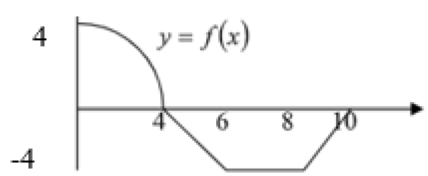
YEAR 12 - MATHEMATICS

HSC Topic 11 – Methods of Integration, Area

MATHEMATICS ADVANCED

LEARNING PLAN			
Learning Intentions Student is able to:	**Learning Experiences** Implications, considerations and implementations:	**Success Criteria** I can:	**Resources**
1. understand that 'the area under a curve' refers to the area between a function and the x-axis, bounded by two values of the independent variable			
2. Find approximations to area under a curve using rectangles above and below the curve.	Find an approximation of the area bounded by $y = x^2$, x–axis, $x = 0$ and $x = 4$, by using inscribed and circumscribed rectangles.	Understand how approximate areas can be found through the use of rectangles.	
3.use the notation of the definite integral $\int_a^b f(x)\, dx$ for the area under the curve $y = f(x)$ from $x = a$ to $x = b$ if $f(x) \geq 0$			
4.use the Trapezoidal rule to estimate areas under curves	where $f(x) \geq 0$, on the interval $a \leq x \leq b$, by dividing the area into a given number of trapezia with equal widths. The formula	demonstrate understanding of the formula:	

– demonstrate understanding of the formula:	$\int_a^b f(x)\,dx = \frac{b-a}{2}\left(f(a)+f(b)\right)$ Or combined trapezia may be found using $\int_a^b f(x)\,dx \approx \frac{b-a}{2n}\,[f(a) + f(b) + 2\{f(x_1) + \ldots + f($ where $a = x_0$ and $b = x_n$, and the are found by dividing the interval $a \leq x \leq b$ into n equal sub-intervals values of $x_0,\ x_1,\ x_2,\ \ldots,\ x_n$ are found by dividing the interval $a \leq x \leq b$ into n equal sub-intervals		
5. use geometric ideas to find the definite integral $\int_a^b f(x)\,dx$	Use area of a circle rather than integration to find area bound by $y=\sqrt{r^2-x^2}$ Eg. $\int_0^2 \sqrt{4-x^2}\,dx$ Eg. Determine $\int_0^{10} f(x)dx$ 	Show that the area bound by the curve $y=\sqrt{16-x^2}$ and the line $y=x$ and the y-axis is $\frac{1}{8}\pi 4^2$ units^2	

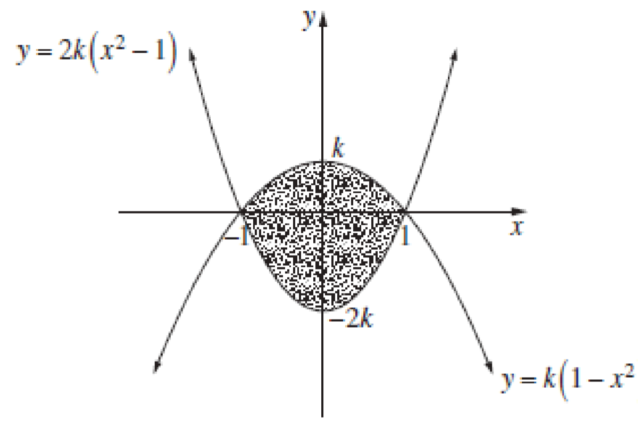
6.understand the relationship of position to signed areas, namely that the signed area above the horizontal axis is positive and the signed area below the horizontal axis is negative.			
7. use the formula for a definite integral $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F(x)$ is the anti-derivative of $f(x)$, to calculate definite integrals	Understand the relationship between the integral and the primitive function. The fundamental Theorem of Calculus		
8.. Integrate x^n by considering integration as the reverse operation of differentiation. 9..Use these rules on indefinite integration: (a) Standard forms. (b $\int cf(x)\,dx = c\int f(x)\,dx$. (c) $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$ (d) Integrals of products handled by first simplifying algebraically:	Egs. $\int (2x+1)^2\,dx$, $\int x\sqrt{x}\,dx$, $\int (3x-2)(4x-3)\,dx$ $\int (\sqrt{x}-1)^2\,dx$.		

