

# Curriculum Units and Learning Outcomes

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| Content Area: Advanced Quantitative Reasoning                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                | Grade Level: 12 |
| Unit 2: Matrices                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             |                 |
| <b>Unit Summary:</b> Matrices are useful for storing and manipulating data. Each matrix has a size that is important in adding, subtracting, multiplying and finding the inverse of matrices. Having basic knowledge of matrices and their manipulation will be very helpful in using a spreadsheet as well as in modeling and solving a variety of problems.                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                |                 |
| <b>Massachusetts Standards:</b> <ul style="list-style-type: none"><li>● <b>AQR.N-VM:</b><ul style="list-style-type: none"><li>C. Perform operations on matrices and use matrices in applications.<ul style="list-style-type: none"><li>○ (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.</li><li>○ (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.</li><li>○ (+) Add, subtract, and multiply matrices of appropriate dimensions.</li><li>○ (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a Commutative operation, but still satisfies the Associative and Distributive properties.</li><li>○ (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.</li><li>○ (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.</li><li>○ (+) Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.</li></ul></li></ul></li><li>● <b>AQR.A-APR:</b><ul style="list-style-type: none"><li>C. Use polynomial identities to solve problems.</li></ul></li></ul> |                 |

- (+) Know and apply the Binomial Theorem for the expansion of  $(x + y)^n$  in powers of  $x$  and  $y$  for a positive integer  $n$ , where  $x$  and  $y$  are any numbers, with coefficients determined for example by Pascal's Triangle.

### Enduring Understandings:

- Matrices are useful to collect, store, retrieve and manipulate all kinds of data.
- Matrix operations use some arithmetic operations but have some differences in their properties.
- Matrix operations can be automated but one needs to understand what is going on behind the button push.
- Matrices can be very useful in finding solutions to simultaneous equation problems.

### Essential Questions:

- How can data be represented in a matrix?
- How can we determine the size of a matrix? How are the size of matrices important in conducting basic matrix operations?
- How do we know if two matrices can be added or subtracted? How do we add or subtract them?
- How are multiplication by a scalar and multiplication of two matrices different? How does the size of the matrices impact whether they can be multiplied? How does the size of each matrix relate to the size of the resulting matrix?
- How does multiplying by the inverse matrix relate to the multiplicative inverse in arithmetic?
- How do we compute the inverse matrix? What is the identity matrix and how is it important in multiplying by an inverse?
- How can matrices be used to find solutions to applications?

### Students will demonstrate KNOWLEDGE of:

- Matrix notation and manipulation (add, subtract, multiply, inverse).
- Solving systems of equations using matrix equations.

### Students will be SKILLED at:

- Adding, subtracting, and multiplying matrices by a scalar.
- Matrix multiplication
- Calculating the inverse of a matrix
- Writing systems of equations as matrix equations.
- Solving matrix equations
- Basic spreadsheet knowledge through completion of a spreadsheet project.

**Estimated Duration: approx 4 weeks**