s^2 + 12s + 8)$$

Put $\frac{dK}{ds} = 0$

$$\therefore -(3s^2 + 12s + 8) = 0$$

$$(3s^2 + 12s + 8) = 0$$

$$s = \frac{-12 \pm \sqrt{12^2 - 4 \times 3 \times 8}}{2 \times 3} = -0.845 \quad \text{or} \quad -3.154$$

Check for K

When $s = -0.845$, the value of K is given by

$$K = -[(-0.845)^3 + 6(-0.845)^2 + 8(-0.845)] = 3.08$$

Since K is positive and real for, $s = -0.845$, this point is actual breakaway point.

When $s = -3.154$, the value of K is given by

$$K = -[(-3.154)^3 + 6(-3.154)^2 + 8(-3.154)] = -3.08$$

Since K is negative for, $s = -3.154$, this is not a actual breakaway point. The breakaway point is marked on the negative real axis as shown in fig 5.23.1.

Step5 : To find angle of departure

Since there are no complex pole or zero, we need not find angle of departure or arrival.

Step 6 : To find the crossing point of imaginary axis

The characteristic equation is given by

$$s^3 + 6s^2 + 8s + K = 0$$

Put $s = j\omega$

$$(j\omega)^3 + 6(j\omega)^2 + 8(j\omega) + K = 0$$

$$-j\omega^3 - 6\omega^2 + j8\omega + K = 0$$

Equating imaginary part to zero

$$-j\omega^3 + j8\omega = 0$$

$$-j\omega^3 = -j8\omega$$

$$\omega^2 = 8$$

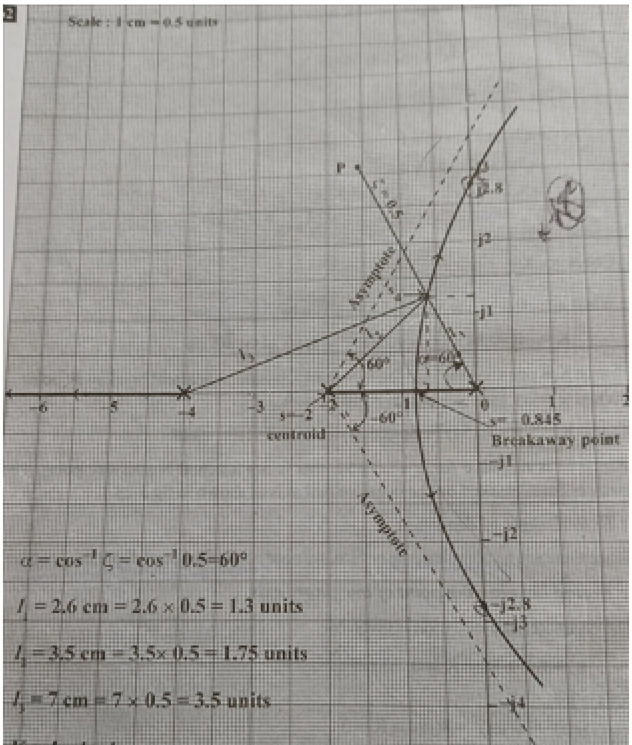
$$\omega = \pm\sqrt{8} = \pm 2.8$$

Equating real part to zero

$$-6\omega^2 + K = 0$$

$$K = 6\omega^2 = 6 \times 8 = 48$$

Scale : 1 cm = 0.5 units



$\alpha = \cos^{-1}\zeta = \cos^{-1}0.5 = 60°$

$l_1 = 2.6\text{ cm} = 2.6 \times 0.5 = 1.3\text{ units}$

$l_1 = 3.5\text{ cm} = 3.5 \times 0.5 = 1.75\text{ units}$

$l_3 = 7\text{ cm} = 7 \times 0.5 = 3.5\text{ units}$

$K = l_1 \cdot l_2 \cdot l_3$

$= 1.3 \times 1.75 \times 3.5 = 7.96 \approx 8$

The crossing point of root locus is ± j2.8. The value of K corresponding to this point is K = 48. (This is the limiting value of K for the stability of the system).

