Unit	8.8
Title	**Square Roots, Irrational Numbers, and the Pythagorean Theorem**
Focus Standards	8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number. 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). 8.EE.A.2 Use square root symbols to represent solutions to equations of the form $x^2=p$, where p is a positive rational number. Evaluate square roots of small perfect squares. Know that $\sqrt{2}$ is irrational. 8.G.B.6 Explain a proof of the Pythagorean Theorem and its converse. 8.G.B.7 Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8.G.B.8 Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.
Supporting Standards	8.EE.A.3 Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. 8.EE.A.4 Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.
Supplementary Extensions	This document offers extension and challenge for deeper discussions, class activities, and assessments. [Pythagorean Theorem] **and** [Square Roots and Irrational Numbers]
Mathematical Goals	• Understand the relationship between square numbers and roots 8.EE.A.2 • Evaluate square roots by approximation and using technology; locate them on a number line 8.NS.2 • Use exponents and radical notation to express square roots (i.e. solutions to $x^2=p$) 8.EE.A.2 • Understand what decimal expansion means and how it relates to rational numbers 8.NS.1 • Convert numbers w/ terminating and repeating decimals into ratios; locate them on a number line 8.NS.1

	• Recognize that some numbers are irrational and what makes them so 8.NS.1; approximate their location on a number line 8.NS.2 • Understand the Pythagorean Theorem 8.G.B and use it to find unknown lengths in right triangles 8.G.B.7 • Explain a proof of the Pythagorean Theorem and its converse 8.G.B.6 • Use the Pythagorean Theorem to find the distance between two points on the coordinate plane 8.G.B.8
The story before this unit *(including prior knowledge)*	As they begin the unit, students will have understood factors, multiples, and ratios as number relationships, and performed operations on rational numbers 7.NS.3. They will have recognized the inverse relationships between addition and subtraction and between multiplication and division, and known that multiplication can be modeled in terms of the area of a rectangle. They will also have made sense of rational numbers in terms of their *absolute size* (can locate them on the number line 7.NS.6), *relative size* (can compare them 7.NS.7), and *form* (can convert fractions to decimals 7.NS.2d). Geometrically speaking, students will have known types of triangles, analyzed the measures of their angles and sides, and found areas of triangles and special quadrilaterals 6.G.1. They will have constructed triangles 7.G.A.2 and drawn other polygons, including on coordinate plane given coordinates of vertices 6.G.3, and represented 3D objects two-dimensionally 6.G.4.
The part of the story happening in this unit	In this unit students will expand their understanding of operations, the number system, and geometry, while at the same time weave a few key threads through them. They will explore what squaring and finding square roots mean numerically (as inverse operations) and geometrically (as a relationship between the areas and side length of a square). To make sense of this relationship, they will construct and deconstruct squares –using concrete models, diagrammatic representations, and numerical expressions– and evaluate square roots. They will attempt to locate square roots on the number line, noting the challenge of being accurate and precise in doing so. In evaluating square roots students will notice numbers are difficult to express because they have decimals that neither repeat nor terminate, cannot be written as ratios of integers, and –given students' knowledge of the number system and geometry at this point– would be impossible to accurately locate on a number line. Students will be primed to think about irrational numbers in several ways: by discussing rational numbers with repeating decimals (i.e. how do we locate them on a number line or convert them to ratios?) and by considering how square roots might be constructed or represented visually (e.g. if $\sqrt{12}$ is an irrational number, how do create a square with that length?). They will also do so by looking at history, i.e. at how early Greek mathematicians believed that every number could be expressed as a ratio of integers, and this belief was challenged when their work with geometry –which they considered to be seamlessly tied to their number system– presented lengths that could not. Students will then transfer their numerical and geometric awareness of square numbers and -roots into a study of right triangles. They will learn about the relationships of the sides of right triangles, use the Pythagorean Theorem to solve for unknown side lengths, and explain a proof of the theorem and its converse. They will apply a working

	knowledge of the Pythagorean Theorem in new contexts: to find the distance between two points on a coordinate system and to solve real-life and mathematical problems in two- and three dimensions.
The story after this unit	Students will use what they learned about square numbers and roots will to explore cube numbers, cube roots, and exponential expressions of higher powers. The foundation laid here will also help them to think about the geometric meanings of such expressions (i.e. square numbers and roots as tied to two-dimensional figures and measures; cube numbers and roots as related to three-dimensional ones) and to use them in problem solving. Facility with the Pythagorean Theorem will enable students to determine unknown lengths in other contexts beyond the coordinate grid. In Unit 8.9, for instance, students will need to use the Theorem to be able to solve volume problems. It will also pave the way for deductive reasoning around other planar and solid figures, right-triangle trigonometry, as well as the unit circle in trigonometric functions.

