Assignment

Complex Analysis I

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[For multiple choice questions, no credit will be given if the problem is not solved]

1. The function $f(z)=\dfrac{e^{z^2}}{z^4}$ has

(a) an essential singularity at z=0, (b) a pole of order 4 at z=0,

(c) a simple pole at z=0, (d) no singularity at z=0.

2. (a) Define analytic function in a region R of the complex plane. State a set of necessary conditions for a function f(z)=u(x,y)+iv(x,y) to be analytic in R.

(b) Prove that $u(x,y)=e^{-x}(x \sin y - y \cos x)$ is harmonic. Using Milne Thomson method find an analytic function f(z)=u+iv, where u is given as above.

3. State and prove Cauchy's integral formula. Use this evaluate $\displaystyle\int_C \frac{e^{2z}+\sin z+z^2}{(z+1)^4}dz$, where C is the circle |z|=2.

4. Expand f(z)=1/(z+1)(z+3) in Laurent's series valid for 0<|z+1|<2.

5. Find a bilinear transformation which maps the points z=1, z=i and z=-1 into the points w=i, w=0 and w= -i. Also find the image |z|≤1 under the transformation.

6. The value of the integral $\displaystyle\oint_C \frac{2z}{z-4}dz$, where C: |z-1|=2 is

(a) 2πi, (b) πi, (c) 4πi, (d) 0.

7. Determine the singularities of the function cotz.

8. Obtain the singularities of f(z) = 1/(sinz-cosz)

9. Find a bilinear transformation which maps the points z=1, z=-i and z=-1 into the points w=0, w=i and w=∞ and show further that the area of the circle |z|=1 is transformed into the half plane above the real axis in w- plane.

10. Let α and β be a complex numbers with |α|<|β|. Let f(z)=$\sum(3\alpha^{n} - \beta^{n})z^{n}$; determine the radius of convergence of f(z).

11. Let f and g be entire functions such that |f(z)|<|g(z)| for all z∈C. Prove that f(z)=kg(z) for some constant k with |k|<1.

12. Find $z\in\overline{D} max|f(z)|$, where f(z)=cosz with D={z=x+iy: 0<x<2π, 0<y<2π }.

13. Let G={z=x+iy: 0≤x≤1 and 0≤y≤π} and f(z)=e^z, for all z∈G. Find the maximum and minimum values of Imf(z) in G.