The complete root locus sketch is shown in fig 5.23.2. The root locus has three branches. One branch starts at the pole at s = – 4 and travel through negative real axis to meet the zero at infinity. The other two root locus branches starts at s = 0 and s = – 2 and travel through negative real axis, breakaway from real axis at s = –0.845, then crosses imaginary axis at s= ± j2.8 and travel parallel to asymptotes to meet the zeros at infinity

TO FIND THE VALUE OF K CORRESPONDING TO $\zeta = 0.5$

Given that $\zeta = 0.5$

Let $\alpha = \cos^{-1}\zeta = \cos^{-1} 0.5 = 60°$

Draw a line OP, such that the angle between line OP and negative real axis is 60° ($\alpha = 60°$) as shown in fig 5.23.2. The meeting point of the line OP and root locus gives the dominant pole, s_d.

$$\left.\begin{array}{c}\text{The value of K corresponding}\\ \text{to the point, } s = s_d\end{array}\right\} = \frac{\text{Product of length of vector from all poles to the point, } s = s_d}{\text{Product of length of vectors from all zeros to the point, } s = s_d}$$

$$= \frac{l_1.l_2.l_3}{1} = 1.3 \times 1.75 \times 3.5 = 7.96 \approx 8$$

Text : Control Systems Engineering, Nagarath M Gopal – Page – 323, Control Systems, Nagoor Kani -558

B. Compare P,PI and PID controllers.

Feature / Controller	Proportional (P)	Proportional-Integral (PI)	Proportional-Integral-Derivative (PID)
Control Action	Proportional to current error ($K_p \cdot e(t)$)	Proportional + Integral ($K_p e(t) + K_i \int e(t)dt$)	Proportional + Integral + Derivative ($K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt}$)
Steady-State Error	Reduced, but not eliminated (offset exists)	Eliminated (zero offset)	Eliminated (zero offset)
Rise Time	Decreased	Can be increased (slower response)	Decreased (faster response)
Overshoot	Increased with high K_p	Can be significant, especially if K_i is high	Reduced significantly

Text : Control Systems Engineering, Nagarath M Gopal – Page – 220

16. A. Sketch the root locus for a unity feedback control system that has an open loop transfer function G(s) = K/(s(s^2+4s+13)

Step 1 : To locate poles and zeros

The poles of open loop transfer function are the roots of the equation, $s(s^2+4s+13)=0$.

The roots of the quadratic are, $s=\dfrac{-4\pm\sqrt{4^2-4\times 13}}{2}=-2\pm j3$

$\therefore$ The poles are 0, $-2+j3$ and $-2-j3$.

The poles are marked by X (cross) as shown in fig 5.22.1

Step 2 : To find the root locus on real axis

There is only one pole on real axis at the origin. Hence if we choose any test point on the negative real axis then to the right of that point the total number of real poles and zeros is one, which is an odd number. Hence the entire negative real axis will be part of root locus. The root locus on real axis is shown as a bold line in fig 5.22.1.

Note : For the given transfer function one root locus branch will start at the pole at the origin and travel through the negative real axis to meet the zero at infinity.

Step 3 : To find angles of asymptotes and centroid

Since there are 3 poles, the number of root locus branches are three. There is no finite zero. Hence all the three root locus branches ends at zeros at infinity. The number of asymptotes required are three.

$$\text{Angles of asymptotes}=\frac{\pm 180^\circ(2q+1)}{n-m}$$

Here $n=3$, and $m=0$.

$$\text{If } q=0,\ \text{Angles}=\pm\frac{180^\circ}{3}=\pm 60^\circ$$

$$\text{If } q=1,\ \text{Angles}=\pm\frac{180^\circ\times 3}{3}=\pm 180^\circ$$

$$\text{If } q=2,\ \text{Angles}=\pm\frac{180^\circ\times 5}{3}=\pm 300^\circ=\mp 60^\circ$$

$$\text{If } q=3,\ \text{Angles}=\pm\frac{180^\circ\times 7}{3}=\pm 420^\circ=\pm 60^\circ$$

