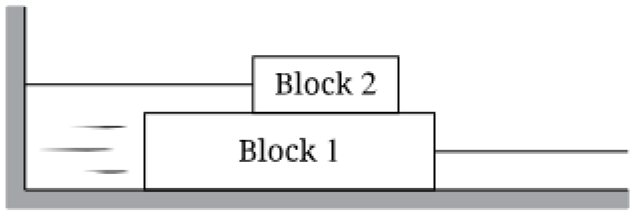
Physics C 2.7 Solutions

Static and Kinetic Friction



1. Block 1 of mass 4.0 kg is on a horizontal surface and is pulled at a constant velocity to the right by a string, as shown in the figure. Block 2 of mass 1.0 kg is on Block 1 and held at rest by another string attached to a wall. The coefficients of kinetic and static friction between all surfaces are 0.20 and 0.40, respectively, and the strings have negligible mass. What is the tension in the string attached to Block 1 ?

Because Block 1 is moving at a constant velocity, its surface is moving against both Block 2 and the ground. Therefore, we can ignore static friction.

If Block 1 is moving at a constant velocity, then all forces must be balanced. The tension (what we're looking for) is pulling Block 1 to the right, and kinetic friction against Block 2 and the ground are both resisting motion, to the right.

The kinetic friction between blocks 1 and 2 is found by multiplying μ_k by the normal force from Block 1, which only holds up the mass of Block 2.

$$F_{k12} = (0.2)(10) = 2\text{ N}$$

The kinetic friction between Blocks 1 the ground is found by multiplying μ_k by the normal force from the ground, which holds up the mass of both blocks 1 and 2.

$$F_{kG1} = (0.2)(50) = 10\text{ N}$$

Add them together: $\mathbf{\Sigma F_k = 12\text{ N}}$

2. A 1.5 kg block is held at rest near the top of a rough incline that makes an angle of 42° from the horizontal. The block is then released and starts moving with a constant acceleration. When the block travels 2.6 m, its velocity is 4.0 m/s. Find the coefficient of kinetic friction between the surface of the incline and the block.

First, we'll find the acceleration of the block using the 3rd kinematics equation:

$$(4.0)^2 = 0^2 + 2a(2.6)$$

$$a = 3.077$$

By using the acceleration, we can find the total force on the block:

$$\Sigma F = (1.5)(3.077) = 4.615\text{ N}$$

Using a little bit of trigonometry, we can find the component of gravitational force which is pointed down the ramp (and is unbalanced):

$$F_{g\parallel} = F_g\sin(\theta) = 15\sin(42°) = 10.037\text{ N}$$

If the total force is only 4.615 N while gravity is 10.037 N, there must be a force in the direction opposite to gravity. That force is kinetic friction, and its magnitude is the difference in the magnitude between total force and gravitational force:

$$|F_k| = |F_{g\parallel}| - |\Sigma F| = 5.422\text{ N}$$

Kinetic friction force is simply μ_k by the normal force. The normal force is also found with trigonometry:

$$F_N = F_g\cos(\theta) = 15\cos(42°) = 11.147 \text{ N}$$

Divide kinetic friction force by normal force:

$$\mu_k = \frac{F_k}{F_N} = \frac{5.422}{11.147} = \mathbf{0.486}$$

3. A string is attached to a block that lies on a horizontal table with a rough surface. In three different trials, the string pulls the block along the table at the same constant velocity to the right. In Trial 1, the string is pulled horizontally. In Trial 2, the string is pulled at an angle θ_0 *below* the horizontal. In Trial 3, the string is pulled at an angle θ_0 *above* the horizontal. In which trial, if any, is the string tension the greatest?

Compared to Trial 1, Trial 3 will decrease the normal force, which decreases the kinetic friction and decreases tension.

Compared to Trial 1, Trial 2 will increase the normal force, which increases the kinetic friction and increases tension.

Trial 2

4. A rectangular block is pushed by a constant force and accelerates along a rough horizontal surface. The block can be oriented to slide along any of three different sides, A, B, and C. Sides A, B, and C have surface areas S_A, S_B, and S_C, respectively, where $S_A > S_B > S_C$, and kinetic friction coefficients μ_A, μ_B, and μ_C, respectively, where $\mu_A < \mu_B < \mu_C$. On which side(s) should the block be placed to have the greatest magnitude of acceleration?

