28. If $y = x\sin(x+y)$, then the value of $y' =$ _____

29. $\frac{d}{dx}\left[x^2 + 3xe^{2x}\right] =$ _____

A. $2x + 6xe^{2x} + 3e^x$ B. $2x + 6xe^{2x}$

C. $2x + 6xe^{2x} + 3e^{2x}$ D. $2x + 3e^{2x}$ E. *None of the above*

30. If $y = x^2\sin\left(\frac{1}{x}\right)$, then $\frac{dy}{dx} =$ _____

A. $2x\sin\left(\frac{1}{x}\right) - x^2\cos\left(\frac{1}{x}\right)$ B. $\frac{-2}{x}\cos\left(\frac{1}{x}\right)$

C. $2x\cos\left(\frac{1}{x}\right)$ D. $2x\sin\left(\frac{1}{x}\right) - \cos\left(\frac{1}{x}\right)$ E. $-\cos\left(\frac{1}{x}\right)$

31. Let $y = \sqrt{3-2x}$, then $\frac{dy}{dx} =$ _____

A. $\frac{1}{2\sqrt{3-2x}}$ B. $-\frac{1}{\sqrt{3-2x}}$ C. $-\frac{(3-2x)^{\frac{3}{2}}}{3}$ D. $-\frac{1}{3-2x}$ E. $\frac{2}{3}(3-2x)^{\frac{1}{2}}$

32. At what value of x is $f(x) = \frac{1}{3}x^3 - 2x^2 - 3x + 5$ have a relative minimum?

A. -1 *only* B. 0 *only* C. 1 *only* D. 3 *only* E. -1 *and* 3.

33. If $f'(x) = \frac{(x+1)x^2}{(x-1)^{\frac{1}{3}}}$, then which interval(s) is the continuous function $f(x)$ is increasing?

A. $(-1,1)$ B. $(-\infty,-1)\cup(1,\infty)$ C. $(-\infty,0)\cup(1,\infty)$ D. $(-\infty,-1)\cup(0,\infty)$ E. $(1,\infty)$

34. Find the derivative $\frac{dy}{dx}$. Simplify your answer, if possible

a) $y = \left(\frac{3x-2}{2x+1}\right)^5$ b) $y = \frac{\left(x^2+2\right)^2}{\left(x^2+x^{-1}\right)^3}$

35. Differentiate the following trigonometric functions.

a. $f(x) = \cot(4x+5)$ b. $f(x) = \cosh^{-1}(\sec x)$ c. $f(x) = \frac{1-\tan 2x}{\sec 2x}$ d. $f(x) = \sqrt{\cos 2x + 1}$

36. Find the derivative for the following functions

(a) $f(x) = \ln\sqrt{1+\sin^2 x}$ (b) $f(x) = e^{\ln\sqrt{x}}$ (c) $f(x) = \sin\left(\ln\left(\tan^{-1}x\right)\right)$

Addis Ababa Science and Technology University

Department of Mathematics

Applied Mathematics IB (Worksheet V)

1. Find the anti-derivative of $f(x)=\frac{1}{1+e^x}$.
2. Evaluate the following integrals

   a. $\int \frac{1}{x\ln x}dx$ b. $\int 3x^2\sqrt{9x^3+1}dx$ c. $\int \frac{x+3}{\sqrt{x-1}}dx$ d. $\int \frac{1}{1+e^{-x}}dx$

   e. $\int x\cos(4x^2+1)dx$ f. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}(e^{\sqrt{x}}+1)}dx$ g. $\int \frac{xdx}{\sqrt{9-x^4}}$ h. $\int \frac{2x+1}{\sqrt{1-x^2}}$
3. Solve the integration of the following

   a. $\int x\sin x dx$ b. $\int x^2\cos x dx$ c. $\int x\sin(2x)dx$ d. $\int \sin(\ln x)dx$

   e. $\int e^{\sqrt{x}}dx$ f. $\int \cos\sqrt{x}dx$ g. $\int x5^x dx$ h. $\int x^3e^{x^2}$
4. Find the following integrals

   a. $\int \cos 4x\cos 3x dx$ b. $\int \cos^3 x\sin^4 x dx$ d. $\int \frac{\cos(1/x)}{x^2}dx$

   d. $\int e^x\sin(e^x)dx$ e. $\int(\sin x+\cos x)^2 dx$ f. $\int e^x\cos(2e^x)\sin(3e^x)dx$
5. Evaluate the following integrals