Note : It is enough if you calculate the required number of angles. Here it is given by first three values of angles. The remaining values will be repetitions of the previous values.

$$\text{Centroid}=\frac{\text{Sum of poles}-\text{Sum of zeros}}{n-m}$$

$$=\frac{0-2+j3-2-j3-0}{3}=\frac{-4}{3}=-1.33$$

The centroid is marked on real axis and from the centroid the angles of asymptotes are marked using a protractor. The asymptotes are drawn as dotted lines as shown in fig 5.22.1.

Step 4 : To find the breakaway and breakin points

$$\left.\begin{array}{r}\text{The closed loop}\\ \text{transfer function}\end{array}\right\}\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)}$$

$$\therefore \frac{C(s)}{R(s)}=\frac{\dfrac{K}{s(s^2+4s+13)}}{1+\dfrac{K}{s(s^2+4s+13)}}=\frac{K}{s(s^2+4s+13)+K}$$

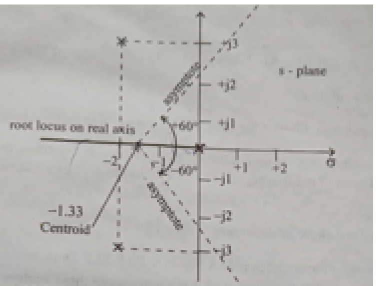


Fig : 5.22.1 : Figure showing the asymptotes, root locus on real axis and location of poles and centroid

The characteristic equation is $s(s^2+4s+13)+K=0$

$$s^3+4s^2+13s+K=0$$

$$\therefore K=-s^3-4s^2-13s$$

On differentiating the equation of K with respect to s we get.

$$\frac{dK}{ds}=-(3s^2+8s+13)$$

$$\text{Put } \frac{dK}{ds}=0$$

$$\therefore -(3s^2+8s+13)=0$$

$$(3s^2+8s+13)=0$$

$$\therefore s=\frac{-8\pm\sqrt{8^2-4\times13\times3}}{2\times3}$$

$$=-1.33\pm j1.6$$

Check for K

When $s=(-1.33+j1.6)$, the value of K is given by

$$K=-(s^3+4s^2+13s)$$

$$=-[(-1.33+j1.6)^3+4(-1.33+j1.6)^2+13(-1.33+j1.6)]$$

$$\neq \text{positive and real.}$$

Also when $s=-1.33-j1.6$ the value of K is not equal to real and positive.

Since the values of K for $s=(-1.33\pm j1.6)$ are not real and positive, the points are not an actual breakaway or breakin points. The root locus has neither breakaway nor breakin point.

Step 5 : To find the angle of departure

Let us consider the complex pole A shown in fig 5.22.2. Draw vectors from all other poles to the pole A as shown in fig 5.22.2. Let the angles of these vectors be θ_1 and θ_2.

Here, $\theta_1=180°-\tan^{-1}(3/2)=123.7°$

$\theta_2=90°$

Angle of departure from the complex pole A $=180°-(\theta_1+\theta_2)$

$=180°-(123.7°+90°)=-33.7°$

The angle of departure at complex pole A* is negative of the angle of departure at complex pole A.

$\therefore$ Angle of departure at pole A* $=+33.7°$

Mark the angles of departure at complex poles using protractor.

