1.

Given x is measured in radians, use small angle approximations to simplify the following expression.

$$\frac{\cos 7x - 1}{x \sin x}$$

(4)

2.

The sum to infinity of a geometric series is 3 times as large as its first term and the third term of the same series is 40.

a. Find the value of the first term of the series.

(5)

b. Determine the exact value of the sum of the first four terms of the series.

(2)

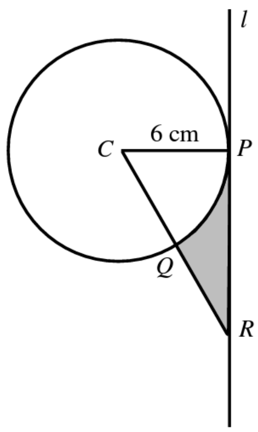
3.

Prove by contradiction that for all real θ

$$\sin\theta + \cos\theta \leqslant \sqrt{2}$$

(5)

4.



The figure above shows a circle of radius 6 cm, centre at point C, and the straight line *l* which is a tangent to the circle at the point P.

The point R lies on *l*.

The straight line segment CR meets the circle at the point Q.

Given that the length of the arc QP is $2\pi\,\text{cm}$, show that the area of the finite region bounded by PR, RQ and QP, shown shaded in the figure, is

$$6\left(3\sqrt{3} - \pi\right)$$

(9)

5.

A curve C has equation

$$y = \frac{1}{x^3 + 1},\ x \in \mathbb{R},\ x \neq -1.$$

a. Determine an equation of the curve which is obtained by translating C by the vector $\begin{bmatrix} 3 \\ 1 \end{bmatrix}$

(2)

b. Describe fully a sequence of two transformations which map the graph of C onto the graph with equation

$$y = \frac{1}{x^3 - 1},\ x \in \mathbb{R},\ x \neq 1.$$

(2)

6.

$$\mathrm{f}(x) = 3x^2 - 18x + 21,\ x \in \mathbb{R},\ x > 4.$$

a. Express $\mathrm{f}(x)$ in the form $A(x + B)^2 + C$, where A, B and C are integers.

(2)

b. Find a simplified expression for $\mathrm{f}^{-1}(x)$.

(3)

c. Determine the domain and range of $\mathrm{f}^{-1}(x)$

(3)

7.

If x is in radians

$$\frac{\mathrm{d}}{\mathrm{d}x}(\cos kx) = -k\sin kx$$

Prove the validity of the above from first principles.

(5)

8.

A sequence of numbers, $u_1, u_2, u_3, u_4, \ldots,$ is defined by

$$u_n = \frac{1}{1-u_{n-1}}.\ u_1 = 2$$

Determine the value of

$$\sum_{n=1}^{20} u_n$$

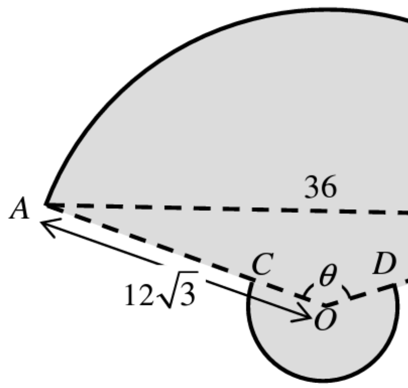
(5)

9.

Show by using algebra, that the sum of the integers between 1 and 600 inclusive, that are **not** divisible by 3, is 120,000.

(5)

10.



The figure above shows a model of the region used by shot putters in to throw the shot. The throwing region consists of a minor circular sector OAB of radius $12\sqrt{3}$ metres subtending an angle θ radians at O. The chord AB is 36 metres.
The shot putter's region COD is a major circular sector of radius $3\sqrt{3}$ metres, where C and D lie on OA and OB, respectively.

a. Show that $\theta = \frac{2\pi}{3}$

(3)

b. Find, in terms of π, the total area of throwing region and shot putter's region.

(4)

c. Show, further that the total perimeter of the throwing region and the shot putter's region, shown shaded in the figure above, is
$6(2\pi + 3)\sqrt{3}$

(3)

11.

The function f is defined by

$$\mathrm{f}(x) = 2 + \sqrt{x},\ x \in \mathbb{R},\ x \geqslant 0$$

a. Evaluate $\mathrm{ff}(49)$

(1)

b. Find an expression for the inverse function, $\mathrm{f}^{-1}(x)$

(2)

c. Sketch in the same set of axes the graph of $y = \mathrm{f}(x)$ and the graph of $y = \mathrm{f}^{-1}(x)$, clearly marking the line of reflection between the two graphs.

(3)

d. Show that $x = 4$ is the only solution of the equation $\mathrm{f}(x) = \mathrm{f}^{-1}(x)$

(3)