UNIT FLOW SUMMARY

UNIT 8.8 (17-22 days)	**Square Roots, Irrational Numbers & the Pythagorean Theorem**
Section 0 (1 day)	**Diagnostic Pre-Unit Assessment**
Section 1 (1 day)	**Square Numbers & Square Roots**
Section 2 (2-3 days)	**Approximating Square Roots**
Section 3 (2-3 days)	**Irrational Numbers**
Section 4 (1 day)	**Mid-Unit Assessment**
Section 5 (2 days)	**The Pythagorean Theorem**
Section 6 (2-3 days)	**Proofs of the Pythagorean Theorem**
Section 7 (2-3 days)	**Applications of the Pythagorean Theorem**
Section 8 (2 days)	**The Pythagorean Theorem on the Coordinate System**
Section 9 (1-2 days)	**Visualizing Square Roots & Irrational Numbers** (Extension)
Section 10 (1 day)	**Summative Assessment**

Section 0: 1 day	**Diagnostic Pre-Unit Assessment**
Pre-Unit Assessment Targets	Diagnose students' ability to ● Multiply and divide rational numbers (esp. decimals, in preparation for work with squares and roots) 7.NS.2c ● Model multiplication of two numbers as a areas of rectangles and interpret areas of rectangles as products 3.MD.C.7 ● Recognize characteristics of right triangles and find their areas 6.G.1 ● Compare rational numbers, understand their absolute values 6.NS.7, and locate them on a number line 6.NS.6 ● Convert terminating decimals to fractions and vice versa 4.NF.C.6 ● Convert fractions to repeating decimals 7.NS.2d ● Add and subtract rational numbers (esp. positive and negative integers, in preparation for work on finding distances between points on the coordinate plane) 7.NS.1d ● Express rational numbers in scientific notation 8.EE.A.3
Sample Activity	*Diagnostic Pre-Unit Assessment*

Section 1: 1 day	**Square Numbers & Square Roots**
Mathematical Goal	● Understand the relationship between square numbers and roots 8.EE.A.2 ● Model that relationship diagrammatically and numerically
Narrative overview of section (and how the standards are achieved)	Students have learned that the area of a rectangle can model the product of two numbers with the side length being the factors. They use this awareness to look at what square numbers and square roots mean. They begin by attempting to construct squares of various side lengths, starting with counting numbers and moving to other positive rational numbers. Then they reverse the process and try to construct squares of certain area measures. In doing so they: 1) distinguish perfect squares (as squares with integral side lengths) from non-perfect squares (with non-integral side lengths), 2) begin to make sense of the value of a square root by connecting it to a visual model, and 3) explore how to find the root of squares by trial and error (they will not have used a calculator to find square roots at this point).
Sample Activity	*Square Dancing*, Mathalicious **In development**

	WHAT: What does it mean to "square" a side, and is it possible to find the root of every number? Students use pebbles to explore the relationship between the root and the square, and define the latter as the product of a number multiplied by itself: i.e. $n \times n = n^2$. By looking at examples and generalizing this rule, they look for and express regularity in repeated reasoning MP8. They transition from this discrete (pebble) model to an area model and discuss how to find the root of a square with a given area 8.EE.A.2. Along the way, students discover that for certain areas, e.g. 2, the roots are hard to "pin down;" they have decimals that continue forever and with no pattern 8.NS.1. Students estimate where numbers such as $\sqrt{2}$ fall on a number line 8.NS.2, and debate whether for those numbers, which they can't express as a fraction or a terminating decimal, precise squares can exist MP3. **WHY:** There are different ways to conceptualize a square. We can think of it as a shape, and as a number (e.g. the square, 4, is the product of 2 x 2). For thousands of years, mathematics was a primarily geometric undertaking; mathematicians such as the Pythagoreans used shapes to better reveal, understand, and articulate the underlying mathematical rules. Introducing terms like "square" and "root" in a concrete geometric context (e.g. pebbles and area) before moving to more abstract ones (symbols and equations) will help students make sense of their meanings by connecting new learning to previous knowledge.
Focus Standards	8.EE.A.2 Use square root to represent solutions to equations of the form $x^2 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares. 8.NS.2 Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).
Supporting Standards	8.NS.1 Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually.
Mathematical Practices	MP3, MP8

