Roll No…….. **Total No. of Printed Pages: 1**

Total No. of Questions: [11]

M.Sc. (Mathematics) (Semester – 4th)

PARTIAL DIFFERENTIAL EQUATIONS

Subject Code: MMAT1418

Paper ID: [19220519]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It carries 16 marks. It consists of 4 questions of 4 marks each.
2. Section B consist of 4 questions of 8 marks each. The student has to attempt any 3 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(4 marks each)**

Q1. Find the complete integral of $p^2 + q^2 = x + y.$

Q2. Solve $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = f\left(\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}, \frac{\partial u}{\partial z}\right)$ by Jacobi method.

Q3. State and prove Lagrange-Green's identity.

Q4. Classify the equation $u_{xx} + u_{yy} = u_z.$

Section – B **(8 marks each)**

Q5. Solve $px + qy = pq$ by Charpit method.

Q6. Reduce the following equation to a canonical form and hence solve it $u_{xx} - 2\sin x\, u_{xy} - \cos^2 x\, u_{yy} - \cos x\, u_y = 0.$

Q7. Use Duhamel's principle to solve the heat equation problem described by
$u_t(x,t) = ku_{xx}(x,t) + f(x,t);\ -\infty < x < \infty, t > 0,$
$u(x,0) = 0,\ -\infty < x < \infty.$

Q8. Find the Green's function for the equation $\frac{\partial^2 u}{\partial x \partial y} + u = 0.$

Section – C **(10 marks each)**

Q9. A string of length *l* is initially at rest in equilibrium position and each of its points is given the velocity, $\left(\frac{\partial y}{\partial t}\right)_{t=0} = b\sin^3\frac{\pi x}{l}.$ Find the displacement function.

Q10. Find the spherical polar form of the equation $\nabla^2 u = 0.$

Q11. Obtain the solution of the interior Dirichlet problem for a sphere using the Green's function method and hence derive the Poisson integral formula.