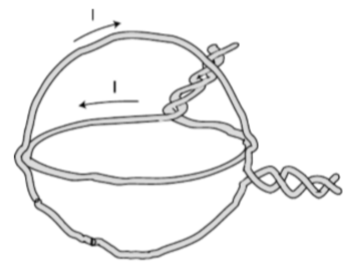
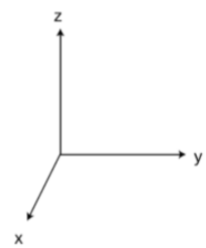
Magnetic Fields - Extension

Magnetic Fields Problems

1) The figure shows a nested pair of circular wire loops used to create magnetic fields. (The twisting of the leads is a practical trick for reducing the magnetic fields they contribute, so the fields are very nearly what we would expect for an ideal circular current loop.) The coordinate system below is to make it easier to discuss directions in space. One loop is in the $y - z$ plane, the other in the $x - y$ plane. Each of the loops has a radius of 1.0 cm, and carries 1.0 A in the direction indicated by the arrow.[1]





   a) The following equation for the magnetic field created by a single loop of current,

$$B = \frac{\mu_0}{2}\frac{I r^2}{\left(r^2+z^2\right)^{3/2}}$$

   where r is the radius of the loop and z is the distance of the point from the plane of the loop. Use it to calculate the magnetic field that would be produced by one such loop, at its centre.

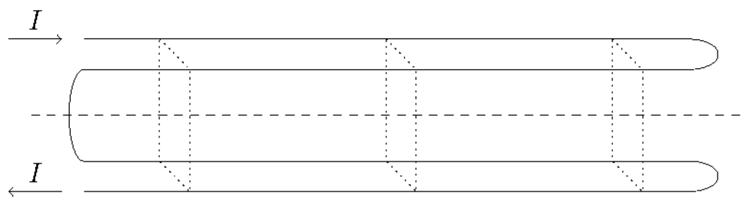
   b) Describe the direction of the magnetic field that would be produced, at its centre, by the loop in the $x - y$ plane alone.

   c) Do the same for the other loop.

   d) Calculate the magnitude of the magnetic field produced by the two loops in combination, at their common centre. Describe its direction.

[1] Taken from the Light and Matter textbook: Crowell, B. (2010) Light and Matter, http://www.lightandmatter.com/lm/

2) Four long wires are arranged, as shown, so that their cross-section forms a square, with connections at the ends so that current flows through all four before exiting. Note that the current is to the right in the two back wires, but to the left in the front wires. If the dimensions of the cross-sectional square (height and front-to-back) are *b*, find the magnetic field (magnitude and direction) along the long central axis.[2]



[2] Taken from the Light and Matter textbook: Crowell, B. (2010) Light and Matter, http://www.lightandmatter.com/lm/

Magnetic Fields - Extension

Magnetic Fields SOLUTIONS

1) **The figure shows a nested pair of circular wire loops used to create magnetic fields. (The twisting of the leads is a practical trick for reducing the magnetic fields they contribute, so the fields are very nearly what we would expect for an ideal circular current loop.) The coordinate system below is to make it easier to discuss directions in space. One loop is in the $y - z$ plane, the other in the $x - y$ plane. Each of the loops has a radius of 1.0 cm, and carries 1.0 A in the direction indicated by the arrow.[3]**

   a) **The following equation for the magnetic field created by a single loop of current,**

$$B = \frac{\mu_0 I}{2}\frac{r^2}{\left(r^2+z^2\right)^{3/2}}$$

where r is the radius of the loop and z is the distance of the point from the plane of the loop. Use it to calculate the magnetic field that would be produced by one such loop, at its centre.

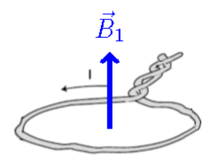
As z is the distance to the plane of the loop, $z = 0$ will place us right on top of the loop, giving us the magnetic field at the centre of the loop:

$$B = \frac{\mu_0 I}{2}\frac{r^2}{\left(r^2\right)^{3/2}} = \frac{\mu_0 I}{2}\frac{r^2}{r^3} = \frac{\mu_0 I}{2r} = \frac{(4\pi\times10^{-7}\ N/A^2)(1.0\ A)}{2\ (0.01\ m)} = 62.8\ \mu T$$

[3] Taken from the Light and Matter textbook: Crowell, B. (2010) Light and Matter, http://www.lightandmatter.com/lm/

b) Describe the direction of the magnetic field that would be produced, at its centre, by the loop in the $x - y$ plane alone.

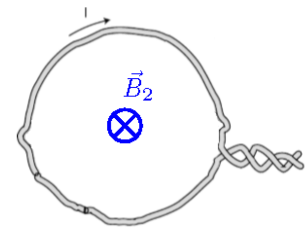
By the right hand rule, we can see that the magnetic field produced by the horizontal loop is in the (positive) z direction, $\vec{B}_1 = (0,\ 0,\ 62.8\ \mu T)$.



c)

d) Do the same for the other loop.

The right hand rule for the vertical loop tells us that the magnetic field it generates is in the negative x direction,

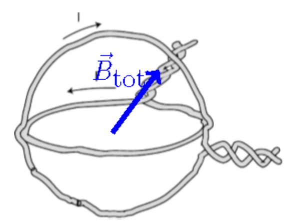


$$\vec{B}_2 = (-\ 62.8\ \mu T,\ 0,\ 0).$$

e) Calculate the magnitude of the magnetic field produced by the two loops in combination, at their common centre. Describe its direction.

