HAI PHONG DEPARTMENT OF EDUCATION AND TRAINING
LEQUYDON HIGHT SCHOOL

LE QUY DON ENGLISH MATHEMATICS AND SCIENCE COMPETITION FOR GRADE 10 STUDENTS
SCHOOL YEAR: 2020 – 2021
Time allowance: 120 minutes

PART I: MULTIPLE – CHOICE (100 marks)

Question 1: The three digit number $\overline{2a3}$ is added to the number 326 to give the three digit number $\overline{5b9}$. If $\overline{5b9}$ is divisible by 9, then $a+b$ equals:

A. 2; B. 4; C. 6; D. 8; E. 9.

Question 2: When the base of a triangle is increased 10% and the altitude to this base is decreased 10% the change in are is

A. 1% increase; B. $\frac{1}{2}\%$ increase C. 0%; D. $\frac{1}{2}\%$ decrease E. 1% decrease

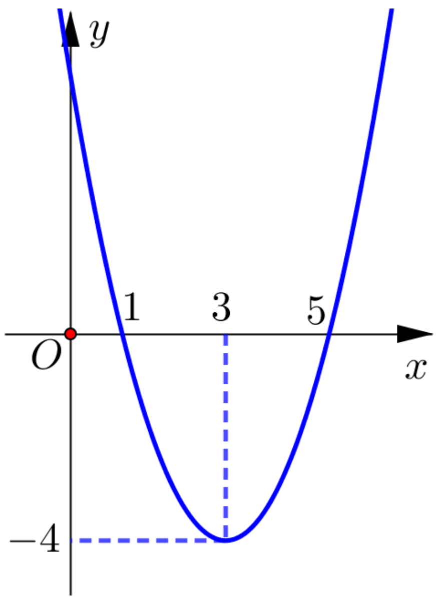
Question 3: How many integers m are there such that the function $f(x) = mx^2 + 2(m-6)x + 2$ increases on the interval $(-\infty;2)$?

A. 1; B. 2; C. 3; D. 4; E. 5.

Question 4: Given triangle ABC. Let J, E be two points so that $2\overrightarrow{JA} + 5\overrightarrow{JB} + 3\overrightarrow{JC} = \overrightarrow{0}$ and $\overrightarrow{AE} = k\overrightarrow{EB}$. If C, E, J are collinear then k is

A. $k = -\frac{2}{5}$; B. $k = \frac{2}{7}$; C. $k = -\frac{2}{7}$; D. $k = \frac{2}{5}$; E. None of the above

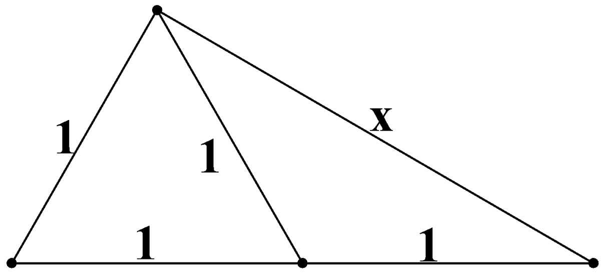
Question 5: The function $y = f(x) = x^2 - 6x + 5$ has the below graph.



Let S be the sum of all integers m such that the equations $(x-1)|x-5|+m=0$ has two distinct roots. Choose the true result.

A. $S=-6$; B. $S=-4$; C. $S=4$; D. $S=5$; E. $S=6$

Question 6: What is the value of x in the diagram?



A. 1; B. $\sqrt{2}$; C. 2; D. $\sqrt{3}$; E. $\sqrt{5}$.

Question 7: Write the numbers 8^{2008} and 125^{2008} cosecutively. What is the number of decimal digits of the resulting numbers ?

