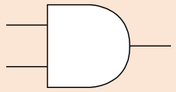
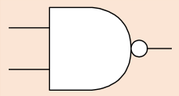
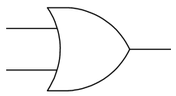
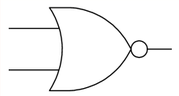
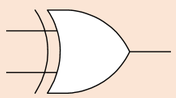
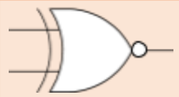
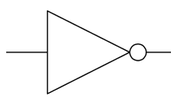
Boolean Algebra

Cheat Sheet

<table>
<tr><th>Name</th><th>Reads as</th><th>Logic Gate</th><th>OCR Notation</th><th>Alternative Notation</th><th>Examples</th><th>Truth table</th><th>Notes</th></tr>
<tr><td>Conjunction</td><td>AND</td><td>  </td><td>∧</td><td>AND

e.g. A AND B</td><td>Both operators in a submarine need to turn their launch keys at the same time for a missile to fire.</td><td><table><tr><th>A</th><th>B</th><th>Output</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>0</td><td>0</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td><td> 
Adding a circle to the end of an AND gate turns it into a NAND gate (Not AND). It simply reverses the output from the gate.</td></tr>
<tr><td>Disjunction</td><td>OR</td><td>  </td><td>∨</td><td>OR e.g A OR B

+ e.g A+B</td><td>The fire alarm will go off if the smoke detector senses the temperature rises to high or if it senses smoke or both.</td><td><table><tr><th>A</th><th>B</th><th>Output</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>1</td></tr></table></td><td> 
Adding a circle to the end of an OR gate turns it into a NOR gate (Not OR). It simply reverses the output from the gate.</td></tr>
<tr><td>Exclusive Disjunction</td><td>XOR</td><td>  </td><td>⊻</td><td>XOR e.g A XOR B

⊕ e.g A ⊕ B</td><td>Opposing football teams will get 3 points for a win if one side scores more, but not if they draw.</td><td><table><tr><th>A</th><th>B</th><th>Output</th></tr><tr><td>0</td><td>0</td><td>0</td></tr><tr><td>0</td><td>1</td><td>1</td></tr><tr><td>1</td><td>0</td><td>1</td></tr><tr><td>1</td><td>1</td><td>0</td></tr></table></td><td> 
Adding a circle to the end of an XOR gate turns it into a XNOR gate (Not XOR). It simply reverses the output from the gate.</td></tr>
<tr><td>Negation</td><td>NOT</td><td>  </td><td>¬</td><td>bar e.g $\bar{A}$
~ e.g ~A
NOT e.g NOT A</td><td>A microwave will stop if the door is not closed.
A house alarm will go off if the door is not closed.</td><td><table><tr><th>A</th><th>Output</th></tr><tr><td>1</td><td>0</td></tr><tr><td>0</td><td>1</td></tr></table></td><td></td></tr>
<tr><td>Equivalence</td><td>If and only if.
Means the same as</td><td></td><td>≡</td><td>—</td><td>The decimal 0.25 is the same as the fraction ¼.
The temperature 25 Celsius is the same as 77 Fahrenheit.</td><td></td><td></td></tr>
</table>