Section 2: 1-2 days	**Approximating Square Roots**
Mathematical Goals	• Evaluate square roots by approximation 8.NS.2 and using technology • Locate square roots on a number line 8.NS.2 • Use exponents and radical notation to express square roots (i.e. solutions to $x^2=p$) 8.EE.A.2
Narrative overview of section (and how the	Here students take a closer look at ways to evaluate square roots (particularly non-perfect ones), going from rougher estimates to finer ones. They start their estimation by referencing the closest perfect squares (e.g. they see

standards are achieved)	that $\sqrt{28}$ is between $\sqrt{25}$ and $\sqrt{36}$ but is closer to the former, so the root will be closer to 5 than to 6). Before using a calculator to find square roots, they notice that they could continue to refine the accuracy of their estimates by adjusting or extending the decimal values and fine-tuning their locations on the number line MP8. For example, in trying to find the $\sqrt{12}$ they might try to find $(3.4)^2$, $(3.46)^2$, then $(3.461)^2$, etc. Students also explore and consider the merits of other techniques for estimating square roots.
Sample Activity 1	*Estimating Square Roots*, Illustrative Mathematics **WHAT:** Students estimate the square root of a non-perfect integer to two-decimal places without a calculator 8.NS.2, 8.EE.2. They use the meaning of square roots to estimate the root of a larger non-perfect integer and to a greater accuracy. **WHY:** The purpose of this task is to move students away from a concrete understanding of square root –as the length of a side of a square with a given area, per the previous section– to seeing roots as *numbers*, whose values can be approximated. This cognitive shift is necessary for students to achieve 8.NS.2 –using rational approximations of irrational numbers– a little later. They will need to think about how to approach the task beyond realizing that the root is between *a* and *b* and is closer to one or the other MP1, and to engage in repeated reasoning MP8 as they try out a strategy or two.
Sample Activity 2	*Square Roots Go Rational*, NCTM Illuminations **WHAT:** Students model the *growth* of a sequence of perfect squares and their roots, record their data on a table, and observe the patterns that emerge in the number of squares that is added every time the side length of the square increases by one unit MP4. They use their analysis of the model to estimate the roots of non-perfect squares 8.NS.2. For instance, to find $\sqrt{27}$ they first notice the difference between the two closest square numbers: 36 (or 6^2) and 25 (or 5^2) are 11 square units apart. They also notice that 27 is 2 square units more than 25, and thus estimate the square root of 27 to be $\frac{2}{11}$ unit away from 5, or $5\frac{2}{11}$. They use their observations to write an algorithm for evaluating square roots. **WHY:** This activity equips students with a reliable process for estimating the value of a non-perfect square root with a rational number. While learning such an algorithm is not required by 8.NS.2, it does provide students who prefers following procedures with a more straightforward method than guess and check. Also, understanding why the algorithm works draws a nice connection to proportional reasoning. By organizing a given non-perfect square into the frame of a square, using that organization to estimate its value, and extending the method to any non-perfect square students look for and employ mathematical structures MP7 and practice repeated reasoning MP8.

Focus Standards	8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations. 8.EE.A.2 Use square root to represent solutions to equations of the form $x^2 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares.
Supporting Standards	
Mathematical Practices	MP1, MP4, MP7, MP8

Section 3: 2-3 days	**Irrational Numbers**
Mathematical Goals	• Understand what decimal expansion means and how it relates to rational numbers 8.NS.1 • Convert numbers w/ terminating and repeating decimals into ratios 8.NS.1 • Locate such numbers on a number line by first writing them as ratios • Recognize that some numbers are irrational and what makes them so 8.NS.1 • Approximate locations of irrational numbers on a number line 8.NS.2
Narrative overview of section (and how the standards are achieved)	Most numbers students have seen so far can be written as ratios of two integers. They have learned to convert numbers with terminating decimals into fractions 4.NF.C.6 (e.g. 0.72 = 72/100) and vice versa, including numbers with repeating decimals 7.NS.2d (e.g. 1/3 = 0.3333…). Here they examine more closely what this idea of rationality means by making sense of why division of two integers might generate repeating decimals and how to convert such numbers with into fractions. They also notice that by virtue of being able to be written as ratios these decimals can be easily expressed and -located on a number line. Students will then contrast these to irrational numbers, which have non-terminating and non-repeating decimals, cannot be converted into ratios (because their entire decimal expansion is long and unknown to us), and –in terms of their decimal expressions– can only be approximated. They will recognize that π and some square roots fall into this realm of numbers. Unlike a rational number, the "address" of an irrational number on a number line is both difficult to name and –until students learn how to construct square roots geometrically– tough to locate accurately. Students make another pass at approximating non-perfect square roots and thinking about the accuracy and precision of their approximations, with irrationality as a new lens.
Sample Activity 1	*Infinite Decimals*, EngageNY (**p. 82-97**)

