

Unit 2 Summary

| Prior Learning | Grade 7, Unit 2 | Later in Grade 7 | Grade 8 & HS |
|--|---|---|--|
| Grades 3–5 <ul style="list-style-type: none"> Fraction operations Graphing points Grade 6 <ul style="list-style-type: none"> Equivalent ratios Unit rates Grade 7, Unit 1 <ul style="list-style-type: none"> Scale factor | <ul style="list-style-type: none"> Proportional relationships (in tables, equations, and graphs) | <ul style="list-style-type: none"> Divide fractions. Apply fractions and percentages to solve problems. | <ul style="list-style-type: none"> Slope and rate of change Linear equations |

Proportional Relationships in Tables

Carpets are sold at a price per square foot, so the ratios for amount of carpet to cost are all equal.

$$\frac{\$15}{10 \text{ sq. ft.}} = \frac{\$30}{20 \text{ sq. ft.}} = \frac{\$75}{50 \text{ sq. ft.}} = \$1.5 \text{ per square foot}$$

This is called a **proportional relationship**.

| Carpet (sq. ft.) | Cost (dollars) |
|------------------|----------------|
| 10 | 15.00 |
| 20 | 30.00 |
| 50 | 75.00 |

In this relationship, every square foot of carpet costs \$1.50.

This number 1.5 is called a **constant of proportionality**.

| Carpet (sq. ft.) | Cost (dollars) |
|------------------|----------------|
| 10 $\times 1.5$ | → 15.00 |
| 20 $\times 1.5$ | → 30.00 |
| 50 $\times 1.5$ | → 75.00 |

Another constant of proportionality in this example is $\frac{2}{3}$.

You get $\frac{2}{3}$ of a square foot of carpet for every dollar spent.

| Carpet (sq. ft.) | Cost (dollars) |
|---------------------------|----------------|
| 10 ← $\times \frac{2}{3}$ | 15.00 |
| 20 ← $\times \frac{2}{3}$ | 30.00 |
| 50 ← $\times \frac{2}{3}$ | 75.00 |

Proportional Relationships in Equations

The cost of carpet is 1.5 times the number of square feet.

We can represent this relationship with the equation:

$$y = 1.5x$$

x represents the number of carpet bought.

y represents the cost of the carpet, in dollars.

In general, the equation for a proportional relationship looks like:

$$y = kx$$

x and y represent the two related quantities.

k represents the constant of proportionality.

Proportional Relationships in Graphs

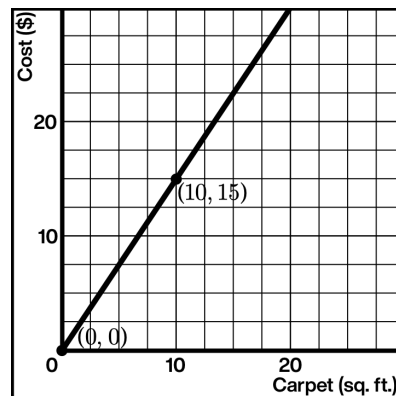
Graphs of proportional relationships:

- Lie on a line.
- Include the point $(0, 0)$, called the origin.

If you buy 10 square feet of carpet, it costs \$15.

If you buy 0 square feet of carpet, it costs \$0.

These are represented by the points $(10, 15)$ and $(0, 0)$.



Using Proportional Relationships

We can identify the constant of proportionality (1.5) in every representation.

Description

Each square foot of carpet costs \$1.50.

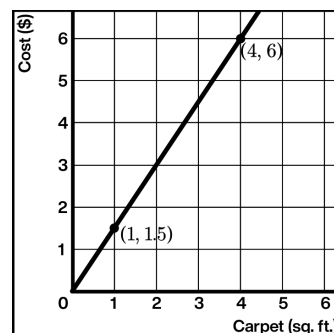
Table

| Carpets (sq. ft.) | Cost (dollars) |
|-------------------|----------------|
| 0 | 0 |
| 1 | 1.50 |
| 4 | 6 |

Equation

$$y = 1.5x$$

Graph



$$\frac{6}{4} = 1.5$$

Try This at Home

Proportional Relationships in Tables

Here is a brief recipe for pineapple soda:

For every 5 cups of soda water, mix in 2 cups of pineapple juice.

1. Create a table that shows at least three possible combinations of soda water and pineapple juice to make pineapple soda.
2. How much pineapple juice would you mix with 20 cups of soda water?
3. How much soda water would you mix with 20 cups of pineapple juice?
4. What is one constant of proportionality for this situation?