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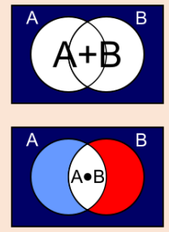
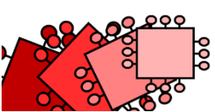
Rule	What does it mean?	In Boolean Algebra in the OCR exam	Examples in English	Notes
De Morgan's Law	Either logical function **AND** or **OR** may be replaced by the other, given certain changes to the equation.	¬ (A ∨ B) ≡ (¬ A) ∧ (¬ B) ***NOT*** *(A* ***OR*** *B) is the same as (****NOT*** *A)* ***AND*** *(NOT B)* This is the same as: ¬ (A ∧ B) ≡ (¬ A) ∨ (¬ B) ***NOT*** *(A* ***AND*** *B) is the same as (****NOT*** *A)* ***OR*** *(****NOT*** *B)*	It cannot be both *winter* **AND** *summer* (at any point in time) Is the same as… (At any point in time) It is **NOT** *winter* **OR** it is **NOT** *summer*	
Distribution	This law allows for the multiplying or factoring out of an expression.	This is the **OR** Distributive law: A∧(B∨C) ≡ (A∧B)∨(A∧C) *A* ***AND*** *(B* ***OR*** *C) is the same as (A* ***AND*** *B)* ***OR*** *(A* ***AND*** *C)* This is the **AND** Distributive law: A∨(B∧C) ≡ (A∨B)∧(A∨C) *A* ***OR*** *(B* ***AND*** *C) is the same as (A* ***OR*** *B)* ***AND*** *(A* ***OR*** *C)*	You can choose 1 main course and either a start or a desert. Is the same as… You can choose 1 main and 1 starter or you can choose 1 main and 1 desert	
Association	This law allows for the removal of brackets from an expression and the regrouping of the variables.	This is the **OR** Association Law: A∨(B∨C) ≡ (A∨B)∨C ≡ A∨B∨C *A* ***OR*** *(B* ***OR*** *C) is the same as (A* ***OR*** *B)* ***OR*** *C is the same as A* ***OR*** *B* ***OR*** *C* This is the **AND** Association Law: A∧(B∧C) ≡ (A∧B)∧C≡ A∧B∧C *A* ***AND*** *(B* ***AND*** *C) is the same as (A* ***AND*** *B)* ***AND*** *C is the same as A* ***AND*** *B* ***AND*** *C*	"Craig and his friends James & Tom are coming to the party" Is the same as.. "James & Tom and their friend Craig are coming to the party" Is the same as… "Craig, James and Tom are coming to the party"	
Commutation	The order of application of two separate terms is not important.	A ∧B ≡ B∧A The order in which two variables are **AND**'ed makes no difference A∨B ≡ B∨A The order in which two variables are **OR**'ed makes no difference	Tom and Jane are going shopping. Is the same as… Jane and Tom are going shopping	
Double negation	NOT NOT A (double negative) = "A"	¬(¬ A) = A	"It's not as if I don't like you" clearly means "I do like you"!	
Absorption	Where the rule applies the second term inside the bracket can always be eliminated and "absorbed" by the term outside the bracket.	X∨(X∧ Y) ≡ X *X* ***OR*** *(X* ***AND*** *Y) is the same as X* X∧(X∨ Y) ≡ X *X* ***AND*** *(X* ***OR*** *Y) is the same as X*	If it will rain, then I will wear my coat. Therefore, if it will rain then it will rain and I will wear my coat.	To be able to apply the Absorption rule: 1. The operators inside and outside the brackets must be different 2. The term outside the brackets must also be inside the brackets

In addition to the 5 laws / rules above which are listed in the OCR specification and the Absorption rule there are also 8 general identities or "rules" which you really should know which will help you gratefully when it comes to simplifying Boolean Expressions.

	Rule	Explanation		Rule	Explanation
AND rules		Remember that with **AND** both terms have to be 1 or **TRUE** for the result to be **TRUE**	**OR** rules		Remember that with **OR** only 1 term has to be 1 or **TRUE** for the result to be **TRUE**
1	X∧ 0 = 0 ∧	X **AND** 0 is the same as 0 *Or to put it another way… X* ***AND FALSE*** *has to equal* ***FALSE*** *(See truth table on reverse side for proof)*	**5**	X ∨ 0 = X	X **OR** 0 is the same as X *Or to put it another way… X* ***OR FALSE*** *has to equal* ***TRUE*** *(See truth table on reverse side for proof)*
2	X 1 = X	X **AND** 1 is the same as X *Or to put it another way… X* ***AND TRUE*** *has to equal* ***TRUE*** *(See truth table on reverse side for proof)*	**6**	X ∨ 1 = X	X **OR** 1 is the same as X *Or to put it another way… X* ***OR TRUE*** *has to equal* ***TRUE*** *(See truth table on reverse side for proof)*
3	X∧ X = X	X **AND** X is the same as X *Or to put it another way… X* ***AND*** *X has to equal X (See truth table on reverse side for proof)*	**7**	X∨ X = X	X **OR** X is the same as X *Or to put it another way… X* ***OR*** *X has to equal X (See truth table on reverse side for proof)*
4	X∧ ¬X = 0	X **AND not** X is the same as 0 *Or to put it another way… X* ***AND NOT(X)*** *has to equal* ***FALSE*** *(See truth table on reverse side for proof)*	**8**	X∨ ¬X = 1	X **OR not** X is the same as 1 *Or to put it another way… X* ***OR NOT(X)*** *has to equal* ***TRUE*** *(See truth table on reverse side for proof)*



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