	WHAT: Building on what they know about expansion of finite decimals, students write infinite repeating decimals in their expanded form (e.g. $0.8\overline{2} = \frac{8}{10^1} + \frac{2}{10^2} + \frac{2}{10^3} + \ldots$) and express them on the number line in terms of intervals of tenths, hundredths, thousandths, and so on. **WHY:** This lesson helps to bridge what students know about the structure of finite decimals and their ties to fractions to make sense of infinite decimals 8.NS.1, MP7, using repeated reasoning on the number line MP8. It invites them to consider what each step in the sequence of an infinite decimal means (that it approaches zero) and prepares students to understand how to approximate the decimal expansion of an irrational number.
Sample Activity 2	*Converting Repeating Decimals to Fractions*, Illustrative Mathematics **WHAT:** By this point students will have recognized that terminating decimals can be written as fractions with multiples of 10 as their denominators. They will also have noticed that some fractions convert to numbers with repeating decimals, though they would likely take these instances to be idiosyncrasies. Here students notice the conditions that prompt the repeating decimal patterns and generalize their observations into a method for reversed conversion (from decimals to ratios) 8.NS.1. **WHY:** This task prompts students to look for possible structures that connect patterns in decimals and their fractions to prime them to the conversion itself 8.NS.1. It also asks them to reason inductively MP7, MP8 and attend to precision MP6: by asking them to translate a certain variety of fractions, examine the patterns, and draw conclusions about their decimal results.
Sample Activity 3	*Converting Repeating Decimals to Fractions*, EngageNY (**p. 122-132**) **WHAT:** Students convert repeating decimals to fractions by writing and solving linear equations. They learn that, since every repeating decimal can be converted in fractions, they belong to the family of rational numbers. Numbers whose decimal expansions do not repeat or terminate cannot be converted; these numbers are thus called irrational. **WHY:** This task helps students reason about how to convert a repeating decimal to a fraction 8.NS.1 and why the conversion method works. Students use what they know about the laws of exponents 8.EE.A.3, 8.EE.A.4 to transform the decimals into simple one-variable linear equations that, when solved, produce numbers in fraction form. Precision –in identifying decimal values, performing calculations, and communicating each part of the process– is integral to the lesson MP6. Students also practice MP3 as they articulate and discuss their flow of

	reasoning.
Sample Activity 4	*Comparison of Irrational Numbers*, EngageNY (**p. 160-168**, excluding a problem involving the Pythagorean Theorem) **WHAT:** Students use rational approximations to compare the size of irrational numbers and locate them on the number line. **WHY:** In Section 2 students have evaluated square roots and approximated their locations on a number line. Here they revisit and fine-tune these skills with a clearer concept of decimal expansion in mind and in the larger context of both irrational and rational numbers 8.NS.2. In explaining their approximation and methods, students activate MP3.
Focus Standards	8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number. 8.NS.2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.
Supporting Standards	8.EE.A.3, 8.EE.A.4
Mathematical Practices	MP2, MP3, MP6, MP7, MP8

Section 4: 1 day	**Diagnostic Mid-assessment or Summative Assessment**
Pre-Unit Assessment Targets	Assess students' ability to: ● Evaluate and compare square roots. Approximate them on a number line. 8.NS.2. ● Convert numbers with terminating or repeating decimals to rational numbers. Locate such decimals on a number line. 8.NS.1. ● Distinguish irrational numbers from rational numbers and articulate what the distinctions mean. 8.NS.1.
Sample Activity	*Mid-Unit Assessment*

Section 5: 1-2 days	**The Pythagorean Theorem**
Mathematical Goals	• Understand the Pythagorean Theorem 8.G.B • Use the theorem to determine unknown lengths in right triangles 8.G.B.7
Narrative overview of section (and how the standards are achieved)	Students now explore the ideas of squares and roots in the context of right triangles. They learn about the unique relationship between the three sides of a right triangle, captured by $a^2 + b^2 = c^2$, and in which finding the values of the variables would require finding square roots. The focus of this section is on noticing that the relationship of the three sides is always true and that some right triangles –which we term "Pythagorean triples"– stand out as being special because of their sides are all integers. Students begin to use the theorem to find missing lengths in simple problems.
Sample Activity 1	*Sizing Up Diagonals*, David Cox **WHAT**: Given certain side lengths, students construct a series of rectangles and their diagonals. They measure the diagonals, analyze the three measurements (length, width, and diagonal), and try to find a set of operations (involving addition, multiplication, and square root) that would consistently relate the three numerical values. They then generalize and articulate the operations as an equation and test it against new rectangles of their choosing. **WHY**: This task guides students to approach the relationship coded by the Pythagorean Theorem through numerical experimentation. By constructing and measuring parts of rectangles and then relating the measures, students form a concrete experience –based on simple geometry and numbers– on which to later conceptualize $a^2 + b^2 = c^2$ 8.G.B. They engage in MP7 and MP8 by collecting data and examining them for potential structures and patterns. Since the operations that could relate all sets of triples may not be immediately apparent, in tackling the task students also practice MP1.
Sample Activity 2	*The Pythagorean Theorem*, Annenberg Learner – Geometry Course (p. **130-132**), or *Sizing Up Squares*, Illustrative Mathematics **WHAT**: The first task is a visual counterpart to Activity 1 above. Students analyze several diagrams in which a right triangle (drawn on dot paper) has a square built on each of its sides. They find the area of each square, show how they go about doing so (especially the tilted one built on the hypotenuse of the triangle), record the area measures, and try to come up with a relationship between the three numbers that hold for all diagrams MP7, MP8. The second task is similar in spirit, but here students verify –instead of induce– that the sum of the areas of the two

