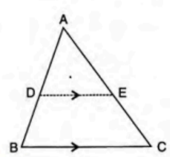
Concept Images these are linked across my Daily log so that i can quickly refer them when needed	Concept Images
These are notes written while learning and possibly for school - Doubles up revision material :)	Handwritten Notes
FlashCards for Physics	Physics QuizLet
FlashCards for Maths	Math FlashCards
History: duplicated from another set and slightly re-edited for personal clarity)	History

Theorem 1: (Basic Proportionality Theorem)

- A line drawn parallel to any side of a triangle divides the other two sides proportionally
- and, conversely, a line in a triangle that divides two sides proportionally is parallel to the third side

Given : In Δ ABC; line DE is drawn parallel to side BC which meets AB at D and AC at E.



To Prove : $\frac{AD}{DB} = \frac{AE}{EC}$

Proof :

	Statement :	**Reason :**
	In Δ ABC and Δ ADE,	
1.	$\angle ABC = \angle ADE$	[Corresponding angles]
2.	$\angle ACB = \angle AED$	[Corresponding angles]
3.	$\angle BAC = \angle DAE$	[Common]
$\therefore$	$\Delta ABC \sim \Delta ADE$	[AAA postulate]
$\therefore$	$\frac{AB}{AD} = \frac{AC}{AE}$	[Corresponding sides of similar triangles are proportional]
$\Rightarrow$	$\frac{AD+DB}{AD} = \frac{AE+EC}{AE}$	
$\Rightarrow$	$1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$	[$\frac{AD}{AD} = 1$ and $\frac{AE}{AE} = 1$]
$\Rightarrow$	$\frac{DB}{AD} = \frac{EC}{AE}$	[Cancelling 1 from both the sides]
$\Rightarrow$	$\frac{AD}{DB} = \frac{AE}{EC}$	[Taking the reciprocal]

Hence Proved.

Theorem 2:

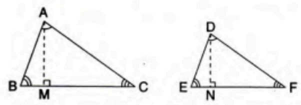
The areas of two similar triangles are proportional to the squares on their corresponding sides, i.e. the ratio between the areas of the similar triangles is equal to the square of the ratio between two sides

The areas of two similar triangles are proportional to the squares on their corresponding sides.

Given : $\Delta ABC \sim \Delta DEF$

such that $\angle BAC = \angle EDF$,

$\angle B = \angle E$ and $\angle C = \angle F$.



To Prove :

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Construction :

Draw $AM \perp BC$ and $DN \perp EF$.

Proof :

Statement	***Reason***
1. Area of $\Delta ABC = \frac{1}{2} BC \times AM$	Area of $\Delta = \frac{1}{2}$ base $\times$ altitude
Area of $\Delta DEF = \frac{1}{2} EF \times DN$	Area of $\Delta = \frac{1}{2}$ base $\times$ altitude

$$\Rightarrow \frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{\frac{1}{2} BC \times AM}{\frac{1}{2} EF \times DN}$$

$$= \frac{BC}{EF} \times \frac{AM}{DN} \quad \text{.....I}$$

2. In ΔABM and ΔDEN :

(i) $\angle B = \angle E$ [Given]

(ii) $\angle AMB = \angle DNE$ [Each angle being 90°]

$\therefore \Delta ABM \sim \Delta DEN$ [By AA postulate]

$$\Rightarrow \frac{AM}{DN} = \frac{AB}{DE} \quad \text{.....II}$$

[Corresponding sides of similar triangles are in proportion]

3. Since, $\Delta ABC \sim \Delta DEF$ [Given]

$$\Rightarrow \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} \quad \text{.....III}$$

[Corresponding sides of similar triangles are in proportion]

$$\therefore \frac{AM}{DN} = \frac{BC}{EF}$$ [From II and III]

Substituting $\frac{AM}{DN} = \frac{BC}{EF}$ in equation I, we get :

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{BC}{EF} \times \frac{BC}{EF} = \frac{BC^2}{EF^2} \quad \text{.....IV}$$

Now combining eq. III and eq. IV, we get :

$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta DEF} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$

Hence Proved.

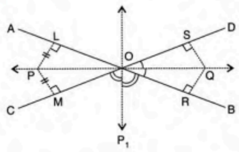
Theorem 3

- The locus of a point equidistant from intersecting lines is the bisector of the angle between the lines

Given : Two straight lines AB and CD intersecting at O. A point P is the interior of angle AOC such that it is equidistant from AB and CD.

To Prove : Locus of P is the bisector of angle AOC.

i.e. (i) P lies on bisector of angle AOC, and *conversely.*



(ii) every other point on the bisector of ∠AOC is equidistant from the intersecting lines AB and CD.

Construction : Draw a line through O and P. Then draw PL perpendicular to AB and PM perpendicular to CD.

(i) Proof :

Statement	*Reason*
In triangles POL and POM :	
1. PL = PM	P is equidistant from AB and CD [Given]
2. ∠PLO = ∠PMO	Each is 90° [By construction]
3. PO = PO	Common
∴ Δ POL ≅ Δ POM	R.H.S.
∴ ∠POL = ∠POM	Corresponding parts of congruent triangles are congruent

Therefore, **P lies on the bisector of angle AOC.**

(ii) Conversely : Let Q be any point on the bisector OP.