A) Side A

B) Side B

C) Side C

D) Impossible to determine without knowing the precise magnitudes of surface areas and friction coefficients

Acceleration is greatest when kinetic friction is least. Kinetic friction is determined only by normal force, which is the same for all three sides, and kinetic friction coefficient. Surface area has no effect.

5. A box is pushed down a ramp with an incline of 11° with respect to the horizontal and then released. The coefficients of static and kinetic friction between the box and the ramp are 0.50 and 0.20, respectively. Which of the following best describes the motion, if any, of the box after it is released?

A) The box will remain at rest on the incline.

B) The box will start sliding down the incline at a constant velocity.

C) The box will start sliding down the incline and then slow down at a constant rate.

D) The box will start sliding down the incline and then continue to speed up at a constant rate.

The fact that the box is pushed down the ramp and then released heavily implies that the box is initially in motion, so answer choice A is out. Because the box is initially in motion, we can ignore static friction.
The gravitational force down the ramp is given by:

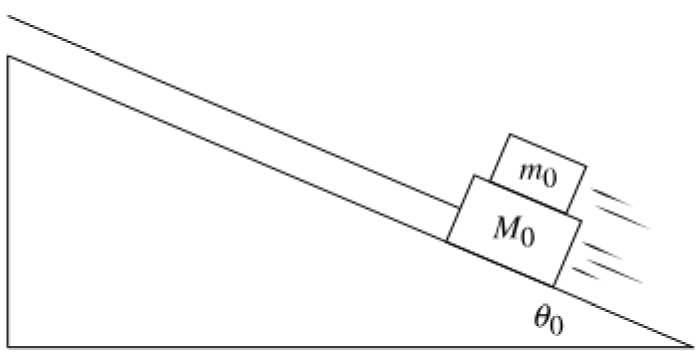
$$F_{g\|} = mg\sin(11°) = 0.191mg$$

The kinetic friction force up the ramp is given by:

$$F_k = \mu_k mg\cos(11°) = 0.196mg$$

Regardless of the values of m and g, kinetic friction force will always be greater than the gravitational force down the ramp. Therefore, the object will accelerate up the ramp, or more precisely, will slow down as it moves down the ramp.

6. A block of mass m_0 is at rest on a ramp inclined at an angle of θ_0 with the horizontal. The coefficient of static friction between the ramp and block is μ_s. The angle of the ramp must be increased by 10° before the block starts to slide. In terms of m_0, g, and θ_0, what is the magnitude of the force of friction exerted on the block when $\theta < 10°$?
When $\theta < 10°$, the block is at rest, so static friction simply balances with the gravitational force down the ramp:
$\boldsymbol{F_s = F_{g\|} = m_0 g \sin\theta_0}$



7. A small block of mass m_0 is on top of a large block of mass M_0 on a ramp that is at an angle θ_0 from the horizontal, as shown in the figure. The large block is pulled by a string as the blocks accelerate together up the ramp. The coefficients of kinetic and static friction between all surfaces are μ_k and μ_s, respectively. In terms of μ_k, μ_s, g, and θ_0, what is the maximum acceleration that the small block can have without slipping along the top surface of the large block?
Static friction from M_0 is the force pushing m_0 up the ramp. Maximum acceleration without slipping happens when static friction is at its maximum value, which means:

$$F_s = m_0 g \mu_s \cos(\theta_0)$$

Meanwhile, gravity is pulling down the ramp:

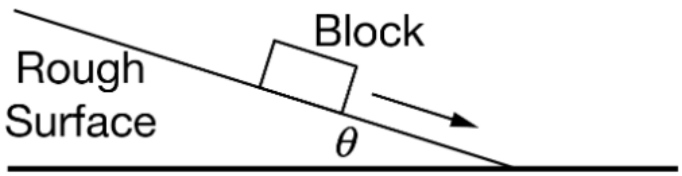
$F_g = -m_0 g\sin(\theta_0)$

The total force is therefore:

$\Sigma F \ = \ m_0 g\mu_s\cos(\theta_0) - m_0 g\sin(\theta_0)$

Because we want acceleration, simply divide force by mass:

$\boldsymbol{a \ = \ g\mu_s\cos(\theta_0) - g\sin(\theta_0)}$



8. A wood block is placed on a rough surface. The surface starts horizontal, and one end is then raised so that the angle the surface makes with the horizontal gradually increases until, at an angle θ, the block begins to move down the surface, as shown in the figure. From this observation, which of the following can be concluded for angles greater than θ ?

