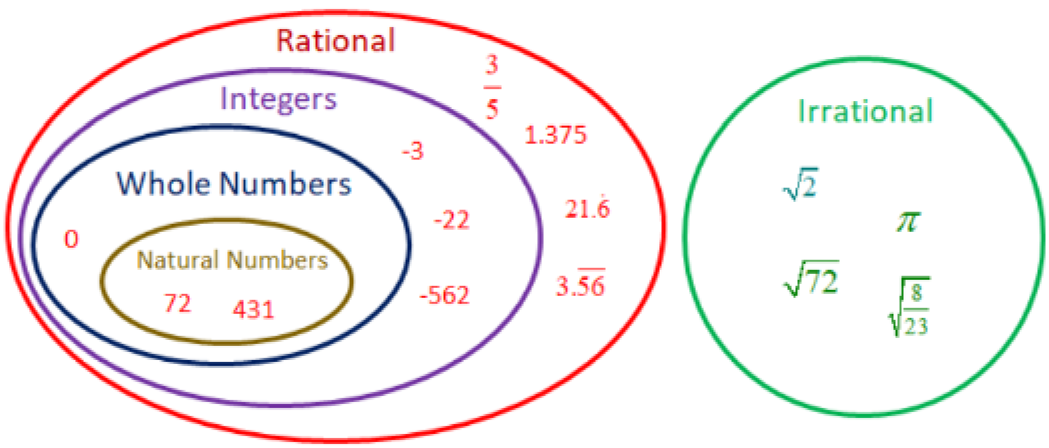
Pre-Algebra Module 1 Study Guide

Real Number System

Examples of rational numbers, integers, whole numbers, natural numbers, and irrational numbers



Rational Numbers: terminate, repeat, include whole numbers, perfect squares, perfect cubes and fractions

Irrational Numbers: non-terminating, do not repeat, cannot be fractions, include pi

Perfect Square: A number

that is made by squaring a number:a * a = a^2

Examples:

2^2	$2 \cdot 2$	4
(-3)^2	$-3 \cdot -3$	9
(1/3)^2	$1/3 \cdot 1/3$	$1/9$
(-1/10)^2	$-1/10 \cdot -1/10$	$1/100$

Perfect Cube: A number that is made by cubing a number: a * a * a = a^3

Examples:

2^3	$2 \cdot 2 \cdot 2$	8
(-3)^3	$-3 \cdot -3 \cdot -3$	-27
(1/3)^3	$1/3 \cdot 1/3 \cdot 1/3$	$1/27$
(-1/10)^3	$-1/10 \cdot -1/10 \cdot -1/10$	$-1/1{,}000$

Perfect Squares 1 to 15

$1^2 = (1)(1) = 1$	$6^2 = (6)(6) = 36$	$11^2 = (11)(11) = 121$
$2^2 = (2)(2) = 4$	$7^2 = (7)(7) = 49$	$12^2 = (12)(12) = 144$
$3^2 = (3)(3) = 9$	$8^2 = (8)(8) = 64$	$13^2 = (13)(13) = 169$
$4^2 = (4)(4) = 16$	$9^2 = (9)(9) = 81$	$14^2 = (14)(14) = 196$
$5^2 = (5)(5) = 25$	$10^2 = (10)(10) = 100$	$15^2 = (15)(15) = 225$

$\sqrt{1} = 1$	$\sqrt{4} = 2$	$\sqrt{9} = 3$	$\sqrt{16} = 4$	$\sqrt{25} = 5$
$\sqrt{36} = 6$	$\sqrt{49} = 7$	$\sqrt{64} = 8$	$\sqrt{81} = 9$	$\sqrt{100} = 10$
$\sqrt{121} = 11$	$\sqrt{144} = 12$	$\sqrt{169} = 13$	$\sqrt{196} = 14$	$\sqrt{225} = 15$

