

Name:

IN THIS LESSON, YOU WILL:

- Write recursive formula for a given sequence
- Use a recursive formula to model debt repayment
- Calculate how a debt amount decreases over time, given the interest rate and regular payment amounts



INTRO

INTERACTIVE: Tower of Hanoi

The Tower of Hanoi is a mathematical puzzle that has inspired many myths about its origins. According to one version, priests were tasked with solving this puzzle using 64 disks and when the puzzle was completed, the world would end. Explore this classic puzzle to see what patterns you can discover!

- [illegible]

4. The table below shows the minimum number of moves required to solve the puzzle with each number of disks. Complete the last two rows: How many moves would it take to complete the puzzle with 9 disks? With 10 disks?

Number of Disks	Minimum Number of Moves
1	1
2	3
3	7
4	15
5	31
6	63
7	127
8	255
9	
10	

5. Describe the pattern for the number of moves as required the number of disks increases. If you prefer, you can draw a representation instead.



LEARN IT

Recursive Sequences

As you add more disks to the Tower of Hanoi, you can use the previous solutions that had fewer disks to help you figure out the puzzle. The pattern we discovered for the minimum number of moves is an example of a recursive sequence.

Sequence: a list of numbers

Term: a number in a sequence

Recursive sequence: a sequence in which a term is defined using a previous term.

Example

Here is a sequence: 11, 13, 15, 17, 19, 21...

We could define this sequence recursively by saying it starts with 11 and each term is 2 greater than the previous term.

1. Write a sequence of numbers where the first term is 5 and each term is 2 times the previous term.



EXPLORE IT

Describing Sequences

Let's practice describing recursive sequences and finding patterns in them.

1. Given the sequence: 20, 10, 5, 2.5, 1.25, 0.625...
 - a. What is the 4th term?
 - b. Describe the pattern used to get from one term to the next.

2. Given the sequence $f(n)$ below.

n	1	2	3	4	5
$f(n)$	3000	2850	2700	2550	?

- a. What is the 3rd term?
- b. What is the 5th term?

- c. Describe the pattern used to get from one term to the next.



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Writing Recursive Formulas

Throughout this lesson, you've been noticing patterns and describing how to get from one term to the next. We can capture that information using a recursive formula.

A recursive formula specifies the first term of the sequence and the steps to get from one term to the next. The formula includes two parts:

- One of the terms in the sequence (usually the first)
- The pattern: how you can find a specific term by using the previous term
 - The notation $f(n-1)$ represents "the previous term"

Example

Here is the recursive formula for the sequence you saw earlier in the Explore It Question 1.

Sequence: 20, 10, 5, 2.5, 1.25, 0.625...

Formula:

$$f(1) = 20 \quad \leftarrow \text{This tells us the first term is 20}$$

$$f(n) = \frac{1}{2} \cdot f(n-1) \quad \leftarrow \text{This tells us to find a specific term by multiplying the previous term by } \frac{1}{2}$$



PRACTICE IT

The Tower of Hanoi

Let's write a recursive formula for the Tower of Hanoi problem in the Intro.

1. **Write a recursive formula for the sequence of minimum moves in the Tower of Hanoi. You can use [this video walkthrough](#) to guide you through the process.**

n Number of Disks	f(n) Minimum Number of Moves
1	1
2	3
3	7
4	15
5	31

2. **Check your formula by using $f(4)$ to find $f(5)$. Show your work.**

ACTIVITY: [MOVE: Matching Recursive Sequences](#)

Let's practice matching recursive formulas with their sequences. Follow your teacher's instructions to complete this activity.



APPLY IT

Follow your teacher's directions to complete the Application Problems.

Teachers, you can find the Application problems linked in the Lesson Guide.