A. 6024; B. 6026; C. 6023; D. 6025; E. 6027;

Question 8: The parabola $y=ax^2+bx+c\ (P)$ has vertex $I\left(\frac{3}{2};\frac{1}{4}\right)$ and intersects the *Ox*-axis at two distinct 2 points $M(x_1;0),N(x_2;0)$ such that $x_1^3+x_2^3=9$. The value of $P=abc$ is

A. $P=4$; B. $P=6$; C. $P=3$; D. $P=8$; E. $P=12$

Question 9: Given ΔABC with right angle A and $AB=a,BC=2a$. Let d be the line through A and parallel to BC. If the point M move on the line d, then the smallest value of $\left|\overrightarrow{MA}+2\overrightarrow{MB}-\overrightarrow{MC}\right|$ is

A. $2a\sqrt{3}$; B. $a\sqrt{3}$; C. $\frac{a\sqrt{3}}{4}$; D. $\frac{a\sqrt{3}}{2}$; E. None of the above

Question 10: If the perimeter of rectangle $ABCD$ is 20 inches, the least value of diagonal AC, in inches, is:

A. 5; B. $\sqrt{50}$; C. 10; D. $\sqrt{200}$; E. 100.

PART II: COMPOSE (200 marks)

Problem 1. If the side lengths a,b and c of a triangle satisfy the conditions $a+b-c=2,$ and $2ab-c^2=4,$ show that the triangle is equilateral.

Problem 2. Find positive integers p and q that are relatively prime to each other such that $p+p^2=q+q^2+3q^3$.

Problem 3. Given aquilateral triangle ABC with sides of length 2*a*. Let M be a point satisfy the condition $\overrightarrow{MB}+3\overrightarrow{MC}=\vec{0}$. Let *K*, *H* be the feet of the perpendiculars drawn from *M* to *AB*, *AC*. Determine the length of vector $\vec{u}=\overrightarrow{MK}+\overrightarrow{MH}$.

Problem 4. Let $a,b,$ and c be positive real numbers such that $a+b+c=\frac{3}{2}$. Find the minimum value of the expression $P=\frac{1+b}{1+4a^2}+\frac{1+c}{1+4b^2}+\frac{1+a}{1+4c^2}$.

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HONGBANG ENGLISH MATHEMATICS AND SCIENCE COMPETITION FOR GRADE 10 STUDENTS
SCHOOL YEAR: 2020 – 2021
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Question 1: The three digit number $\overline{2a3}$ is added to the number 326 to give the three digit number $\overline{5b9}$. If $\overline{5b9}$ is divisible by 9, then $a+b$ equals:

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Question 2: When the base of a triangle is increased 10% and the altitude to this base is decreased 10% the change in are is

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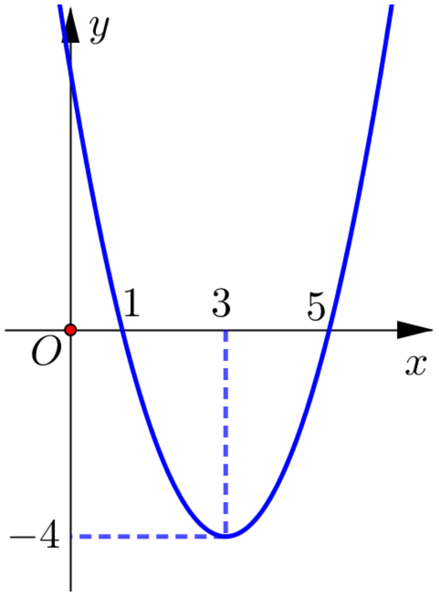
Question 3: How many integers m are there such that the function $f(x)=mx^2+2(m-6)x+2$ increases on the interval $(-\infty;2)$?

A. 1; B. 2; C. 3; D. 4; E. 5.

Question 4: Given triangle ABC. Let J, E be two points so that $2\overrightarrow{JA}+5\overrightarrow{JB}+3\overrightarrow{JC}=\vec{0}$ and $\overrightarrow{AE}=k\overrightarrow{EB}$. If C, E, J are collinear then k is

A. $k=-\frac{2}{5}$; B. $k=\frac{2}{7}$; C. $k=-\frac{2}{7}$; D. $k=\frac{2}{5}$; E. None of the above

Question 5: The function $y = f(x) = x^2 - 6x + 5$ has the below graph.