   a. $\int \frac{4x-1}{(x-1)(x+2)}dx$ b. $\int \frac{1}{(x-1)^2(x+4)}dx$ c. $\int \frac{2x+3}{x^3+3x}dx$ d. $\int \frac{\sqrt{x+4}}{x}dx$
6. Solve the integration of the following

   a. $\int \frac{x}{\sqrt{1-x^2}}dx$ b. $\int \tan^{-1}x dx$ c. $\int \frac{dx}{\sqrt{2x-x^2}}$ d. $\int \frac{dx}{x^3\sqrt{x^2-16}}$
7. Evaluate the following definite integrals

   a. $\int_0^1 \frac{xdx}{(x^2+1)^2}$ b. $\int_4^5 \frac{dx}{(x-3)^2}$ c. $\int_0^1 x^2\ln(2+x^3)dx$

   d. $\int_{\pi/4}^{\pi/2}\cos x dx$ e. $\int_0^2(2-|x-1|)dx$ f. $\int_{-1}^2|x^2-x|dx$
8. Evaluate the following improper integrals

   a. $\int_0^1 \frac{dx}{x^3+x}$ b. $\int_0^\infty \frac{dx}{e^x+1}$ c. $\int_{-\infty}^\infty \frac{dx}{x^2-6x+10}$
9. The region bounded by the curves $y=x$ *and* $y=x^2$ is rotated about the x-axis. Find the volume of the resulting solid.
10. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y=x(x-1)^2$ *and* $y=0$.
11. Find the length of the curve $y^2=x^3$ *b/n the points* $(1,1)$ *and* $(4,8)$
12. Find the length of the curve $y=\ln(\cos x)$

Name:

Part-I: Give short answer only on the space provided

1. If $\frac{d}{dx}(f(2x)) = x^2$ then $f'(x) =$ ____
2. The critical number(s) of $f(x) =$ [illegible] is/are ____
3. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, the smallest possible va[lue of $f(4)$] is ____
4. If $\lim_{x \to \infty} (\ldots)^x = e^6$, then the value of a is ____
5. Find the point on the line $y = 4x + 7$ that is closest to the origin. ____
6. The value of $\int_0^3 |x^2 - 4| dx$ is ____
7. Let $G(x) = \int$ [illegible] dt then $G'(x) =$ ____
8. If $f(1) = 12$, f' is continuous and $\int_1^4 f'(x)dx = 11$ then $f(4)$ ____
9. Let $f'(x) = xf(x)$ and $f(0) = 2$ then $f'(2) =$ ____
10. The anti derivative of $f(x) = \frac{1}{1+\sin x}$ is ____

Show all the necessary steps clearly and neatly on the

$f(x) = \frac{2x^2}{x^2-1}$ then find

[illegible] horizontal asymptote

[illegible] monocity

[illegible]d inflection point/s

[illegible] x^3 and the line x

ive short answer only on the space provided

If $\frac{d}{dx}(f(2x)) = x^2$ then $f'(x) =$

The critical number(s) of $f(x) = x^2e^{-3x}$ is/are

If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, the sma

is

If $\lim_{x\to\infty}\left(\frac{x+a}{x-a}\right)^x = e$, then the value of a is

Find the points on the line $y = 4x + 7$ that is closes

The value of $\int_0^3 |x^2 - 4|dx$ is

Let $G(x) = \int_{tanx}^{3} \frac{t^2}{1+t^2}dt$ then $G'(x) =$

If $f(1) = 12$, f' is continuous and $\int_1^4 f'(x)dx = 1$

Let $f'(x) = xf(x)$ and $f(0) = 2$ then $f'(2) =$

The anti derivative of $f(x) = \frac{1}{1+sinx}$ is

ow all the necessary steps clearly and neatl

d. $f(x) = \cos^2(\ln x)$ (e) $f(x) = \ln(\ln x)$ f) $f(x)=10^{10x}$ g) $f(x)=\frac{3^x}{5^x}$

37. Calculate $\frac{dy}{dx}$ at the point (1,1)

a) $xy^3 - 3x^2 = xy + 5$ b) $e^{xy} + y\ln x = \cos 2x$

38. Find the first derivative of the following functions

a) $y = \arcsin(2x-5)$ b) $y = \arctan(2x^3)$ c) $y = \tanh(x^2+1)$

d. $y = \cos^{-1}(x^2)$ e) $f(x) = \arcsin h(5x)$ f) $g(x) = \arccos h(e^x)$

39. Evaluate the following limit

a. $\lim_{x\to 0^+} x\ln(\sin x)$ b. $\lim_{x\to 0}\frac{\cos x-\cos 3x}{x^2}$ c. $\lim_{x\to \frac{\pi}{2}}\frac{\ln(\sin x)}{1-\sin x}$ d. $\lim_{x\to 0}(1+2x)^{\frac{1}{\sin x}}$ e. $\lim_{x\to 0}\frac{\sqrt{1+\tan x}-\sqrt{1+\sin x}}{x^3}$