Proportional Relationships in Equations

5. Write an equation that represents the relationship in the recipe above, using s for cups of soda water and p for cups of pineapple juice.
6. Write a second equation that represents this relationship.
7. Select all the equations that represent a proportional relationship:
 - ☐ $K = C + 283$
 - ☐ $m = \frac{1}{4}j$
 - ☐ $V = s^3$
 - ☐ $h = \frac{14}{x}$

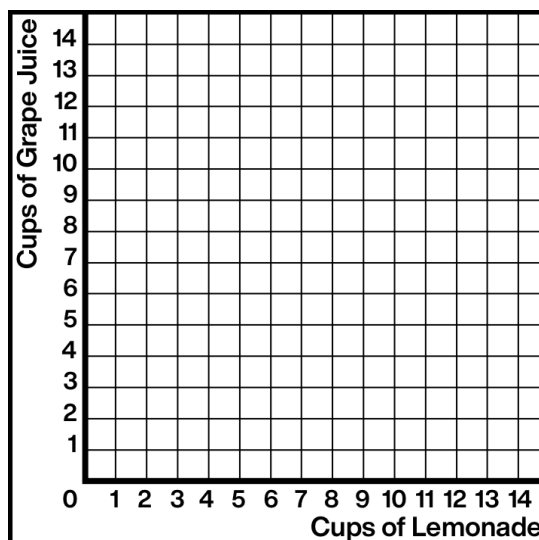
□ $c = 6.28r$

Proportional Relationships in Graphs

Here is a brief recipe for Grape-Ade:

For every 6 cups of lemonade, mix in 3 cups of grape juice.

8. Create a graph that represents the relationship between the amounts of lemonade and the amounts grape juice in different-sized batches of Grape-Ade.
9. Choose one point on your graph. Explain what that point means in a sentence.
10. What is a constant of proportionality for this relationship? Circle where you see the constant of proportionality in the graph.



Using Proportional Relationships

11. Describe a proportional relationship between quantities that you might encounter in your life.
12. What is a constant of proportionality in the relationship from Problem 4?
What does this number mean?

Solutions:

1. *Responses vary.*

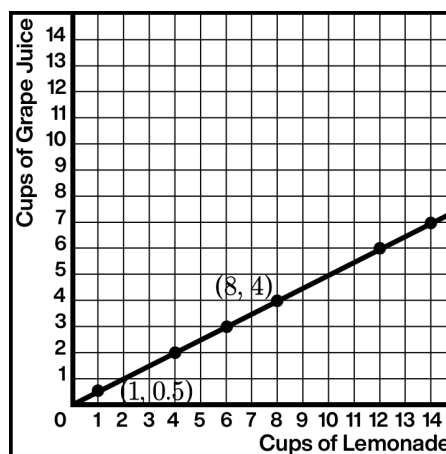
| Cups of Soda Water | Cups of Pineapple Juice |
|--------------------|-------------------------|
| 5 | 2 |
| 10 | 4 |
| 2.5 | 1 |

2. 8 cups of pineapple juice would be needed for 20 cups of soda water. One way to think about this is that you need 4 times the recipe because $5 \cdot 4 = 20$, and so $2 \cdot 4 = 8$ cups. Another way to think about it is that there are $\frac{2}{5}$, or 0.4, cups of pineapple juice per cup of soda water. Therefore, you would need $\frac{2}{5} \cdot 20 = 8$ cups of pineapple juice.
3. 50 cups of soda water would be needed for 20 cups of pineapple juice. There are $\frac{5}{2}$, or 2.5, cups of soda water per cup of pineapple juice. $\frac{5}{2} \cdot 20 = 50$ cups of soda water.
4. Both $\frac{2}{5}$ and $\frac{5}{2}$ are constants of proportionality for this situation.
5. Two equations that represent this situation are $p = 0.4s$ and $s = 2.5p$, where s represents the number of cups of soda water and p represents the number of cups of pineapple juice used.

7. $\checkmark m = \frac{1}{4}j$

$\checkmark c = 6.28r$

8.



9. The point (8, 4) means that you can make Grape-Ade using 8 cups of lemonade and 4 cups of grape juice.
10. The constant of proportionality is 0.5 or $\frac{1}{2}$. You can see this as the second coordinate of the point (1, 0.5), or in the simplified ratio $\frac{4}{8} = \frac{1}{2}$.
11. *Responses vary.*
- Miles driven on a new tank vs. gallons of gas used
 - Number of toy cars purchased vs. cost
 - Amount of flour used in cookies vs. number of cookies baked
12. *Responses vary.* The meaning of the constant of proportionality often involves “per” or “for every.”



Unit 7.2, Family Resource

6. See above.

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