(e) Integrals of quotients: (f) Integrals of the form $\int (ax+b)^n \, dx$, including fractional values of n such as $\int \sqrt{ax+b} \, dx$ (g) Differentiate and hence integrate a function	e) Integrals such as $\frac{x^3+5}{x^2}$, $3x+\frac{\sqrt{x}}{x}$ which initially need to be manipulated algebraically. (f) Understand that the rule only works when the function inside the brackets is linear. (g) (i) Differentiate $y=\sqrt{9-x^2}$ with respect to x. (ii) Hence, or otherwise, find $\int \frac{6x}{\sqrt{9-x^2}} dx$.		
Area			
Determine that the definite integral $\int_a^b f(x) dx$ gives the exact area bound by the curve $y=f(x)$ and the x-axis from $x=a$ to $x=b$.	Reinforce that the definite integral represents the infinite sum of rectangles as $\delta x \to 0$ $\int$ symbol is an elongated S, for sum. Show by a consideration of the area of rectangles above and below the curve that the		

	definite integral gives the exact area under the curve. The exact area is the infinite sum of rectangles where the width $\delta x \to 0$.		
Perform calculations involved in finding			
(i) Areas on one side of the *x*-axis andbounded by two ordinates, the *x*-axis and the curve.		Find area enclosed by the *x*-axis $y = x^3 - 4x^2$ between $x = 1$ and $x = 3$.	
(ii) Areas for curves which cut the x-axis.		Find area enclosed by the *x*-axis $y = x^2 - 5x + 6$ between $x = 0$ and $x = 4$.	
(iii) Areas bounded by 2 curves and which are entirely on one side at the x-axis.		Area bounded by $y = x^2$ and $y = x^3$.	
(iv) Areas which involve addition and subtraction of integrals.		Area bounded by $y = 2x$ and $y = x^2 - 5x + 6$.	
(v) Find the area enclosed involving trigonometric, exponential and logarithmic functions.	g. Determine the volume formed when the region bounded by $y = e^x$, $x = \ln 2$ and the coordinate axes is rotated about the *x*-axis. e.g. Determine the volume formed when the region bounded by the *x*-axis , $y = \frac{1}{x+1}$, $x = 1$ and $x = 4$ is rotated about the *x*-axis. e.g. Determine the area bound by $y = \ln x$ and the *x*-axis from $x = 1$ to $x = 3$.	e.g. Find area bounded by $y = \sin x$, $y = \cos x$ between $x = \frac{\pi}{4}$ and $x = \frac{3\pi}{4}$. HSC QUESTIONS INVOLVING LOGS AND EXPONENTIALS	

	$A=\int_1^3 \ln x\,dx$ however the integral of $\ln x$ is not in the course. This area should be found by considering the area of the rectangle subtracting the area bound by the *y*-axis. Area of Rectangle $=3\times\ln 3$ Area bound by *y*-axis $\int_0^{\ln 3} e^x\,dx$ $\therefore$ Area bound by *x*-axis $3\ln 3-\left[e^x\right]_o^{\ln 3}$ $3\ln 3-\left[e^{\ln 3}-e^0\right]$ $3\ln 3-\left[3-1\right]$ $3\ln 3-2$ units^2		
HSC Area Questions			

17 14 d The shaded region shown is enclosed by two parabolas, each with x-intercepts at $x = -1$ and $x = 1$.
The parabolas have equations $y = 2k(x^2 - 1)$ and $y = k(1 - x^2)$, where $k > 0$.
Given that the area of the shaded region is 8, find the value of k.



3 Solution

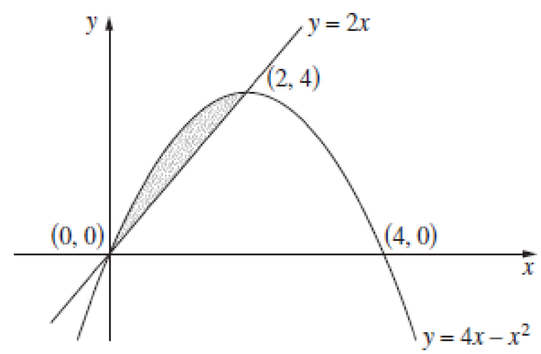
15 7 The diagram shows the parabola
$y = 4x - x^2$ meeting the line $y = 2x$ at
(0, 0) and (2, 4). Which expression gives the area of the shaded region bounded by the parabola and the line?

(A) $\int_0^2 x^2 - 2x \, dx$

(B) $\int_0^2 2x - x^2 \, dx$

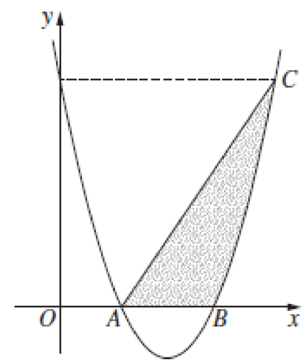
(C) $\int_0^4 x^2 - 2x \, dx$

(D) $\int_0^4 2x - x^2 \, dx$



1 Solution

15 **16a** The diagram shows the curve with equation $y = x^2 - 7x + 10$. The curve intersects the x-axis at points A and. The point C on the curve has the y-coordinate as the y-intercept of the curve.