40. Find $\int \sin x \sin(\cos x)\,dx$ =

A. $\sin(\cos x)+C$ *B.* $\sin(\sin x)+C$

C. $-\cos(\sin x)+C$ *D.* $\cos(\cos x)+C.$ *E. None of these*

41. $\int x^2 \sin x\,dx$ =

A. $-x^2\cos x - 2x\sin x - 2\cos x + C$ *B.* $-x^2\cos x + 2x\sin x - 2\cos x + C$

C. $-x^2\cos x + 2x\sin x + 2\cos x + C$ *D.* $-\frac{x^3}{3}\cos x + C$ *E.* $2x\cos x + C$

42. The anti-derivative of $f(x)=\frac{1}{1+e^{-x}}$ is

A. $1+e^{-x}$ B. $\ln(1+e^{-x})$ C. $\ln(1+e^{x})$ D. e^x+e^{-x}

43. The value of

A. $\int_0^1 \frac{x\,dx}{(x^2+1)^2}$ b. $\int_4^5 \frac{dx}{(x-3)^2}$ c. $\int_0^1 x^2\ln(2+x^3)\,dx$

44. Find the following integrals

a. $\int \frac{1}{x\ln x}dx$ b. $\int 3x^2\sqrt{9x^3+1}\,dx$ c. $\int \frac{x+3}{\sqrt{x-1}}dx$ d. $\int \frac{1}{1+e^x}dx$

45. Evaluate the following integrals

a. $\int \frac{1}{x^2-1}dx$ b. $\int \frac{1}{2+x-x^2}dx$ c. $\int \frac{1}{1+\sqrt{x+1}}dx$ d. $\int \frac{1}{1-x^4}dx$

46. Use the basis information to Sketch the graph of $f(x)=\frac{x}{(x+1)^2}$

47. The value of $\int_0^{\frac{\pi}{4}} \sin x\,dx =$ _____

A. $-\frac{\sqrt{2}}{2}$ B. $\frac{\sqrt{2}}{2}$ C. $-\frac{\sqrt{2}}{2}-1$ D. $1-\frac{\sqrt{2}}{2}$ E. $\frac{\sqrt{2}}{2}-1$.

48. $\int x^2\cos(x^3)dx =$ _____

A. $-\frac{1}{3}\sin(x^3)+C$ B. $\frac{1}{3}\sin(x^3)+C$

C. $-\frac{x^3}{3}\sin(x^3)+C$ D. $\frac{x^3}{3}\sin(x^3)+C$. E. $\frac{x^3}{3}\sin\left(\frac{x^4}{4}\right)+C$

49. $\int\frac{1}{x^2-6x+8}dx =$

A. $\frac{1}{2}\ln\left|\frac{x-4}{x-2}\right|+C$ B. $\frac{1}{2}\ln\left|\frac{x-2}{x-4}\right|+C$

C. $\frac{1}{2}\ln|(x-2)(x-4)+C|$ D. $\frac{1}{2}\ln|(x-4)(x+2)|+C$ E. $\ln|(x-4)(x-2)+C|$

50. $\int\tan x\ln(\cos x)\,dx =$ _____ $\frac{d}{dx}\left(\int_1^{\int_1^x e^{x^2}du} t^t dt\right) =$ _____.

51. Find $\int_a^{a^2}\frac{dx}{x^2}$ *when* $a\neq 0$.

A) $\frac{a-1}{a}$ B) $\frac{a+1}{a^2}$ C) $\frac{a-1}{a^2}$ D) $\frac{a^2}{a-1}$ E) None of these

52. Solve the integration of the following

a. $\int x\sin x dx$ b. $\int x^2\cos x dx$ c. $\int x\sin(2x)dx$ e. $\int\sin^2 x dx$

53. Find

a. $\int\cos 4x\cos 3x dx$ b. $\int\cos^3 x\sin^8 x dx$ c. $\int x^2 2^x dx$ d. $\int\frac{x}{\sqrt{1-x^2}}dx$