A) There is no frictional force on the block.

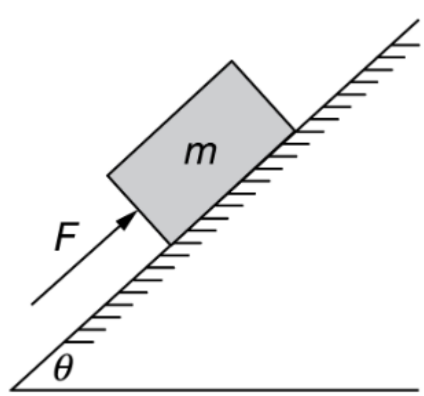
B) The gravitational force is equal to the frictional force.

C) The component of the gravitational force along the surface is greater than the frictional force.

D) The normal force is equal to the gravitational force.

E) The component of the normal force along the surface is greater than the gravitational force.

In order for the block to accelerate down the ramp, the force down the ramp (gravity) must be greater than the force opposing motion (friction).



9. A block of mass m is being pushed up a rough inclined plane by a force of magnitude F. The magnitude of the friction force between the block and the surface is f. The plane makes an angle θ with the horizontal. Which of the following equations can be used to solve for the acceleration of the block?

A) $F - f + mg\cos\theta = ma$

B) $F + f - mg\cos\theta = a$

C) $F - f - mg\sin\theta = ma$

D) $F - mg\sin\theta = ma$

E) $F - f = ma$

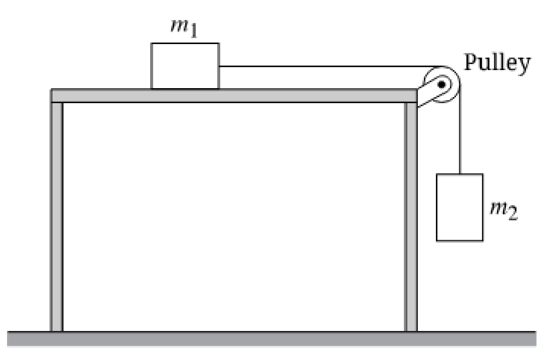
10. A block is given an initial velocity up an incline that makes an angle of 33° from the horizontal. As it is sliding up the incline, the block is slowing down with a constant acceleration of magnitude a. If the coefficient of kinetic friction between the block and the incline is 0.4, what is a ?

Both gravity and friction are pointed down the ramp, so total force is:

$$\Sigma F = F_g + F_k = mg\sin(33°) + 0.4mg\cos(33°)$$

In order to find acceleration, divide force by mass:

$$a = g\sin(33°) + 0.4g\cos(33°) = \mathbf{8.801\ m/s^2}$$



11. A block of mass m_1 rests on a rough horizontal tabletop, as shown in the figure. The block is connected by a string to a second block of mass m_2, which hangs below a pulley at the edge of the table. The coefficient of static friction between the tabletop and the first block is μ_s. The masses of the string and the pulley are negligible, and the pulley can rotate with negligible friction in its axle. In terms of μ_s and m_2, what is the minimum mass of m_1 that will prevent the blocks from moving?

The static friction force on m_1 must be equal to the gravitational force on m_2:

$$\mu_s m_1 g = m_2 g$$

$$\boldsymbol{m_1} = \frac{m_2}{\mu_s}$$

1. 12 N
2. 0.486
3. Trial 2
4. A
5. C
6. $m_0 g \sin\theta_0$
7. $\mu_s g \cos\theta_0$ - $g \sin\theta_0$
8. C
9. C
10. 8.625 m/s^2
11. $\frac{m_2}{\mu_s}$