	smaller squares equal that of the largest. **WHY**: These tasks illustrate the Pythagorean relationship in geometric terms 8.G.B. They allow students to see that the squares of the two perpendicular side lengths of a right triangle add up to be the square of the length of its hypotenuse. Finding the areas of the orthogonal squares (built on the legs of the right triangle) is simple enough, but doing so for the tilted square is less straightforward. Students will need reason about it by deconstructing the square or constructing auxiliary lines, applying 6.G.1 and practicing MP1. The reasoning students engage in to calculate the areas of tilted squares will prime them for subsequent work on proving the theorem 8.G.B.6.
Sample Activity 3	*The Pythagorean Theorem: Square Areas*, Mathematics Assessment Project (**p. T-1–T-6**) **WHAT**: Students construct on dot paper a series of squares that are tilted at given x/y ratios. For instance, the first side of a 1/3 square can be drawn by connecting one point on the paper to another that is 1 unit over and 3 units up; the other three sides can be built on it. They then determine the area of each tilted square, organize the tilt-ratio and area data on a table, and analyze them for patterns MP7, MP8. **WHY**: This task guides students to derive the Pythagorean relationship 8.G.B through a combination of geometric, numeric, and algebraic reasoning MP2 and prepares them to prove the theorem 8.G.B.6 (see Activity 1 in the next section). While their explorations focus on the relationship on the side length of the square to its tilt ratio, students see that the relationship can be generalized to right triangles. If students have no prior experience with dissection of tilted quadrilaterals on a grid (such as in Activity 2 above), this task is also an exercise in MP1 and MP4.
Focus Standards	8.G.B. Understand the Pythagorean Theorem
Supporting Standards	8.G.B.6
Mathematical Practices	MP1, MP2, MP4, MP7, MP8.

Section 6: 2-3 days	**Proofs of the Pythagorean Theorem**
Mathematical Goals	• Explain a proof of the Pythagorean Theorem and its converse 8.G.B.6
Narrative overview of section (and how the standards are	After recognizing that $a^2 + b^2 = c^2$ holds in Section 5, students now think about why it does. They consider possible visual and algebraic explanations for the Pythagorean Theorem and its converse by studying some existing proofs. They will also look at how the theorem and its converse can be used to solve certain types of mathematical

achieved)	problems that go beyond finding a missing length.
Sample Activity 1	*The Pythagorean Theorem: Square Areas*, Mathematics Assessment Project (starting on **p. T-7**) **WHAT:** This task follows Activity 3 of the previous section. Having noticed that the squaring the x- and y values (i.e. the vertical and horizontal lengths) of a right triangle on a grid and adding them produces the square of the hypotenuse length, students now look for an explanation for why this is true. **WHY:** Given the preceding explorations, in which they deconstructed or reconstructed triangles and quadrilaterals to find their areas, students will now have a good geometric context and a set of tools for showing that $a^2 + b^2 = c^2$ is always true, regardless of what the values of *a*, *b*, and *c* might be 8.G.B.6. They activate MP2 as they transfer their quantitative work above to an abstract variable-only setting and MP3 as they articulate their reasoning.
Sample Activity 2	*Proving the Pythagorean Theorem*, Hilary Smith Risser **WHAT:** Students derive (and prove) the Pythagorean Theorem by constructing a composite figure of a few triangles, finding its area using two different methods, pairing the two expressions as an equation, and transforming the equation to illustrate the theorem 8.G.B.6. **WHY:** This task guides students through a simple yet intriguing path for proving the Pythagorean Theorem, motivating them to make multiple cross-topic and cross-grade connections. Students consolidate and reinforce familiar skills and ideas as they set up the premise of the proof (drawing and finding areas of triangles and quadrilaterals), determine details about figure they drew (applying facts about congruence, triangle angle sum, and straight angle), and write and rewrite equations to establish that $a^2 + b^2 = c^2$ (applying properties of equality and operations). Along the way, they practice abstract reasoning MP2, constructing a logical progression of statements MP3, and mathematical modeling MP4.
Sample Activity 3	*Proof of Pythagorean Theorem Using Similar Triangles*, CPM **WHAT:** In this task students decompose a right triangle into two smaller triangles, establish their similarity, and use that fact to show that the Pythagorean Theorem is true. **WHY:** This task integrates the work in this unit 8.G.B with what students have learned about similarity 8.G.A.4 in Unit 8.2 to confirm the Pythagorean Theorem and achieve 8.G.B.6. The prompts in the task requires students to carefully make sense of all parts of the problem MP1, attend to and communicate carefully the details in the drawn figures MP6, and reason abstractly MP2, in order to construct a logical argument for proving the theorem MP3. The