Perfect Cubes –5 to 5

$(-1)^3 = (-1)(-1)(-1) = -1$	$1^3 = (1)(1)(1) = 1$
$(-2)^3 = (-2)(-2)(-2) = -8$	$2^3 = (2)(2)(2) = 8$
$(-3)^3 = (-3)(-3)(-3) = -27$	$3^3 = (3)(3)(3) = 27$
$(-4)^3 = (-4)(-4)(-4) = -64$	$4^3 = (4)(4)(4) = 64$
$(-5)^3 = (-5)(-5)(-5) = -125$	$5^3 = (5)(5)(5) = 125$

$\sqrt[3]{-1} = -1$	$\sqrt[3]{-8} = -2$	$\sqrt[3]{-27} = -3$	$\sqrt[3]{-64} = -4$	$\sqrt[3]{-125} = -5$
$\sqrt[3]{1} = 1$	$\sqrt[3]{8} = 2$	$\sqrt[3]{27} = 3$	$\sqrt[3]{64} = 4$	$\sqrt[3]{125} = 5$

Square Root: a number which produces a specified quantity when multiplied by itself.

Examples:

$\sqrt{4}$	± 2
$\sqrt{9}$	± 3
$\sqrt{\frac{1}{9}}$	$\pm\frac{1}{3}$
$\sqrt{\frac{1}{100}}$	$\pm\frac{1}{100}$

Remember a negative sign in front of the square root means your final answer is negative.

$-\sqrt{25} = -5$

Steps for Estimating the Square Root of a Nonperfect Square

Step 1 Use perfect squares to estimate which whole numbers the nonperfect square is between on a number line.

Step 2 Use the whole numbers you found in Step 1 to create a number line split into tenths.

Step 3 Use the number line and guess and check to determine which two decimal numbers the nonperfect square is between. (The decimal number that the nonperfect square is closest to is the square root estimated to the tenths place.)

Step 4 Use the decimal numbers you found in Step 3 to create a number line split into hundredths.

Step 5 Use the number line and guess and check to determine which two decimal numbers the nonperfect square is between. (The decimal number the nonperfect square is closest to is the square root estimated to the hundredths place.)

Steps for Estimating the Cube Root of a Nonperfect Cube

Step 1 Use perfect cubes to estimate which integers the nonperfect cube is between on a number line.

Step 2 Use the integers you found in Step 1 to create a number line split into tenths.

Step 3 Use the number line and guess and check to determine which two decimal numbers the nonperfect cube is between. (The decimal number it is closest to is the cube root estimated to the tenths place.)

Step 4 Use the decimal numbers you found in Step 3 to create a number line split into hundredths.

Step 5 Use the number line and guess and check to determine which two decimal numbers the nonperfect cube is between. (The decimal number it is closest to is the cube root estimated to the hundredths place.)

Exponent Laws with Integer Exponents

(where *m* and *n* are integers and *a* and *b* are real numbers)

Product of Powers Law To multiply factors that have the same base, keep the base the same and add the exponents. $a^m \cdot a^n = a^{m+n}$ Example: $5^2 \cdot 5^4 = 5^{2+4} = 5^6$	**Quotient of Powers Law** To divide expressions that have the same base, keep the base the same and subtract the exponents. $\frac{a^m}{a^n} = a^{m-n}$ Example: $\frac{3^8}{3^6} = 3^{8-6} = 3^2$
Power of a Power Law To raise an expression with an exponent to another exponent, keep the base the same and multiply the exponents. $(a^m)^n = a^{m(n)}$ Example: $(4^2)^3 = 4^{2(3)} = 4^6$	**Zero Exponent Law** When the base has an exponent of zero, it is equal to 1. $a^0 = 1$, where *a* ≠ 0 Example: $7^0 = 1$
Power of a Product Law To raise a product to an exponent, raise each factor to the exponent. $(ab)^m = a^m \cdot b^m$ Example: $(5^2 \cdot 3^3)^4 = (5^2)^4 \cdot (3^3)^4$	**Power of a Quotient Law** To raise a quotient to an exponent, raise the numerator and denominator to the exponent. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, where *b* ≠ 0 Example: $\left(\frac{5^2}{3^3}\right)^4 = \frac{(5^2)^4}{(3^3)^4}$

Negative Exponent Law

To simplify an expression with a negative exponent, take the reciprocal of the base and rewrite the exponent as positive.

