

Unit 2 Simplifying Expressions and Solving Equations

		Knowledgehook- www.khmath.com	Learning Goals
Page 02	Lesson 01: Introduction to Algebra	Simplifying Expressions ☆☆☆ Challenge 6 ☆☆☆ Challenge 7	I understand the difference between simplifying expressions and solving equations.
Page 03	Lesson 02: Single-Variable Expressions		I can solve single-variable expressions.
Page 04	Lesson 03: Gathering Like Terms		I can simplify like terms.
Page 06	Lesson 04: Adding and Subtracting Polynomials	Simplifying Expressions Challenge 9	I can add and subtract polynomials with up to two variables.
Page 08	Lesson 05: Multiplying Polynomials	Simplifying Expressions ☆☆☆ Challenge 10	I can multiply a polynomial by a monomial involving variables.
	Mid-Chapter Assessment (quiz)		
Page 10	Lesson 06: Solving Equations - by addition and subtraction		I can solve equations through inverse operations.
Page 12	Lesson 07: Solving Equations - by division and multiplication		I can solve equations through inverse operations.
Page 15	Lesson 08: Solving Equations - using signs and expansion		I can solve equations with negative signs. I can solve equations by multiplying polynomials by monomials.
Page 18	Lesson 09: Solving Equations - with like terms		I can solve equations by adding like terms.
Page 20	Lesson 10: Solving Equations to solve problems	Solving Equations ☆☆☆ Challenge 2 Solving Equations ☆☆☆ Challenge 4	I can substitute values into equations to solve problems.
Page 23	Review and extra practice		
	Assessment (test)		

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Or, download free iOS app DragonBoxEDU
(the free one)

Goal: complete chapter 8



Lesson 01: Difference between solving equations and simplifying expressions

Simplifying Expressions:

-
- Example of expression: $3x - 2x + 5x - 6x$
- Example of expression in an equation: $2x + 3x + 4 = 24$

Solving Equations:

-
- Example: $2x + 3 = 5$
 $x = 1$

PRACTICE:

Are these expressions or equations? Circle one for each.

$y = 3x + 7$	expression	equation
$4x - 2x + 7 - 11$	expression	equation
$7x + 1 = 7$	expression	equation
$6m - 7t + 1 - 11m$	expression	equation
$12 - 11t + 4t - 17$	expression	equation

Terms in Algebra

- Variable
- Coefficient

$3x$ - variable is x, coefficient is 3

$-x^2$ - variable is x, coefficient is -1 and exponent 2

PRACTICE:

Circle the coefficient for each:

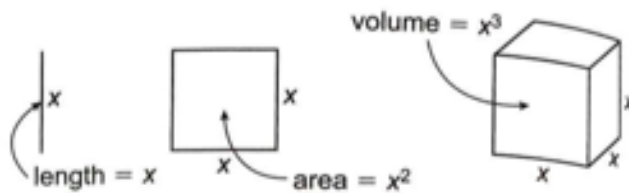
$2x$ $-5x^2$ $-t$ $19y$

Circle the variable for each:

$2x$ $-5x^2$ $-t$ $19y$

Lesson 02: Single-Variable Expressions

An expression with only one variable is called a monomial. Examples: $3x$, $5x^2$, $10y$, $-y^3$, $6t$



- Like terms have the same variable AND exponent
- Like terms can be added

You can group like terms by adding them.

$$\begin{array}{r} 2x^3 + 3x^3 \\ = \end{array}$$

$$\begin{array}{r} x^2 + 3x^2 \\ = \end{array}$$

$$\begin{array}{r} 5x + x \\ = \end{array}$$

$$\begin{array}{r} 3 + 6 \\ = \end{array}$$

PRACTICE:

1. Circle the like terms

a. $4x$, $6t$, $4x^2$, $-7x$, $-x$

b. $9t$, $6x$, $7y$, $-11t$

2. Create an algebraic expression with a set of like terms:

3. Simplify each expression:

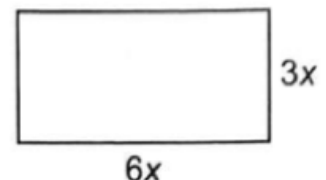
a) $5x^2 + 3x^2 + 6x^2$ b) $7x + 8x + 2x + x$ c) $2x^3 + 5x^3$