Step 6 : To find the crossing point on imaginary axis

The characteristic equation is given by

$$s^3+4s^2+13s+K=0$$

$$\text{Put } s=j\omega$$

$$(j\omega)^3+4(j\omega)^2+13(j\omega)+K=0$$

$$-j\omega^3-4\omega^2+13j\omega+K=0$$

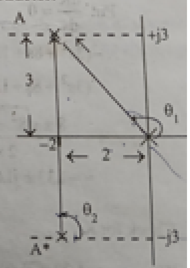
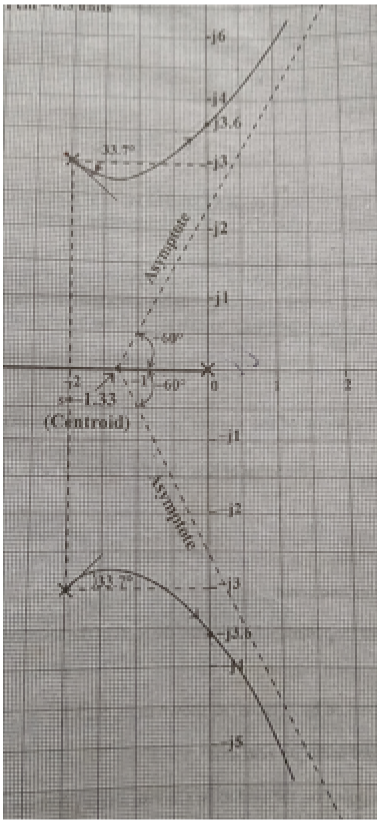


Fig 5.22.2.



On equating imaginary part to zero, we get	On equating real part to zero, we get
$-\omega^3 + 13\omega = 0$	$-4\omega^2 + K = 0$
$-\omega^3 = -13\omega$	$K = 4\omega^2$
$\omega^2 = 13$	$= 4(13) = 52.$
$\omega = \pm\sqrt{13}$	
$= \pm 3.6$	

The crossing point of root locus is $\pm j3.6$. The value of K at this crossing point is $K = 52$. (This is the limiting value of K for the stability of the system).

The complete root locus sketch is shown in fig 5.22.3. The root locus has three branches one branch starts at the pole at origin and travel through negative real axis to meet the zero at infinity. The other two root locus branches starts at complex poles (along the angle of departure), crosses the imaginary axis at $\pm j3.6$ and travel parallel to asymptotes to meet the zeros at infinity.

Text : Control Systems Engineering, Nagarath M Gopal – Page – 553, Control Systems, Nagoor Kani -458

b. Apply routh-hurwitz criterion to the characteristic equation $s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10 = 0$ and investigate the stability of the system.

Routh table

Comment on Stability Analysis (Marginally Stable)

Text : Refer problem in Control Systems Engineering, Nagarath M Gopal – Page – 280, Refer problem in Control Systems, Nagoor Kani -471

17. A. Design a phase lag compensator for the unity feedback control system given by $Gs) = K/(s + 4)(s + 80)$. It is desired to have a phase margin to be at least 33^0 and the velocity error constant $K_v = 30$ per sec.

Calculation of gain K= 9600

Magnitude Table for Bode Plot

Phase Table for Bode Plot

Construction of Bode Plot

Finding PM from Bode plot = 12^0

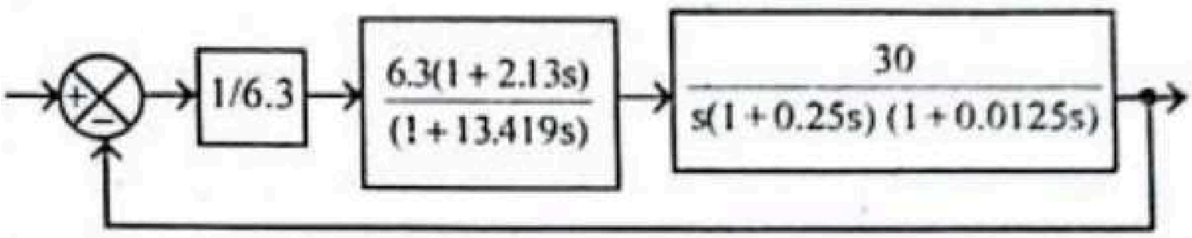
Requires PM=33^0

Calculation of new Gain cross over frequency = 4.7 rad/sec for new PM

Determine the value of β = 6.3

Determine the value of T = 2.13

Block diagram of Compensated system along with transfer function



$$G_o(s) = \frac{1}{6.3} \times \frac{6.3\,(1+2.13s)}{(1+13.419s)} \times \frac{30}{s(1+0.25s)\,(1+0.0125s)}$$

$$= \frac{30\,(1+2.13s)}{s\,(1+13.419s)\,(1+0.25s)\,(1+0.0125s)}$$

Text : Refer problem in Control Systems Engineering, Nagarath M Gopal – Page – 420