$a^{-m} = \frac{1}{a^m}$, where *a* ≠ 0

Example: $5^{-2} = \frac{1}{5^2}$

$\left(\frac{a}{b}\right)^{-m} = \left(\frac{b}{a}\right)^m$, where *a*, *b* ≠ 0

Example: $\left(\frac{2}{3}\right)^{-4} = \left(\frac{3}{2}\right)^4$

The reverse of these laws are also true.

Steps for Order of Operations

Step 1 Simplify inside parentheses ( ) or other grouping symbols [ ], | | using order of operations.

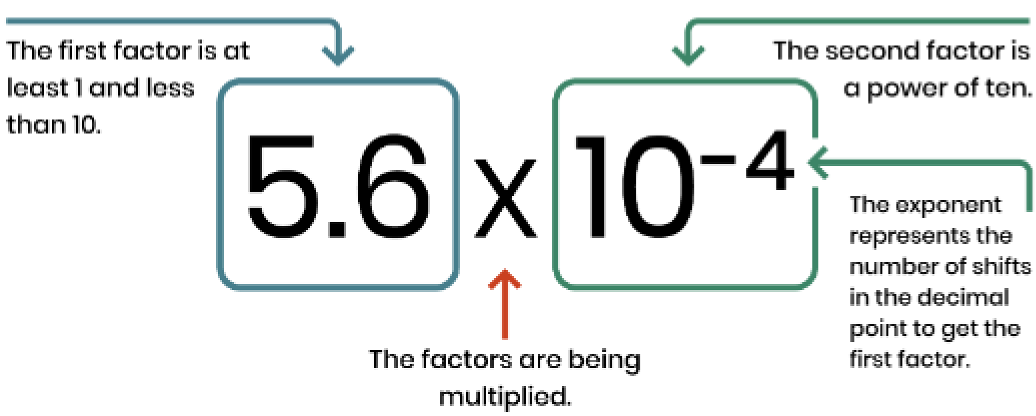
Step 2 Simplify terms with exponents and radicals.

Step 3 Multiply or divide from left to right.

Step 4 Add or subtract from left to right.

PEMDAS

Scientific Notation



Positive Powers of 10	Negative Powers of 10
$10^0 = 1$	$10^{-1} = 0.1$
$10^1 = 10$	$10^{-2} = .01$
$10^4 = 10{,}000$	$10^{-4} = .0001$
$10^5 = 100{,}000$	$10^{-5} = .00001$

Steps for Rewriting Numbers in Scientific Notation

Step 1 Shift the decimal point to create a number that is at least 1 and less than 10.

Step 2 Count the number of places that the decimal point shifted to create this number.

Step 3 Write the new number as a product of a number and a power of ten. The number of places that the decimal point shifted is the exponent on the 10.

- If you shifted left, the exponent is positive.
- If you shifted right, the exponent is negative.

Remember, you can check if the sign of the exponent is correct by putting the number back in its original form.

Convert 250,000 into scientific notation. Convert .0025 into scientifc notation

Step 1: Create a number between 1-9. 2.5 (same for 250,000 and .0025)

Step 2: Count number of places the decimal point moved to create this number.

2 5 0, 0 0 0 .0 0 2 5

5 4 3 2 1 1 2 3

Step 3: Write the number as a product of a number and power of 10.

2.5×10^5 2.5×10^{-3}

Steps for Adding and Subtracting Numbers in Scientific Notation

Step 1 The exponents on the second factors must be the same. If they are not, rewrite one of the expressions so the exponents are the same.

Step 2 Rewrite the sum or difference by factoring out the common factor, the power of ten.

Step 3 Add or subtract the first factors.

Step 4 Rewrite the final answer in scientific notation, if necessary.

Steps for Adding and Subtracting in Scientific Notation

$(2.456*10^5) + (6.0034 * 10^8)$

$(4.801*10^5) - (2.21*10^6)$

1. **Determine the smallest power of 10.**

$(2.456105) + (6.0034 * 10^8)$

10^5 is the smallest power of 10.
Therefore, 10^8needs decrease to match 10^5.