d) $x^2 + 4x^2 + 3x^2 + 5x^2$

4. Find the error in the simplification.

$$4x + 2x^3 + 6x = 12x^3$$

5. Determine the simplified expression that represents the perimeter of the rectangle.



6. Fill in the missing terms to make the two sides of the equation equal.

a) $6x + \underline{\hspace{1cm}} + 4x = 13x$

b) $9x^3 + 3x^3 + \underline{\hspace{1cm}} + 7x^3 = 24x^3$

c) $5x^2 + \underline{\hspace{1cm}} + 3x^2 + 6x^2 = 19x^2$

d) $7x + \underline{\hspace{1cm}} + 3x + 6x = 16x$

7. Mary says x^2 and x^3 are like terms. Do you agree? Explain.

8. Draw a diagram to represent each of x , x^2 and x^3 .

Lesson 03: Gathering Like Terms

A polynomial is an expression of more than two terms

- MONO (1) - monomial = 1 term
 - Examples:
- BI (2) - binomial = 2 terms
 - Examples:
- POLY (many) - polynomial = many terms
 - Examples:

Like terms have the same variable and exponents

- Examples of like terms --> $3x$ and $4x$ $9x^2$ and $2x^2$
- Like terms can be added --> $3x + 4x = 7x$
- Like terms that are the same number, but different signs are called zero pairs --> $2x + -2x = 0$
- To simplify a polynomial, you gather like terms and remove zero pairs

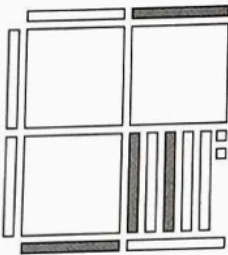
Using algebra tiles

- Large square = x^2
- Long rectangle = x
- Small square = 1
- One color is positive and the other negative ($-x^2$, $-x$, -1)

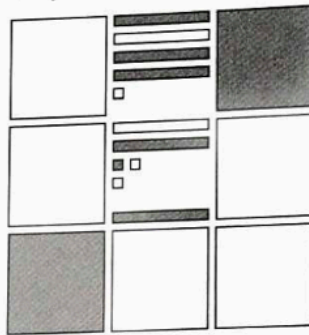
Example

Which two polynomials are equivalent?

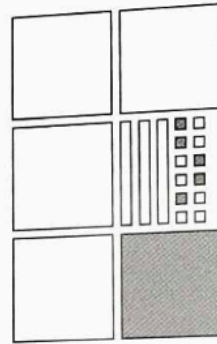
Polynomial A:



Polynomial B:



Polynomial C:

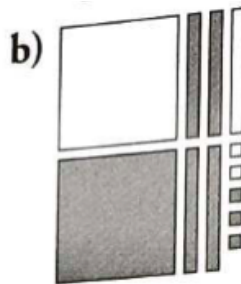
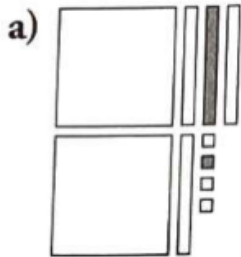


PRACTICE:

1. Gather like terms $3x^2, -2, 5x^3, 7, -2x, -4x^3, 5x, 1, 4x^2, -x$

2. Simplify each polynomial. Write a simplified expression for each.

Simplify each polynomial.



3. Simplify. a) $5x^2 + 2x - 6x - 3x^2$

4. Find the error.

a) $4x - 2x^2 - 2x^2 - x - 3$ is simplified to $3x - 3$.

b) $4x + 2x^2 + 5 + 6x + 1$ is simplified to $12x^2 + 6$.