	step-by-step guidance and informal approach for the task help to make the proof accessible to Grade 8 students. For a less-scaffolded, more-formal task, consider *G-SRT Pythagorean Theorem*, Illustrative Mathematics.
Sample Activity 4	*Proving the Converse of Pythagorean Theorem*, Hilary Smith Risser **WHAT:** Students construct two triangles, establish both hypotheses and known facts about their measures, and discover –through contradiction– that a triangle whose sides meet satisfy $a^2 + b^2 = c^2$ cannot be a non-right triangle. **WHY:** This lesson enables students to verify firsthand the converse of the Pythagorean Theorem 8.G.B.6 through simple constructions and informal analysis. Students practice MP2 and MP3 as they form a line of reasoning using decontextualized facts and known theorems (the Pythagorean and SSS Congruence theorems) and as they look for inconsistency in an argument, all of which help to prime them for more formal proofs down the road.
Focus Standards	8.G.B.6. Explain a proof of the Pythagorean Theorem and its converse.
Supporting Standards	
Mathematical Practices	MP1, MP2, MP3, MP4, MP6

Section 7: 2 days	**Applications of the Pythagorean Theorem**
Mathematical Goals	• Use the theorem to determine unknown lengths in right triangles in mathematical and real-world problems 8.G.B.7.
Narrative overview of section (and how the standards are achieved)	Students now use the Pythagorean Theorem and its converse to solve a variety of two- and three-dimensional problems in mathematical and real-world contexts. Interesting and rich applications are abundant here. Students will benefit from exposure to as wide a range as time permits.
Sample Activity 1	*Spider Box*, Illustrative Mathematics **WHAT:** A spider wanders around the outside of a box, its path forming a series of diagonal lines on the surface of the box. Students are to visualize the path, map it out two-dimensionally (as a net), test their nets, and find out the total distance the spider has traveled.

	WHY: This task serves a dual purpose: to visualize and represent three-dimensional objects two-dimensionally (building on students' work on 6.G.4), and to apply the Pythagorean Theorem 8.G.B.7. By creating and testing concrete models to help them make sense of a problem before performing calculations, students engage in MP4 and MP5.
Sample Activity 2	*51 Foot Ladder*, Mathalicious **WHAT:** In 2005, when discussing the proposed border wall between the United States and Mexico, then- Arizona governor Janet Napolitano said, "You show me a fifty foot wall. I'll show you a fifty-one foot ladder." But would a 51-foot ladder actually be long enough to climb a 50-foot wall? In this lesson, students use the Pythagorean Theorem to explore how high ladders -- from those in the home to those on fire trucks -- can safely reach 8.G.B.7. **WHY:** When students first learn the Pythagorean Theorem, they often spend much of their time finding the missing length of the hypotenuse, *c*. However, it's important that they be able to find the length of a missing leg, too. This real-world activity will help students understand which length corresponds to which side, and how the Pythagorean Theorem can be applied in everyday life. In the process, they will have many opportunities to engage in MP1, as the problems are phrased in natural ways that require students to decipher given information, and MP4, as it's necessary to make simplifying assumptions and abstractions to create diagrams drawn from the real-world situation.
Sample Activity 3	*Viewmongus*, Mathalicious **WHAT:** Televisions (indeed, all screens) are measured diagonally. When Best Buy lists a TV as having a 42-inch screen, it means 42 inches on the diagonal. Yet even though this is advertised as the screen's size, the diagonal only tells us part of the story about how much screen there is. Students are given the aspect ratio and diagonal length, and asked to determine the screen's width, height, and area. For an 80-inch TV with an aspect ratio of 16:9 -- there are 16 units of width for every 9 units of height, i.e. the width is 1.78x the height -- students could use either $(16u)^2 + (9u)^2 = 80^2$ or $(1.78h)^2 + h^2 = 80^2$ to find the screen's dimensions. **WHY:** This lesson requires some higher-level problem solving, as students must access their understanding of ratios and similar figures 8.G.A.4 as well as apply the Pythagorean Theorem 8.G.B.7. There are multiple correct pathways to solving, and some require some sophisticated algebra such as correctly applying an exponent to a monomial base. Students must also to make sense of different variables. Depending on how much of the work is scaffolded by the teacher, this lesson is a great opportunity to engage in MP1.