B. Explain the effect of introduction of a phase lead network to an existing system

Improve the transient response

Increase the system's stability margin.

Text : Control Systems Engineering, Nagarath M Gopal – Page – 435

18. A. Sketch the bode plot for the following open loop transfer function and determine the system gain K for a gain cross over frequency to be 5 rad/sec.

$$G(s) = \frac{Ks^2}{(1 + 0.2s)(1 + 0.02s)}$$

The sinusoidal transfer function G(jω) is obtained by replacing s by jω in the given s-domain transfer function

$$G(j\omega) = \frac{K(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$$

Let K=1, $G(j\omega) = \frac{(j\omega)^2}{(1+0.2j\omega)(1+0.02j\omega)}$

Magnitude plot

The corner frequencies are

ω_{c1} = 1/0.2 = 5rad/sec and ω_{c2} = 1/0.02 = 50 rad/sec

the various terms of G(jω) are listed in the increasing order of their corner frequency.

Term	Corner frequency rad/sec	Slope db/dec	Change in slope db/dec
$\frac{1}{1+(j\omega)^2}$	-	+40	40
$\frac{1}{1+j0.2\omega}$	ω_{c1} = 1/0.2 = 5	-20	40-20=20
$\frac{1}{1+j0.02\omega}$	ω_{c2} = 1/0.02 = 50	-20	20-20=0

Choose a low frequency ω_l such that $\omega_l < \omega_{c1}$ and choose a high frequency ω_h such that $\omega_h > \omega_{c2}$

Let ω_l=0.5 rad/sec and ω_h=100 rad/sec

Let $A = |G(j\omega)|$ in db

Let us calculate A at $\omega_l, \omega_{c1}, \omega_{c2}$ and ω_h.

$$\text{At } \omega = \omega_l,\quad A = 20\log|(j\omega)^2| = 20\log(\omega)^2 = 20\log(0.5)^2 = -12 \text{ db}$$

$$\text{At } \omega = \omega_{c1},\quad A = 20\log|(j\omega)^2| = 20\log(\omega)^2 = 20\log(5)^2 = 28 \text{ db}$$

$$\text{At } \omega = \omega_{c2},\quad A = \left[\text{slope from } \omega_{c1} \text{ to } \omega_{c2} \times \log\frac{\omega_{c2}}{\omega_{c1}}\right] + A_{(at\ \omega=\omega_{c1})}$$

$$= 20 \times \log\frac{50}{5} + 28 = 48 \text{ db}$$

$$\text{At } \omega = \omega_h,\quad A = \left[\text{slope from } \omega_{c2} \text{ to } \omega_h \times \log\frac{\omega_h}{\omega_{c2}}\right] + A_{(at\ \omega=\omega_{c2})}$$

$$= 0 \times \log\frac{100}{50} + 48 = 48 \text{ db}$$

Let the points a, b, c and d be the points corresponding to frequencies ω_l, ω_{c1}, ω_{c2} and ω_h respectively on the magnitude plot. In a semilog graph sheet choose a scale of 1unit = 10db on y-axis. The frequencies are marked in decades from 0.1 to 100 rad/sec on logarithmic scales in x-axis. Fix the points a, b, c and d on the graph. Join the points by straight lines and mark the slope on the respective region.