5. Fill in the missing terms to make the two sides equal.

a) $6x + 8 + \underline{\hspace{1cm}} + 4x + 1 = 13x + 9$

b) $5x^2 + 3x + \underline{\hspace{1cm}} + 3x^2 + 6x^3 = 10x^3 + 8x^2 + 3x$

Lesson 04: Adding and Subtracting Polynomials

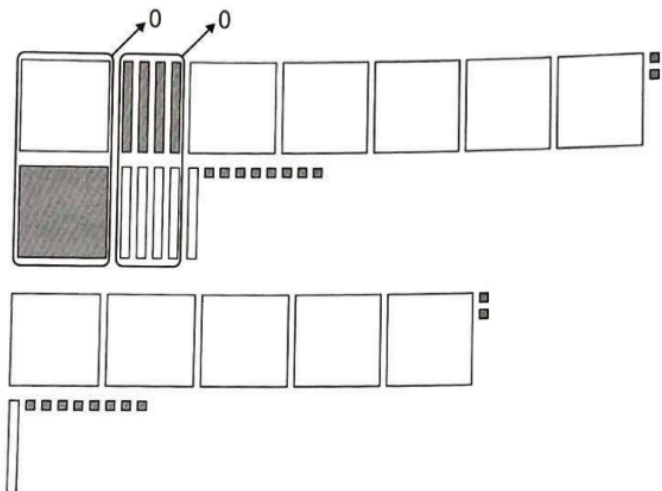
KEEP IN MIND

- You can add polynomials by representing each polynomial using algebra tiles and then simplifying using zero pairs.
- You can add polynomials by grouping like terms from each polynomial and adding each group's coefficients.

Example

Determine the sum of $6x^2 - 4x - 2$ and $-x^2 + 5x - 8$.

Solution 1 Using Algebra Tiles



$$5x^2 + x - 10$$

Solution 2 Using Symbols

$$\begin{aligned}
 & [6x^2 - 4x - 2] + [-x^2 + 5x - 8] \\
 &= [6x^2 + (-1x^2)] + [-4x + 5x] + [-2 + (-8)] \\
 &= 5x^2 + 1x + (-10) \\
 &= 5x^2 + x - 10
 \end{aligned}$$

When subtracting polynomials you can add the OPPOSITE.

$$\begin{aligned}(4x^2 - 5x + 7) - (3x - 1) \\ = 4x^2 - 8x + 8\end{aligned}$$

PRACTICE:

1. Add

a) $(3x - 7) + (x^2 - 2x + 7)$

b) $(5x^2 + x + 2) + (3x^2 + 2x + 5)$

c) $(3x^3 + 2x^2 - 5x + 1) + (4 + 6x - 7x^2 - 4x^3)$

2. Subtract

a) $(-x^2 + 3x - 1) - (3x^2 - 4x + 2)$ b) $(2x^2 + x) - (5x^2 - x + 6)$

Lesson 05: Multiplying Polynomials

You can multiply two monomials by multiplying the coefficients of the terms and writing the product of the variables using a single exponent.

Example:

$$(2x)(-5x^2)$$

$$= -10x^3$$

More Examples:

a. $(x)(2x)$

b. $3x(4x)$

c. $3x(4x^2)$

d. $3(3x)$

e. $-2(3x^3)$

f. $(5x)(-2x)$

You can determine the product of a monomial and a polynomial by applying the distributive property - multiplying each term of the polynomial by the monomial.

Example:

$$4x(x^2 - 3x + 2)$$

$$= 4x(x^2) + 4x(-3x) + (4x)(2)$$

$$= 4x^3 - 12x^2 + 8x$$

More Examples:

a. $4x(2x - 3)$

b. $(-2x^2)(x + 6)$

c. $5x(2x^2 + 3x - 4)$

PRACTICE:

2. Multiply.

a) $(x)(x) = \underline{\hspace{2cm}}$

b) $(x)(x^2) = \underline{\hspace{2cm}}$

c) $(4)(4x) = \underline{\hspace{2cm}}$

d) $(6x)(2x) = \underline{\hspace{2cm}}$

e) $5(2x^2) = \underline{\hspace{2cm}}$

f) $-4(2x^3) = \underline{\hspace{2cm}}$

g) $(5x)(-3x) = \underline{\hspace{2cm}}$

h) $(-2x^2)(-4x) = \underline{\hspace{2cm}}$

3. Multiply.

a) $(5x)(3x - 2)$

b) $(-2x)(x + 7)$

c) $(3x)(2x^2 + 4x - 1)$

d) $(6x^2)(-5x - 4)$

4. Determine the missing polynomials.

a) $(\underline{\hspace{2cm}})(3x - 1) = 6x^2 - 2x$

b) $(\underline{\hspace{2cm}})(-4x + 5) = 20x^2 - 25x$

c) $(2x^2)(\underline{\hspace{2cm}} + \underline{\hspace{2cm}}) = 14x^3 + 2x^2$

2. Multiply.

a) $(x)(x) = \underline{\hspace{2cm}}$

e) $5(2x^2) = \underline{\hspace{2cm}}$

b) $(x)(x^2) = \underline{\hspace{2cm}}$

f) $-4(2x^3) = \underline{\hspace{2cm}}$

c) $(4)(4x) = \underline{\hspace{2cm}}$

g) $(5x)(-3x) = \underline{\hspace{2cm}}$

d) $(6x)(2x) = \underline{\hspace{2cm}}$

h) $(-2x^2)(-4x) = \underline{\hspace{2cm}}$

3. Multiply.

a) $(5x)(3x - 2)$

b) $(-2x)(x + 7)$

c) $(3x)(2x^2 + 4x - 1)$

d) $(6x^2)(-5x - 4)$

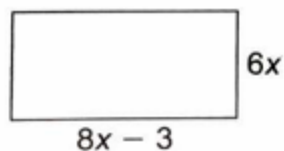
4. Determine the missing polynomials.