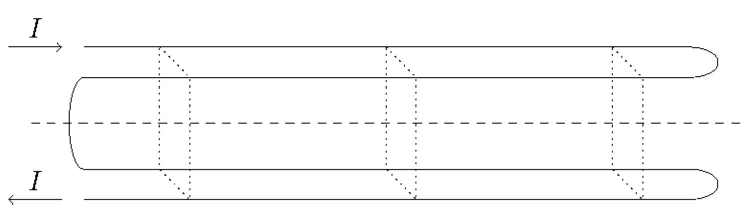
The total magnetic field at the centre of both loops is:

$$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2 = (-\ 62.8\ \mu T,\ 0,\ 62.8\ \mu T)$$

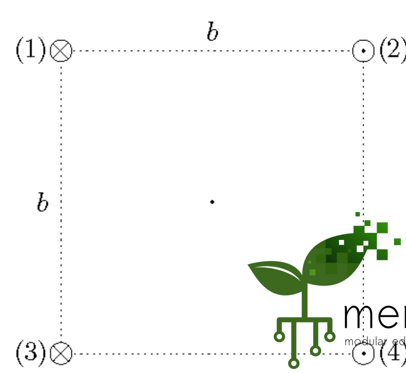


The magnetic field points at an angle, with an upward (positive z) component and a component that goes into the vertical loop (negative x direction).

2) Four long wires are arranged, as shown, so that their cross-section forms a square, with connections at the ends so that current flows through all four before exiting. Note that the current is to the right in the two back wires, but to the left in the front wires. If the dimensions of the cross-sectional square (height and front-to-back) are b, find the magnetic field (magnitude and direction) along the long central axis.[4]



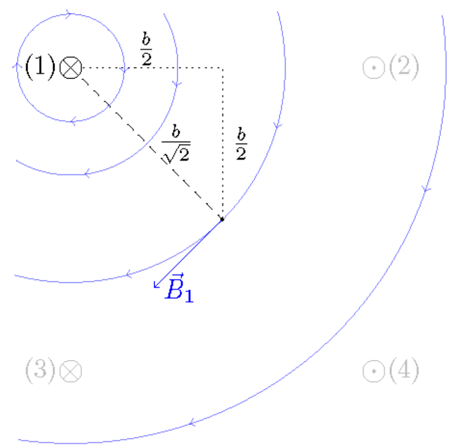
If we look at the wires from the left side we would see their arrangement in the shape of a square. We want to



[4] Taken from the Light and Matter textbook: Crowell, B. (2010) Light and M
http://www.lightandmatter.com/lm/

calculate the magnetic field at the centre of the square, which will be the sum of the magnetic fields generated by each of the four pieces of wire.

Let's start with the top left wire (labelled 1) and ignore the rest for the moment. By the right hand rule, we know that the magnetic field lines will rotate clockwise around the wire, so at the centre of the square, the magnetic field will point downwards and to the left at a 45° angle. The magnitude of this magnetic field is



$$B = \frac{\mu_0 I}{2\pi d}$$

where d is the distance between the wire and the centre of the square, which we can find by using the Pythagorean theorem

$$d = \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{b}{2}\right)^2} = \sqrt{2\frac{b}{4}^2} = \frac{b}{\sqrt{2}}$$

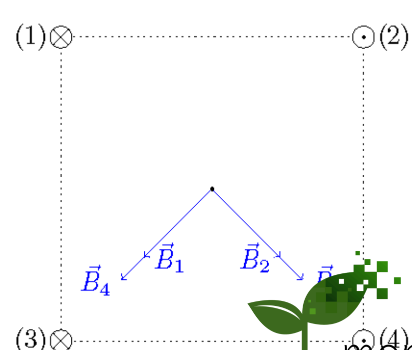
This gives us a magnetic field of

$$B_1 = \frac{\sqrt{2}\mu_0 I}{2\pi b}\ (pointing\ 45°\ down\ and\ to\ the\ left)$$

Or, in vector format

$$\vec{B}_1 = \frac{\sqrt{2}\mu_0 I}{2\pi b}(-\ \cos cos\ 45°\ , -\ \sin sin\ 45°\ ) = \frac{\sqrt{2}\mu_0 I}{2\pi b}\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) = \left(-\frac{\mu_0 I}{2\pi b}, -\frac{\mu_0 I}{2\pi b}\right)$$

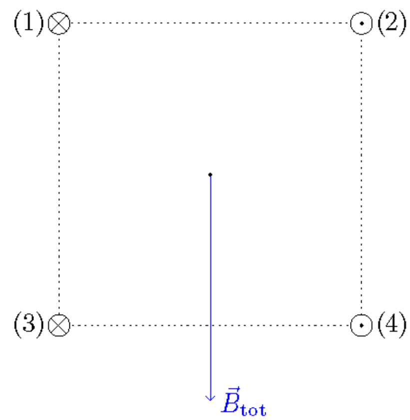
We can repeat similar calculations for the other wires, and we will see that they all have the same magnitude (as they are at the same distance from the centre of the



square). From the right hand rule, we also see that $\vec{B}_4$ points in the same direction as $\vec{B}_1$, and that $\vec{B}_2$ and $\vec{B}_3$ point 45° down and to the right:

$$\vec{B}_1 = \vec{B}_4 = \left(-\frac{\mu_0 I}{2\pi b}, -\frac{\mu_0 I}{2\pi b}\right)$$

$$\vec{B}_2 = \vec{B}_3 = \left(\frac{\mu_0 I}{2\pi b}, -\frac{\mu_0 I}{2\pi b}\right)$$



This gives us a total magnetic field of

$$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2 + \vec{B}_3 + \vec{B}_4 = \left(0, -\frac{2\mu_0 I}{\pi b}\right)$$

We see that the x components cancel, so the total magnetic field points downwards.