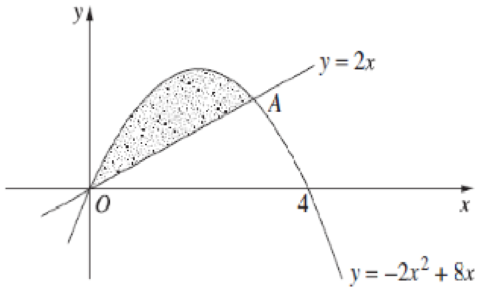
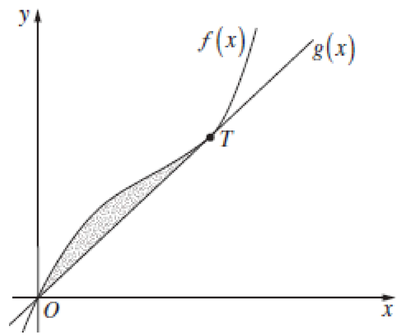
Solution

(i) Find the x-coordinates of points A and B. **1**

(ii) Write down the coordinates of C. **1**

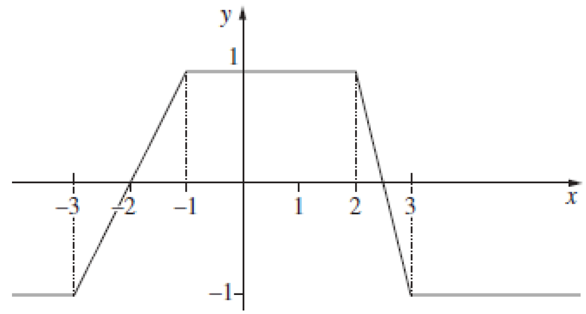
(iii) Evaluate $\int_0^2 \left(x^2 - 7x + 10\right) dx.$ **1**

(iv) Hence, or otherwise, find the area of the shaded region. **2**

	14 12d (d) The parabola $y = -2x^2 + 8x$ and the line $y = 2x$ intersect at the origin and at the point A.  (i) Find the x-coordinate of the point A. **1** (ii) Calculate the area enclosed by the parabola and the line. **3** [Solution]
	13 13b The diagram shows the graphs of the functions $f(x) = 4x^3 - 4x^2 + 3x$ and $g(x) = 2x$. The graphs meet at O and at T. (i) Find the x-coordinate of T. **1** (ii) Find the area of the shaded regions between the graphs of the functions $f(x)$ and $g(x)$. **3**  [Solution]

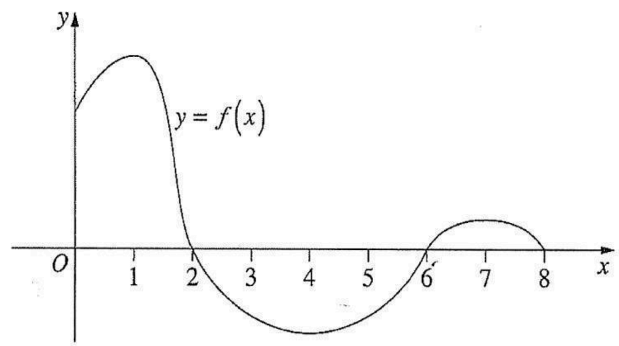
13 **14d** The diagram shows the graph $y = f(x)$.

What is the value of a, where $a > 0$, so that $\int_{-a}^{a} f(x)\,dx = 0$



1 [Solution]

12 **10** The graph of $y = f(x)$ has been drawn to scale for $0 \le x \le 8$. Which of the following integrals has the greatest value?



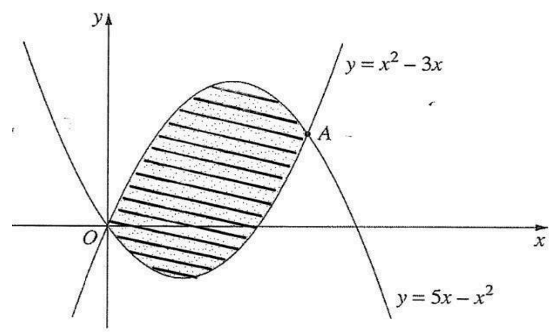
(A) $\int_0^1 f(x)\,dx$

(B) $\int_0^2 f(x)$

(C) $\int_0^7 f(x)$

(D) $\int_0^8 f(x)$

1 [Solution]

	12	**13b**	The diagram shows the parabolas $y = 5x - x^2$ and $y = x^2 - 3x$. The parabolas intersect at the origin O and the point A. The region between the two parabolas is shaded. (i) Find the x-coordinate of the point A. (ii) Find the area of the shaded region.	 	Solution **1** **3**

Established Goals(Syllabus Outcomes):

› applies calculus techniques to model and solve problems MA12-3

› applies the concepts and techniques of indefinite and definite integrals in the solution of problems MA12-7

› chooses and uses appropriate technology effectively in a range of contexts, models and applies critical thinking to recognise appropriate times for such use MA12-9

› constructs arguments to prove and justify results and provides reasoning to support conclusions which are appropriate to the context MA12-10