$(4.801*10^5) - (2.21*10^6)$

10^5 is the smallest power of 10.
Therefore, 10^6 needs to decrease to match 10^5.

2. **Decrease the larger power of 10 until it matches the smaller one found in Step 1 - move the decimal point of the number with the larger power of 10 to the right until it matches the smaller power.**

Decrease the power by 3 because 8 - 5 = 3.
Move the decimal to the right 3 places.

$$6.0034*10^8 = 6003.4*10^5$$

1 2 3

Decrease the power by 1 because 6 - 5 = 1.
Move the decimal point to the right 1 place.

$$2.21*10^6 = 22.1*10^5$$

1

3. Add or subtract the number factors and keep the power of 10.

$(2.456 + 6003.4) * 10^5$
$6005.856 * 10^5$

$(4.801-22.1) * 10^5$
$-17.299 * 10^5$

4. Convert to scientific notation if needed.

$6005.856 * 10^5$
$(6.005856 * 10^3) * 10^5$
$6.005856 * (10^3*10^5)$
6.00595610^8

$-17.299 * 10^5$
$(-1.7299101) * 10^5$
$-1.7299 * (10^1 * 10^5)$
$-1.7299 * 10^6$

Determine which of the numbers has the ***smaller*** exponent.

1. Change this number by moving the decimal place to the ***left*** and ***raising*** the exponent, until the exponents of both numbers agree. Note that this will take the lesser number out of standard form.

2. Add or subtract the coefficients as needed to get the new coefficient.

3. The exponent will be the exponent that both numbers share.

4. Put the number in standard form.

Steps for Multiplying Numbers in Scientific Notation

Step 1 Rewrite the expression by grouping the first factors together and the second factors together using the commutative and associative properties of multiplication.

Step 2 Multiply the first factors. Then multiply the second factors using the Product of Powers Law.

Step 3 Rewrite the final answer in scientific notation, if necessary.

Steps for Dividing Numbers in Scientific Notation

Step 1 Rewrite the expression by grouping the first factors together and the second factors together using the associative property of multiplication.

Step 2 Divide the first factors. Then divide the second factors using the Quotient of Powers Law.

Step 3 Rewrite the final answer in scientific notation, if necessary.

Multiplying and Dividing in Scientific Notation

Group the numerical factors together (multiply or divide)

$(3.1 * 10^7) * (7 * 10^4)$
$(3.1 * 7) \quad * \quad (10^7 * 10^4)$
$21.7 * \quad (10^7 * 10^4)$

$(3.9 \div 10^7) * (1.3 \div 10^4)$
$(3.9 \div 1.3) * (10^7 \div 10^4)$
$3 * (10^7 \div 10^4)$

Use the properties of exponents to simplify powers of 10 (product or quotient rule)

$21.7 * \quad (10^7 * 10^4)$
$21.7 * \quad (10^{7+4})$
$21.7 * (10^{11})$

$3 * (10^7 \div 10^4)$
$3 * (10^{7-4})$
$3 * (10^3)$

Convert to scientific notation(if needed)

$21.7 * (10^{11})$
$(2.17 * 10^1) * 10^{11}$
$2.17 * (10^1 * 10^{11})$
$2.17 * 10^{12}$

Conversion not needed for the division in this problem

Rules for determining significant digits:

1. Non-zero digits are always significant.
2. Any zeros between two significant digits are significant.
3. A final zero or trailing zeros in the decimal portion only are significant.

Adding and Subtracting Numbers in Scientific Notation

In addition and subtraction, the number of significant digits in the final answer is based on the number of digits in the least precise number given. This means the number of digits after the decimal point determines the number of digits that can be expressed in the answer.

Example: If you added or subtracted three numbers: 3.1, 2.006, and 5.6765, the value 3.1 has the least number of digits after the decimal point. The final answer would be given to the tenths place.

Multiplying and Dividing Numbers in Scientific Notation

In multiplication and division, the number of significant digits is based on the number that has the fewest number of digits.

Example: If you multiplied or divided three numbers: 3.1, 2.006, and 5.6765, the value 3.1 has the fewest number of digits. The final answer would be given only to 2 significant digits.