PHASE PLOT

The phase angle of G(jω) as a function of ω is given by

$$\phi = \angle G(j\omega) = 180^\circ - \tan^{-1}0.2\omega - \tan^{-1}0.02\omega$$

The phase angle of G(jω)are calculated for various values of ω and listed in table

 TABLE2

ω rad/sec	$\tan^{-1} 0.2\omega$ deg	$\tan^{-1} 0.02\omega$ deg	$\phi = \angle G(j\omega)$ deg
0.5	5.7	0.6	173.7 ≈ 174
1	11.3	1.1	167.6 ≈ 168
5	45	5.7	129.3 ≈ 130
10	63.4	11.3	105.3 ≈ 106
50	84.3	45	50.7 ≈ 50
100	87.1	63.4	29.5 ≈ 30

On the same semilog sheet choose a scale of 1unit = 20°, on the y-axis on the right side of semilog sheet. Mark the calculated phase angle on the graph sheet. Join the points by a smooth curve.

CALCULATION OF K

Given that the gain crossover frequency is 5 rad/sec. At ω = 5 rad/sec the gain is 28 db. If gain crossover frequency is 5 rad/sec then at that frequency the db gain should be zero. Hence to every point of magnitude plot a db gain of −28db should be added. The addition of −28db shifts the plot downwards. The corrected magnitude plot is obtained by shifting the plot with K = 1 by 28db downwards. The magnitude correction is independent of frequency. Hence the magnitude of −28db is contributed by the term K. The value of K is calculated by equating 20 logK to −28 db.

$$\therefore 20 \log K = -28 \text{ db}$$

$$\log K = \frac{-28}{20} \qquad \therefore K = 10^{-\frac{28}{20}} = 0.0398$$

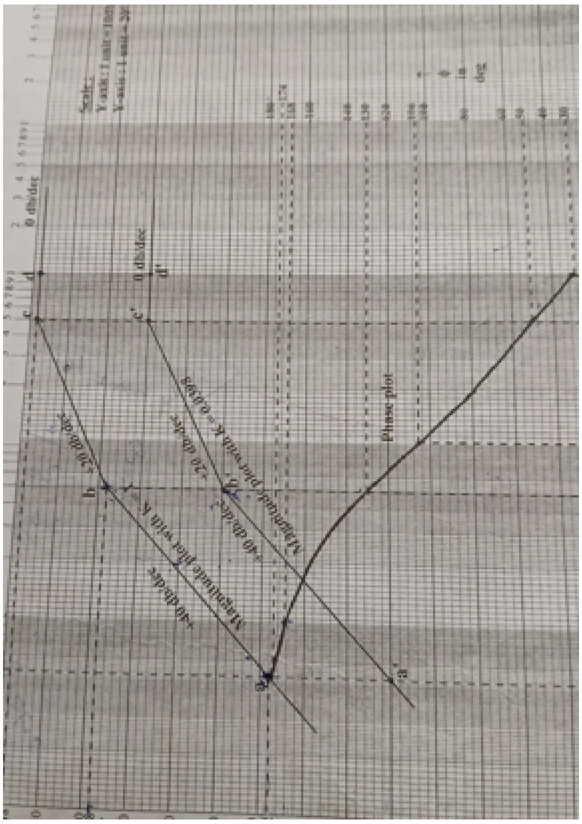
The magnitude plot with K = 1 and 0.0398 and the phase plot are shown in fig 4.1.1.

NOTE

The frequency ω = 5 rad/sec is a corner frequency. Hence in the exact plot the db gain at ω = 5 rad/sec will be 3db less than the approximate plot. Therefore for exact plot the 20logK will contribute a gain of −25db

$$\therefore 20 \log K = -25 \text{ db}$$

$$\log K = \frac{-25}{20} \qquad \therefore K = 10^{\frac{-25}{20}} = 0.0562.$$



Text : Refer problem in Text : Modern Control Engineering, Ogata – Page - 484, Control Systems, Nagoor Kani -370