54. Evaluate the following .

Part I: Write the short and simplified answer for each of the following questions on the space provided. (1.5 pts for each)

1. Let $f(x) = \begin{cases} 8 - x, & \text{If } x < -2 \\ x + 4, & \text{if} x \geq -2. \end{cases}$ then

   (a) $\lim_{x \to -3} f(x) =$ _____

   (b) $\lim_{x \to -2^+} f(x) =$ _____

2. $\begin{vmatrix} 1 & 1 & 2 & 1 \\ 4 & 5 & 7 & 2 \\ 1 & 1 & 2 & 1 \\ 14 & 6 & 7 & 10 \end{vmatrix} =$ _____

3. If $f(x) = x^2 + 3x$, then $f(2) =$ _____

4. $\lim_{x \to 2} f(x) = 2$ & $\lim_{x \to 2} g(x) = -4$, then $\lim_{x \to 2} \frac{x^2 g(x)}{f(x)} =$ _____

Part III: Show all the necessary steps clearly and neatly.

1. Using Cramer's Rule to solve the following systems of linear equation. (3pts)

$$\begin{aligned} x - y + z &= -2 \\ 2x + 3y + 4z &= 1 \\ x + y - z &= 0 \end{aligned}$$

2. Let $A = \begin{pmatrix} a & 2 \\ 3 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 4 \\ b & 2 \end{pmatrix}$, and $C = \begin{pmatrix} -1 & c \\ 3 & 2 \end{pmatrix}$ be 2×2 matrices such that, $2A - 3B = 4C$.
   Find the values of a, b and c (3pts).

3. Find the value of k such that $f(x) = \begin{cases} x^2, & \text{if } x \leq 4 \\ k - x^2 - 5, & \text{if } x > 4. \end{cases}$ is continuous at 4 (3pts)

4. If $A = \begin{pmatrix} 1 & 0 & 1 \\ 1 & -3 & 0 \\ 0 & 4 & -3 \end{pmatrix}$, then find (4pts)

   (a) Cofactor of matrix A

   (b) Adjoint of matrix A

   (c) Determinate of A by using cofactor expansion

   (d) Inverse of matrix A (A^{-1})

5. Find $\lim_{x \to 0} \frac{\sqrt{4 + x} - 2}{x}$ (2pts)

69. If $A+B=\begin{bmatrix}-1 & 1 & 0\\ 3 & 0 & -2\\ 1 & 2 & 1\end{bmatrix}$ and $A-B=\begin{bmatrix}5 & 1 & -1\\ 1 & 4 & 0\\ 2 & 1 & -1\end{bmatrix}$. Then find AB

70. *Let* $A=\begin{pmatrix}2 & 3 & 4\\ 4 & 3 & 1\\ 1 & 2 & 4\end{pmatrix}$,

a. Find det(A)

b. Find Adj(A)

c. Find A^{-1}

71. Find the eigenvalue and eigenvector of the matrix *Let* $A=\begin{pmatrix}1 & 2\\ 3 & 2\end{pmatrix}$.

72. Determine $\alpha, \beta, and\ \gamma$ *such that* $A=\begin{pmatrix}2 & \alpha & 3\\ 4 & 5 & \beta\\ \gamma & 1 & 7\end{pmatrix}$ is a symmetric matrix.

73. Use $\epsilon-\delta$ definition show that $\lim_{x\to 2}\left(\frac{1}{2}x+3\right)=4$

74. If $\|u+v\|=5$ *and* $\|u-v\|=1$, *then find* $u\cdot v$

75. Find the value of n so that vectors 2i+3j-2k, 5i+nj+k, and -i+2j+3k may be coplanar

76. Let $f(x)=\begin{cases}\frac{b}{3x}, & if\ x<1\\ a+b, & if\ x=1\\ 2+ax, & if\ x>1\end{cases}$ is continuous at x=1. Find a and b

77. Evaluate the following limit

a. $\lim_{x\to\infty}\frac{\sin x}{x}$

b. $\lim_{x\to 0} x^2\sin\left(\frac{1}{x^2}\right)$

c. $\lim_{x\to-\infty}\frac{\sqrt{4x^2+3}}{1-2x}$

d. $\lim_{x\to 0}\frac{x\cot(3x)\sin(4x)}{\sin(3x)}$

11. If $\cos(xy)=y^2$, then $\frac{dy}{dx}=$ _____.

12. If $f(x)=e^{4x}$, then $f^{(5)}(0)=$

A) $\frac{6}{4^5}$ B) 6×4^5 C) 4^5 D) $(4^5)^{4x}$ E) None of these

13. Find all the local extrema of $f(x)=x-\cos x$ in $[-2\pi,2\pi]$

14. Let $f(x)=(x^2-1)^3$, then find the critical points and inflection points

15. $\frac{d}{dx}\tan\sqrt{4x}=$ _____.