a) $(\underline{\hspace{2cm}})(3x - 1) = 6x^2 - 2x$

b) $(\underline{\hspace{2cm}})(-4x + 5) = 20x^2 - 25x$

c) $(2x^2)(\underline{\hspace{2cm}} + \underline{\hspace{2cm}}) = 14x^3 + 2x^2$

6. Determine an expression for the area of this rectangle.

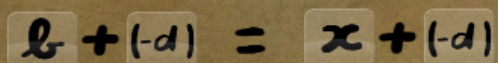


Lesson 06: Solving Equations by Addition and Subtraction



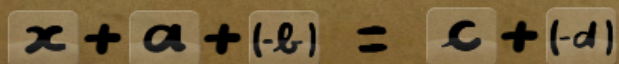
A photograph of a rectangular piece of brown paper with five small, light-colored square tiles placed on it. The tiles are arranged to form the equation $x + c = e$. The first tile contains the letter 'x', the second contains a plus sign '+', the third contains the letter 'c', the fourth contains an equals sign '=', and the fifth contains the letter 'e'.

$$x + c = e$$



A photograph of a rectangular piece of brown paper with six small, light-colored square tiles placed on it. The tiles are arranged to form the equation $b + (-d) = x + (-d)$. The first tile contains the letter 'b', the second contains a plus sign '+', the third contains the letter 'd' with a minus sign '-' in front of it, the fourth contains an equals sign '=', the fifth contains the letter 'x', and the sixth contains the letter 'd' with a minus sign '-' in front of it.

$$b + (-d) = x + (-d)$$



A photograph of a rectangular piece of brown paper with nine small, light-colored square tiles placed on it. The tiles are arranged to form the equation $x + a + (-b) = c + (-d)$. The first tile contains the letter 'x', the second contains a plus sign '+', the third contains the letter 'a', the fourth contains a plus sign '+', the fifth contains the letter 'b' with a minus sign '-' in front of it, the sixth contains an equals sign '=', the seventh contains the letter 'c', the eighth contains a plus sign '+', and the ninth contains the letter 'd' with a minus sign '-' in front of it.

$$x + a + (-b) = c + (-d)$$

More Examples:

$$t - 3 = 15$$

$$b - 12 = 14 + 3$$

$$v + 18 = 47$$

$$c + 6 = 43 + 8$$

PRACTICE:

Solve each equation.

a

1. $b - 8 = 15$

b

$x - 14 = 36$

c

$c - 3 = 28 + 4$

2. $42 = r - 12$

$80 = e - 26$

$20 + 11 = f - 14$

3. $163 = a - 27$

$9 \times 9 = m - 38$

$t - 28 = 102$

4. $117 = w - 83$

$200 - 25 = g - 83$

$h - 75 = 100 + 56$

Solve each equation.

a

1. $d + 12 = 48$

b

$36 + e = 84$

c

$f + 14 = 18 + 18$

2. $38 = j + 13$

$27 = 9 + h$

$20 + 34 = 27 + l$

3. $12 + w = 76$

$114 = x + 38$

$300 - 30 = y + 50$

Lesson 07: Solving Equations by Division and Multiplication

Division

$$a \cdot x = \frac{b}{c} + (-d)$$

$$f \cdot x = i + 2 + f$$

$$2 \cdot x = 3 + e$$

$$4m = 52$$

Multiplication

$$\frac{x}{a} = \frac{b}{c} + (-d)$$

$$\frac{x}{c} = 3 + 4$$

$$\frac{x}{4} = g + \frac{b}{2} + (-g)$$

$$\frac{a}{5} = 35$$

PRACTICE:

Solve each equation.

$$1. \quad 3w = 12$$

$$3b = 51$$

$$8m = 100 - 4$$

$$2. \quad 72 = 2a$$

$$54 = 3c$$

$$96 - 20 = 4r$$

$$3. \quad 6e = 84$$

$$25s = 75$$

$$4d = 75 - 7$$

$$4. \quad 14x = 42$$

$$75 = 15m$$

$$3y = 100 - 28$$

Solve each equation.