Sample Activity 4	*Bird and Dog Race*, Illustrative Mathematics **WHAT:** If a bird and a dog were to race from one street intersection to another in a city arranged in square grid, who will win the race? Students visualize the paths that the bird and the dog would take, think about what they might need to compare to predict the winner, and devise a plan for calculating these. To predict a winner students need to interpret given information, project what information is needed to compare (time), analyze what calculations might need to be performed (first distance and then time), and finally carrying them out. **WHY:** This multi-step task serves several aims. It gives students a chance to apply the Pythagorean Theorem in combination with another familiar math concept (speed-time-distance relationship), prompting them to practice modeling MP4 and to bridge quantitative- and abstract reasoning MP2. Because solving the problem requires finding the distance between two points on a grid 8.G.B.7, it also primes students for working with the coordinate system 8.G.B.8 in the Section 8.
Sample Activity 5	*Applying the Pythagorean Theorem in Mathematical Context*, Illustrative Mathematics **WHAT:** Students determine if a triangle inside of a rectangle (formed by connecting three points on the perimeter of the rectangle)– is a right triangle by analyzing known measurements in the figure. To find out, they will need to think about conditions that determine a right triangle and devise the steps to see if those conditions exist MP4. **WHY:** This task pushes students to tap into their geometric toolkit, to choose a way to solve a mathematical problem MP4, and to offer a logical argument to back their decisions and conclusions MP3. Given the focus of the unit, students might be inclined to apply both the Pythagorean Theorem and its converse 8.G.B.7 immediately upon seeing the problem. But because the question can be answered without the Pythagorean Theorem, it can also be used to help students examine their methods.
Focus Standards	8.G.B.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
Supporting Standards	
Mathematical Practices	MP1, MP2, MP3, MP4, MP5

Section 8: 1-2 days	**The Pythagorean Theorem on the Coordinate System**

Mathematical Goals	• Use the Pythagorean Theorem to find the distance between two points on the coordinate system 8.G.B.8
Narrative overview of section (and how the standards are achieved)	Students transfer their developing knowledge of Pythagorean Theorem, which they used to find side lengths of discrete triangles, to a context. They apply the theorem to determine distances between two points, recognizing that right-angle relationships are inherent here given the grid of coordinate plane and building upon their prior work on slope triangles (which involved finding horizontal and vertical differences between points along a line).
Sample Activity 1	*What's the Distance?*, Illustrative Mathematics *In development* **WHAT:** Students calculate the distance between pairs of points –first from a graph and then from given coordinates that would be inconvenient to graph. They take their observations to generate an expression for calculating the distance between any two points given their coordinates 8.G.B.8. **WHY:** This activity aims to uncover the distance formula by guiding students to generalize the approach they use to find the segment length between specific pairs of points. The first part (in which graphs are supplied) invites students to notice that right triangles could be drawn, to find the horizontal and vertical lengths of the triangle, and then to apply the Pythagorean Theorem to solve the problems. They later transfer this strategy to points not that are not on graphs, increasing the cognitive load. As students perform repeated calculations and look for a systematic approach for figuring the distance between any two points, they engage in MP8.
Sample Activity 2	*Is This A Rectangle?*, Illustrative Mathematics **WHAT:** Students determine if a quadrilateral on a coordinate plane is a rectangle given the coordinates of its four vertices. To establish that the figure has four right angles they will need to construct a logical set of arguments to confirm or deny that condition. **WHY:** This task gives students a rich opportunity to integrate a number of concepts they have learned and strategically choose methods that would help them solve the problem –by using the Pythagorean Theorem 8.G.B.6, 8.G.B.7, 8.G.B.8, auxiliary constructions, knowledge of rigid motions and congruence, or a combination of other geometric and algebraic methods MP4. Given its open-ended nature, this multi-step problem stretches students' creativity and perseverance in problem solving MP1. Students also engage in MP3 as they attempt to back up their answer with a convincing and sound argument.
Focus Standards	8.G.B.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. 8.G.B.8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Supporting Standards	
Mathematical Practices	MP1, MP3, MP4