B. Explain Nyquist stability criterion.
If G(s)H(s) contour in the G(s)H(s) plane corresponding to Nyquist contour in s-plane encircles the point -1+j0 in the anti-clockwise direction as many times as the number of right half s- plane poles of G(s)H(s). Then the closed loop stable.
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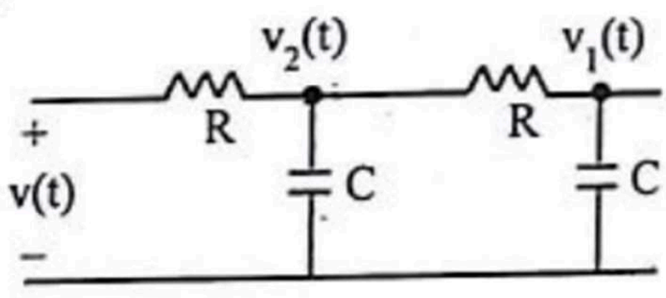
19. A. Obtain the state model of the electrical network shown in the fig. by choosing $V_1(t)$ and $V_2(t)$ as state variables.



Applying Krichoff's current law in node 1 and 2 and replacing $v_1(t)$ by x_1 and $v_2(t)$ by x_2 and input v(t) by u, we get

$$\frac{x_1 - x_2}{R} + C\frac{dx_1}{dt} = 0$$

$$\frac{x_2 - x_1}{R} + \frac{x_2}{R} + C\frac{dx_2}{dt} = \frac{u}{R}$$

Find the state equations of the system

$$\dot{x}_1 = -\frac{1}{RC}x_1 + \frac{1}{RC}x_2$$

$$\dot{x}_2 = \frac{1}{RC}x_1 - \frac{2}{RC}x_2 + \frac{1}{RC}u$$

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} -\frac{1}{RC} & \frac{1}{RC} \\ \frac{1}{RC} & \frac{-2}{RC}\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ \frac{1}{RC}\end{bmatrix}\begin{bmatrix}u\end{bmatrix}$$

The output, $y = v_1(t) = x_1$

$\therefore$ The output equation is $y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

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B. A system is characterised by the transfer function $(s)/U(s)=2/s^3 + 6s^2 + 11s + 6$.Find the state and output equations in matrix form and also test the controllability and observability of the given system.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix} u(t)$$

$$Q = [b : Ab : A^2 b]$$

$$Q = \begin{bmatrix} 0 & 0 & 2 \\ 0 & 2 & -12 \\ 2 & -12 & 50 \end{bmatrix}$$

$|Q| \neq 0$ System is controllable

$$Q' = \left[C^T : A^T C^T : A^{T^2} C^T\right]$$

$$Q' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

System is observable

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20. A. A linear time invariant system is described by the state equation

$$X(t) = \ [-\ 1\ 2\ \ -\ 1\ \ -\ 3\ ]X(t) + [0\ 1\ ]r(t)$$

Find the state transition matrix. If the initial state vector is $X(0) = [1\ 0\ ]$. Obtain the zero input response.

$$State\ transition\ Matrix$$

$$\Phi(t) = \begin{bmatrix} e^{-2t}(cost + sint) & e^{-2t}(2sint) \\ -e^{-2t}(sint) & e^{-2t}(cost - sint) \end{bmatrix}\text{-----}$$

The zero input response is $ZIR = \begin{bmatrix} e^{-2t}(cost + sint) \\ -e^{-2t}(sint) \end{bmatrix}$--

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B. A system is described by the following transfer function

$$G(s) = \frac{20(10s+1)}{s^3+3s^2+2s+1}$$

Find the state and output equation of the system.

Solution

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -3 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix} u$$

$$\frac{y(s)}{X_1(s)} = 10s + 1$$

$$y(s) = (10s + 1)\,X_1(s)$$

$$y(t) = X_1 + 10\,X_2$$

$$y(t) = \begin{bmatrix} 1 & 10 & 0 \end{bmatrix} X(t)$$

Text Refer the problem Control Systems Engineering, Nagarath M Gopal – Page – 594