

On the Impossibility of General Parallel Fast-forwarding of Hamiltonian Simulation

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Hamiltonian simulation is one of the most important problems in the field of quantum computing. There have been extended efforts on designing algorithms for faster simulation, and the evolution time T for the simulation greatly affect algorithm runtime as expected. While there are some specific types of Hamiltonians that can be fast-forwarded, i.e., simulated within time $\mathcal{O}(T)$, for some large classes of Hamiltonians (e.g., all local/sparse Hamiltonians), existing simulation algorithms require running time at least linear in the evolution time T . On the other hand, while there exist lower bounds of $\Omega(T)$ *{circuit size}* for some large classes of Hamiltonian, these lower bounds do not rule out the possibilities of Hamiltonian simulation with large but "low-depth" circuits by running things in parallel. As a result, physical systems with system size scaling with T can potentially do a fast-forwarding simulation. Therefore, it is intriguing whether we can achieve fast Hamiltonian simulation with the power of parallelism.

In this work, we give a negative result for the above open problem in various settings. In the oracle model, we prove that there are time-independent sparse Hamiltonians that cannot be simulated via an oracle circuit of depth $\mathcal{O}(T)$. In the plain model, relying on the random oracle heuristic, we show that there exist time-independent local Hamiltonians and time-dependent geometrically local Hamiltonians on n qubits that cannot be simulated via an oracle circuit of depth $\mathcal{O}(T/n^c)$, where the Hamiltonians act on n qubits, and c is a constant. Lastly, we generalize the above results and show that any simulators that are geometrically local Hamiltonians cannot do the simulation much faster than parallel quantum algorithms.