Let S be the sum of all integers m such that the equations $(x-1)|x-5| + m = 0$ has two distinct roots. Choose the true result.

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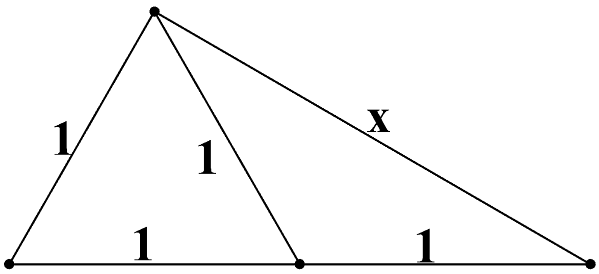

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Question 8: The parabola $y = ax^2 + bx + c\ (P)$ has vertex $I\left(\frac{3}{2};\frac{1}{4}\right)$ and intersects the *Ox*-axis at two distinct 2 points $M(x_1;0), N(x_2;0)$ such that $x_1^3 + x_2^3 = 9$. The value of $P = abc$ is

A. $P = 4$; B. $P = 6$; C. $P = 3$; D. $P = 8$; E. $P = 12$

Question 9: Given ΔABC with right angle A and $AB = a, BC = 2a$. Let d be the line through A and parallel to BC. If the point M move on the line d, then the smallest value of $\left|\overrightarrow{MA} + 2\overrightarrow{MB} - \overrightarrow{MC}\right|$ is

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Question 10: If the perimeter of rectangle $ABCD$ is 20 inches, the least value of diagonal AC, in inches, is:

A. 5; B. $\sqrt{50}$; C. 10; D. $\sqrt{200}$; E. 100.

PART II: COMPOSE (200 marks)

Problem 1. If the side lengths a, b and c of a triangle satisfy the conditions $a + b - c = 2,$ and $2ab - c^2 = 4,$ show that the triangle is equilateral.

Problem 2. Find positive integers p and q that are relatively prime to each other such that $p + p^2 = q + q^2 + 3q^3$.

Problem 3. Given aquilateral triangle ABC with sides of length 2a. Let M be a point satisfy the condition $\overrightarrow{MB} + 3\overrightarrow{MC} = \overrightarrow{0}$. Let K, H be the feet of the perpendiculars drawn from M to AB, AC. Determine the length of vector $\vec{u} = \overrightarrow{MK} + \overrightarrow{MH}$.

Problem 4. Let $a, b,$ and c be positive real numbers such that $a + b + c = \frac{3}{2}$. Find the minimum value of the expression $P = \frac{1 + b}{1 + 4a^2} + \frac{1 + c}{1 + 4b^2} + \frac{1 + a}{1 + 4c^2}$.

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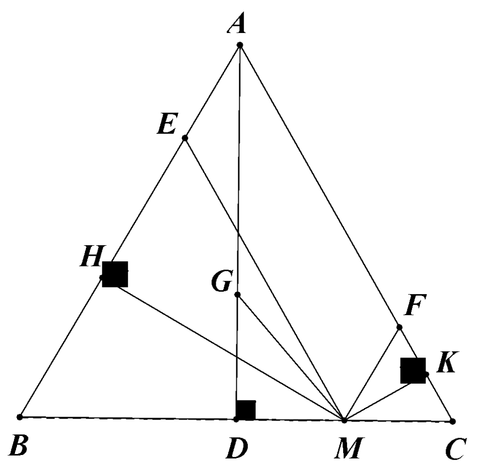
SCHOOL YEAR: 2020 – 2021

PART I: MULTIPLE – CHOICE (100 marks)

ANSWERS AND MARKS

Question	1	2	3	4	5	6	7	8	9	10
Answer	C	E	C	A	B	D	D	B	C	B
Marks	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0	10,0

PART II: COMPOSE (200 marks)