$$1. \quad \frac{a}{8} = 7$$

$$\frac{b}{13} = 9$$

$$\frac{c}{4} = 6 + 12$$

$$2. \quad 16 = \frac{r}{8}$$

$$8 = s \div 7$$

$$2 \times 9 = \frac{t}{5}$$

$$3. \quad g \div 17 = 9$$

$$15 = \frac{h}{5}$$

$$7 \times 6 = \frac{j}{3}$$

$$4. \quad \frac{m}{15} = 17$$

$$23 = \frac{n}{28}$$

$$p \div 19 = 3 \times 9$$

Lesson 08: Solving Equations with Signs and Expansion

$$(-1)(-1)x = a$$

$$(-1)x + c = d$$

$$0 + b = (-b)e \cdot x$$

$$-(x + 1) = 2$$

$$a \cdot \left(\frac{x}{a} + b \right) = \frac{c}{d}$$

$$c \cdot \left(\frac{x}{c} + \frac{g}{c} \right) = i$$

$$(-d) = x \cdot \left(\frac{e}{x} + f \right)$$

$$5 (8x + 8) = 280$$

$$7(9x - 15) = 588$$

$$10 (10x - 6) = 540$$



Solve the equations.

(1) $7(7x + 16) = 455$

(2) $8(6x - 5) = 440$

(3) $9(5x - 4) = 189$

(4) $5(8x + 8) = 280$

(5) $6(7x + 9) = 306$

(6) $6(7x + 7) = 546$

(7) $5(10x + 14) = 320$

(8) $6(9x + 17) = 372$

(9) $5(8x + 12) = 460$

(10) $9(6x - 7) = 315$

(11) $10(7x - 16) = 190$

(12) $7(6x + 17) = 497$

Lesson 09: Solving Equations with Like Terms

$$2 \cdot x + 3 = 4 \cdot x + (-2)$$

$$6 \cdot x + 7 + 2 \cdot x = 4 \cdot x + 9 \cdot x + 8$$

$$(-6) \cdot x + x + (-4) = (-2) \cdot x + 3$$

PRACTICE:



Solve the equations.

(1) $2x + 3(6x - 12) = 144$

(2) $6x + 4(5x - 9) = 172$

(3) $4x + 6(5x - 9) = 184$

(4) $5x + 5(5x - 15) = 195$

(5) $6x + 2(4x + 15) = 198$

(6) $7x + 3(4x + 10) = 125$

(7) $5x + 5(2x + 3) = 120$

(8) $7x + 2(5x - 12) = 129$

(9) $6x + 2(x - 4) = 40$

(10) $4x + 3(2x + 12) = 96$

Lesson 10: Using Equations to Solve Problems

Examples:

1.

The formula

$$B = 100 - \frac{h}{156}$$

gives water's boiling point, B , in degrees Celsius at an altitude of h metres.

What is the approximate **boiling point of water** at an altitude of 5951 m?

- A 38 °C
- B 62 °C
- C 96 °C
- D 156 °C



2.

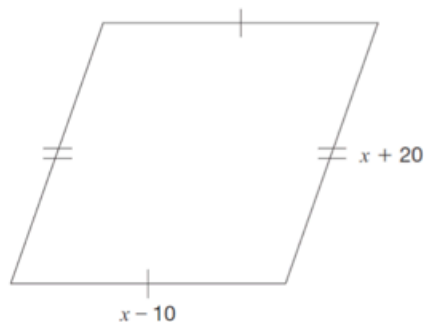
A rocket is fired upward from the ground. The equation below shows the relationship between h , the height of the rocket above the ground in metres, and t , the time in seconds.

$$h = 60t - 5t^2$$

Which of the following is the height of the rocket after 4 seconds?

- a 35 m
- b 44 m
- c 160 m
- d 240 m

3. Pauline builds a fence around her garden, which is shaped like a parallelogram, as shown below.



Pauline uses 100 metres of fencing along the perimeter of the garden. Find the dimensions of her garden. Show your work.

PRACTICE:

Stephen's earnings are calculated according to the formula

$$\text{Earnings} = 7t + \frac{d}{20}$$

where t is the number of hours Stephen works and d is the dollar value of the clothes he sells in the week.