Section 9: 1-2 days	**Visualizing Square Roots & Irrational Numbers** (Extension)
Mathematical Goals	• Recognize one or more geometric representations / constructions of square roots.
Narrative overview of section (and how the standards are achieved)	This optional section is intended to deepen and consolidate students' understanding of square roots, irrational numbers, and the Pythagorean Theorem. In Section 1, students make sense of squares and roots as the inverse relationships. In Sections 2 and 3, they grapple with the magnitude of square roots and irrational numbers. Here they try to reconcile the geometric and numerical explanations of square roots (as a subset of irrational numbers): they consider whether square roots can be constructed precisely. If square roots cannot be exactly specified as a decimal, how can we draw squares with those side lengths and obtain squares whose areas are integers? Students learn that although it seems like it should be impossible to draw a side length of, say, $\sqrt{18}$ because of its irrational nature, it is possible to construct a square with an area of exactly 18 square units and obtain a side that is $\sqrt{18}$ units long, either by using the Pythagorean Theorem or another method.
Sample Activity 1	*One Hundred Squares*, Rachael Eriksen Brown and Alison Owens **WHAT:** In Part 1, students draw as many squares as possible whose areas are integers between 1 and 100 using dot paper or Geoboards (physical or digital). In Part 2, they express the lengths of a series of line segments connecting two points on a dot paper, either in terms of square roots or in terms of the slants of the lines (ratios of the vertical and horizontal distances between the two points). **WHY:** This task can serve many conceptual goals of the unit. Placed here, it loops students back to the opening exploration in Section 1 (on the connections between the size of a square root and both the number and the visual model used to represent it) and strengthens other key ideas since then, especially the meaning of irrational numbers 8.NS.1. By now students will have understood irrational numbers as being hard to pinpoint on a number line and having decimals that can only be approximated. They consider a potential tension: "We know the $\sqrt{4}$ is 2 because $2^2 = 4$, and because we can show it by drawing a square with a side length of 2 units. We know $\sqrt{5}$ is irrational because it cannot be expressed as a number with a terminating or repeating decimal. But does it also mean we cannot construct a square with an area that is exactly 5 square units? Is such a square possible?"

	To construct squares of particular units of area students need to persevere in problem solving MP1 and consider available tools MP5. For example, here they may be inclined to use the Pythagorean Theorem 8.G.B.7 to construct the squares in Part 1 or to use a pattern they noticed MP7, MP8 about the slopes of the sides of squares to find the lengths of segments in Part 2.
Sample Activity 2	*Right Triangle Spiral*, Math Forum **WHAT**: Students construct a series of right triangles, starting with an isosceles triangle whose legs are 1-unit long. They build each subsequent right triangle using the length of the hypotenuse of the preceding triangle for one leg and 1 unit for the other. They apply the Pythagorean Theorem to find the length of each triangle's hypotenuse and in so doing obtain an accurate measure of the square root of every integer starting from 2. (For ex. the hypotenuse of the 1st triangle = $\sqrt{1^2 + 1^2} = \sqrt{2}$; the hypotenuse of the 2nd triangle = $\sqrt{2 + 1^2} = \sqrt{3}$; hypotenuse of 3rd triangle = $\sqrt{3 + 1^2} = \sqrt{4}$; etc.) **WHY**: Placed here, this activity offers students another avenue to bridge the numerical and geometric expressions of irrational numbers 8.NS.1: using Pythagorean Theorem 8.G.B.7 and active composition. By creating the lengths of $\sqrt{2}$, $\sqrt{3}$, etc. in drawings, students engage in repeated reasoning MP8, recognize the structure behind the right triangles they are building MP7, and observe that the square root of any integer can be accurately constructed even when they cannot be easily written as decimals. NOTE: A tactile alternative to this pencil-and-paper activity is to have students construct their triangles from 1-unit-wide strips of paper, using that width as a starting point for measuring and cutting out all other triangles.
Focus Standards	8.NS.1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number.
Supporting Standards	8.G.B.7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
Mathematical Practices	MP1, MP4, MP5, MP7, MP8

Section 10: 1 day	**Summative Assessment**

Pre-Unit Assessment Targets	Assess students' ability to: ● Explain a proof of the Pythagorean Theorem and its converse 8.G.B.6. ● Find missing side lengths in right triangles in both two- and three-dimensional contexts 8.G.B.7. ● Find the distance and midpoint between any two points on the coordinate plane 8.G.B.8.
Sample Activity	*Unit Assessment*

Sample Lessons	8.NS.1	8.NS.2	8.EE.A.2	8.G.B	8.G.B.6	8.G.B.7	8.G.B.8	MP1	MP2	MP3	MP4	MP5	MP6	MP7	MP8
1.1	X	X	X							X					X
2.1		X	X					X							X
2.2		X	X								X			X	X
3.1	X													X	X
3.2	X												X	X	X
3.3	X									X			X		
3.4		X													
5.1				X				X						X	X
5.2				X	X			X						X	X
5.3				X	X			X	X		X			X	X
6.1					X				X	X					
6.2					X				X	X	X				
6.3				X	X			X	X	X			X		

6.4					X				X	X					
7.1						X					X	X			
7.2						X		X			X				
7.3						X		X							
7.4						X	X		X		X				
7.5						X				X	X				
8.1							X								X
8.2					X	X	X	X		X	X				
9.1	X					X		X			X			X	X
9.2	X					X		X				X		X	X