16. $\frac{d}{dx}x^{\sqrt{x}}=$ _____.

17. $\frac{d}{dx}\left(\int_1^{2x-1}\ln(\ln t)dt\right)=$ _____.

18. Let $f(x)=\tan^{-1}(\sin^{-1}x^2)$ find $f'(x)$

19. $\lim_{x\to\infty}\left(1+\frac{1}{e^{x-1}}\right)^{e^x}=$ _____.

A) e^{-e} B) e^{-1} C) e D) e^e E) none of theses

20. Find The equation of the tangent line to the ellipse $x^2+2y^2=6$ at $(2,1)$

21. The slope of the line tangent to the curve $y^3+x^2y^2-3x^3=9$ at $(1,2)$ is _____

22. The critical point(s) of $f(x)=x^2e^{-x}$ is(are):_____

23. Find the interval of monotonic and the infection point(s) of $f(x)=2x^3-6x^2-144x+1$

24. The equation of the line tangent to the graph of a differentiable function f at $x=0$ is $y=3x+4$

.Determine the value of $\lim_{x\to0}\frac{xf(x)}{\sin(2x)}$

25. Let f be a strictly increasing continuous function with $f(3)=5$, find $\lim_{x\to3}\frac{(f(x))^2-25}{f(x)-5}$

26. The derivative of the function $f(x)$ is $f'(x)=(x-1)^2(x-2)$, then which of the following is true

A) f is increasing for all x B) f has local maximum at $x=1$

C. f has local maximum at $x=2$ D. f has local minimum at $x=2$ E. None

27. If $f(x)=x^2\ln(x^3)$, then $f'(x)=$ _____

A. $3x+\ln(x^4)$ B. $3x(1+\ln(x^2))$ C. $\frac{1}{x}$ D. $2x$ E. $2x\ln(3x^3)$

a. $\int_{\pi/4}^{\pi/2} \cos x\,dx$ b. $\int_0^2 (2-|x-1|)dx$ c. $\int \cos x(1-\sin x)^6\,dx$

b. $\int e^x \sin(e^x)dx$ e. $\int (\sin x+\cos x)^2\,dx$ f. $\int e^x \cos(2e^x)\sin(3e^x)dx$

55. If $x\sin(\pi x)=\int_0^{x^2} f(t)dt$, where f is continuous , then find $f(4)$

56. Determine the interval on which the curve $F(x)=\int_0^x \frac{1}{1+t+t^2}\,dt$ is concave upward.

57. Determine the scalar k so that the vectors $a=2i-kj+3k$ and $b=3i+2j+4k$ are orthogonal.

58. Verify the vector $c=b-\frac{a\cdot b}{|a|^2}a$ is orthogonal to a.

59. Suppose that a , b , and c are mutually orthogonal vectors. Let $d=3a-2b+3c$ and $|a|=4$, $|b|=3$ and $|c|=4$. Find the norm of d.

60. Find the direction cosine of $u=i+2j+3k$.

61. Let $a=\sqrt{3}i+2j-3k$ and $b=4i-j+2k$. find $pr_a b$ and $pr_b a$

62. Find the parametric and symmetric equation of the line through the points (5, 3, 1) and (2, 1, 1).

63. Find the equation of the line perpendicular to the plane x+2y=3 and passing through (1, 1, 1)

64. Find the point of intersection of the line x=3-t, y= 2+t, z= 5t and the plane x-y+2z=9.

65. Find the line of intersection of the planes x+y-z-2=0 and 3x-4y+5z-6=0.

66. Let $A=i-3j+2k$ and $B=-2i+j-5k$ and $C=2i-j-10k$

a. Find the unit vector perpendicular to both

b. Find the volume of parallelepiped determined by A,B and C.

67. Let R^3 be the real vector space

a. Show that $w=\{(x,y,z)/\ x+y=2z\}$ is subspace of R^3

b. B={(1,0,0), (1,1,0), (1,1,1)} generate R^3

68. Use Gauss-Jordan method solve the system

$$\begin{cases} 2x+y-z=3 \\ x+y+z=1 \\ x-2y-3z=4 \end{cases}$$