One week, Stephen works for 15 h and sells \$980 worth of clothes. How much does he earn?

- a \$54.25
- b \$105
- c \$154
- d \$176

A steel bar will expand when it is heated and contract when it is cooled. The relationship between the length of the bar, L , (mm) and the temperature, T , ($^{\circ}\text{C}$) is given by

$$L = 5000 + 0.12(T - 20).$$

What is the length of the steel bar when the temperature is 45°C ?

- a 5001 mm
- b 5002 mm
- c 5003 mm
- d 5004 mm

A tap is leaking into a pail. The height of the water in the pail is represented by the equation $h = 0.5t + 2$, where h represents the height of water in the pail, in cm, and t represents the amount of time the tap has been leaking, in minutes.

What is the height of water in the pail if the tap has been leaking for 56 minutes?

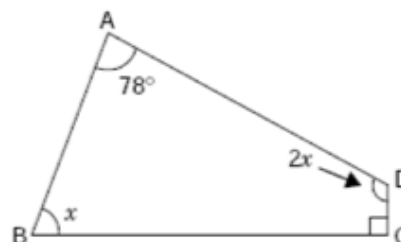
- a 28 cm
- b 30 cm
- c 108 cm
- d 114 cm

Marco is designing a new sail for his windsurfer.

He uses the quadrilateral below as the design of one part of the sail.

- a) Determine the value of x in the quadrilateral by solving the equation.

$$78 + x + 2x + 90 = 360$$



The total number of laps Tessa has run, l , after any number of days, n , is given by the formula

$$l = \frac{n(n+1)}{2}$$

Use this formula to determine **the total number of laps** Tessa will have run after **30 days**.

REVIEW

2.1 Single-Variable Expressions

1. Simplify each expression.

a) $7 + 1 + 1 + 5 + 3$

c) $5x + 4x + 4x + 3x + x$

b) $6x^2 + 2x^2 + x^2 + x^2 + 9x^2$

d) $2x^3 + 5x^3 + 3x^3 + 6x^3 + 7x^3$

2.2 Gathering Like Terms

2. Simplify.

$$2x^2 + 4x^3 - 7x - 6 - 9x^2 + 4x^3 - 1$$

2.3 Adding Polynomials

3. Add.

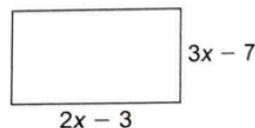
a) $(3x^2 + 7x + 6) + (4x^2 + 2x + 5)$

b) $(7x^3 + 5x^2 + 2) + (2x^2 - 9x - 2)$

c) $(8x^2 - 2x - 9) + (-9x^2 + 4x - 1)$

d) $(10x^3 - 4x - 3) + (-2x^3 + 10x^2 - 2)$

4. Determine an expression for the perimeter of the rectangle.



2.4 Subtracting Polynomials

5. Subtract.

a) $(7x^2 + 5x - 4) - (-3x^2 + 5)$

b) $(2x^3 + 5x^2 - 4x) - (-3x^2 + 5)$

c) $(-2x^3 - x - 5) - (5x^3 - 6x^2 - 2)$

6. What expression was subtracted from $3x^2 + 4x - 3$ to get $4x^2 - 2x + 3$?

7. What expression was added to $3x^2 + 4x - 3$ to get $4x^2 - 2x + 3$?

2.5 Multiplying Polynomials

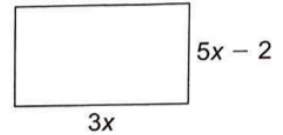
8. Multiply.

a) $(7x)(-3x)$

b) $(-6)(-3x^2)$

c) $(-3x)(-4x - 3)$

9. Determine a simplified expression for the area of the rectangle.

**2.6 Solving Equations: Part 1**

10. Using examples, explain how inverse operations might be used to solve an equation. Use the equations $x - 5 = 9$, $x + 5 = 9$, $3x = 9$, and

$\frac{x}{3} = 9$ in your explanations.

2.7 Solving Equations: Part 2

11. Solve each equation.

a) $2x - 3 = 5$

b) $2(3x - 5) = 8$

12. Provide an example of an equation you could solve by either using the distributive property or not using the distributive property.

2.8 Solving Equations: Part 3

13. Solve each equation, and verify your solution.

a) $2x - 3x = 4$

b) $3x + 2 = -x - 6$

14. Show that the following expressions are equivalent.

a) $2x + 4$

b) $2(x + 2)$

c) $3x - (x - 4)$