Problem	Answer	Marks
1 (50 marks)	Take the square of $a+b-2=c$ and use $c^2=2ab-4$ to get $a^2+b^2-4a-4b+8=0$	**20**
	that is, $(a-2)^2+(b-2)^2=0$. This forces $a=2$ and $b=2$.	**20**
	So $c=a+b-2=2$. The conclusion follows.	**10**
2 (50 marks)	We observe that $p+p^2$ is even for any positive integer p. Therefore in any solution q must be even. By rewriting the given equation as $$p(1+p)=q(1+q+3q^2)$$ we obtain $(1+q+3q^2)\,\vdots\,p$ and $p+1\,\vdots\,q$.	**20**
	We may also rewrite the equation as $(p-q)(p+q+1)=3q^3$ which implies $p>q$. Since $GCD(p-q,q)=GCD(p,q)=1$, we can conclude that $(p+q+1)\,\vdots\,q^3$ and therefore $q^3-q-1\le p\le 1+q+3q^2$	**15**
	which leads to $q^3-3q^2-2q-2\le 0$ Thus $q\le 3$ and since q is positive and even, $q=2$. We obtain $(p,q)=(5;2)$ as the only solution.	**15**
3 (50 marks)		**20**

	Through *M*, draw parallel line to *AC* and intersects *AB* at point *E*. Through *M*, draw parallel line to *AB* and intersects *AC* at point *F*. We have aquilateral triangles BME, CMF and *H*, *K* are midpoints of segments *BE*, *CF*, respectively. $\Rightarrow \overrightarrow{MH} = \frac{1}{2}\left(\overrightarrow{MB} + \overrightarrow{ME}\right)$, $\overrightarrow{MK} = \frac{1}{2}\left(\overrightarrow{MC} + \overrightarrow{MF}\right)$ On the other hand, *MEAF* is a paralleogram $\Rightarrow \overrightarrow{ME} + \overrightarrow{MF} = \overrightarrow{MA}$ Then $\vec{u} = \overrightarrow{MK} + \overrightarrow{MH} = \frac{1}{2}\left(\overrightarrow{MA} + \overrightarrow{MB} + \overrightarrow{MC}\right) = \frac{3}{2}\overrightarrow{MG}$, with *G* is center of triangle ABC. Let *D* is midpoint of *BC* $\Rightarrow MD = \frac{BC}{4} = \frac{a}{2}; AD = \frac{BC\sqrt{3}}{2} = a\sqrt{3}$. We have *MDG* is right triangle $\Rightarrow$ $MG = \sqrt{GD^2 + DM^2} = \sqrt{\left(\frac{a}{2}\right)^2 + \left(a\sqrt{3}\right)^2} = \frac{a\sqrt{13}}{2}$. Vậy $\left\|\vec{u}\right\| = \left\|\overrightarrow{MK} + \overrightarrow{MH}\right\| = \frac{3}{2}MG = \frac{3}{2}.\frac{a\sqrt{13}}{2} = \frac{3\sqrt{13}a}{4}$.	
	From $MC \perp FP$ we infer that Q is the orthocenter of triangle MPF So $PQ \perp FM$.	**10**
	Traingles POQ and NOE are equal, so $QP // EN$. Consequently, $FM \perp EN$.	**20**
4 (50 marks)	We have $(2a-1)^2 \geq 0 \Leftrightarrow 1 + 4a^2 \geq 4a$ Hence $\frac{1+b}{1+4a^2} = b + 1 - \frac{4a^2(b+1)}{1+4a^2} \geq b + 1 - \frac{4a^2(b+1)}{4a} = b + 1 - a - ab$ (1)	**15**
	Similiarly, we get $\frac{1+c}{1+4b^2} \geq c + 1 - b - bc$ (2); $\frac{1+a}{1+4c^2} \geq a + 1 - c - ca$ (3)	**15**
	Adding the above inequalities yields (1), (2), (3), we get	**20**

	$P \geq 3 - (ab + bc + ca) \geq 3 - \frac{(a + b + c)^2}{3} = \frac{9}{4}$ Thus $MinP = \frac{9}{4}$ when $a = b = c = \frac{1}{2}$.	