Now to show that Q is equidistant from AB and CD, draw QR and QS perpendiculars to AB and CD respectively.

Clearly, Δ OQR ≅ Δ OQS [By A.A.S. or A.S.A.]

⇒ QR = QS [C.P.C.T.C.]

⇒ **Q is equidistant from AB and CD**

The same results can be proved by taking a point, in the interior of angle COB or in the interior of angle AOD, etc.

Hence the theorem is proved.

Theorem 4

- The locus of a point equidistant from two points is the perpendicular bisector of the line joining the two points

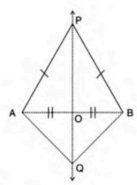
The locus of a point equidistant from two given points is the perpendicular bisector of the line joining the two points.

Given : Two fixed points A and B. P is a moving point equidistant from A and B, *i.e.* PA = PB.

To Prove : Locus of moving point P is the perpendicular bisector of line AB.

i.e. (i) P lies on the perpendicular bisector of AB and *conversely;*

(ii) every other point on this perpendicular bisector is equidistant from A and B.



Construction : Join AB. Locate 'O' the mid-point of AB. Join P and O.

(i) Proof :

Statement	*Reason*
In triangles AOP and BOP :	
1. PA = PB	Given
2. AO = BO	'O' is the mid-point of AB
3. PO = PO	Common
∴ Δ AOP ≅ Δ BOP	S.S.S.
∴ ∠AOP = ∠BOP = 90°	Since, ∠AOP + ∠BOP = 180°.

Therefore, **P lies on the perpendicular bisector of AB.**

(ii) *Conversely* : Let Q be any other point on line PO, the perpendicular bisector of AB.

Clearly, Δ AOQ ≅ Δ BOQ [By S.A.S.]

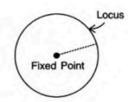
∴ AQ = BQ [Corresponding parts of congruent triangles are congruent]

⇒ **Every point on the perpendicular bisector is equidistant from A and B.**

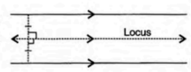
Hence the theorem is proved.

Points About Loci

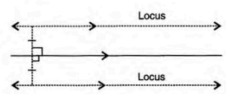
3. The locus of a point, in a plane and at a fixed distance from a given fixed point, is the circumference of the circle with the given fixed point as centre and given fixed distance as radius.



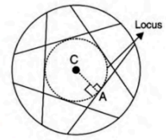
4. The locus of a point, which remains equidistant from two given parallel lines, is a line parallel to the given lines and midway between them.



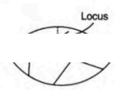
5. The locus of a point, which is at a given distance from a given line, is a pair of lines parallel to the given line and at the given distance from it.



6. The locus of the mid-points of all equal chords in a circle is the circumference of the circle concentric with the given circle and having radius equal to the distance of equal chords from the centre.

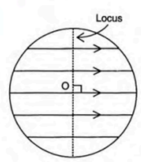


Remember :
Equal chords of a circle are equidistant from its centre.

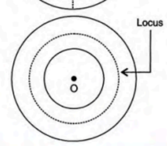


Remember :
Equal chords of a circle are equidistant from its centre.

7. The locus of mid-points of all parallel chords in a circle is the diameter of the circle which is perpendicular to the given parallel chords.



8. The locus of a point equidistant from two concentric circles is the circumference of the circle concentric with the given circles and midway between them.



The locus problems, concerning circles, should be attempted after completing the chapter on **circles.**

Remember :
1. To describe the locus of a moving point, state the kind of geometrical figure obtained and its position.
2. Every point satisfying the given condition(s) lies on the locus.
3. Every point on the locus satisfies the given condition(s).
4. The locus can be a straight or a curved line (or lines).

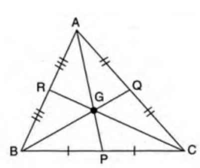
Certain Points related to Triangles

16.6 Important :

1. Each triangle has *three medians* which intersect each other at one point only. This point of intersection is called **centroid** of the triangle.

   In the figure alongside AP, BQ and CR are the medians of triangle ABC and are intersecting at point G.

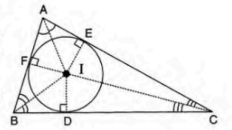
   ∴ **G** is the **centroid** of triangle ABC.



Centroid of a Δ always divides each median in the ratio 2 : 1

i.e. AG : GP = 2 : 1; BG : GQ = 2 : 1 and CG : GR = 2 : 1.

2. In each triangle, the bisectors of the interior angles meet at a point. This point of intersection is called **incentre** of the triangle and is equidistant from the sides of the triangle.

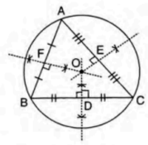


In the figure alongside, AI, BI and CI are the bisectors of angles at A, B and C respectively.

∴ I is the **incentre** of the triangle ABC.

Clearly : I is equidistant from the sides of the triangle, *i.e.* ID = IE = IF = radius of the incircle.

3. The perpendicular bisectors of the three sides of a triangle are concurrent (*i.e.* they intersect each other at the same point). This point of intersection is called **circumcentre** of the triangle and is equidistant from the vertices of the triangle.

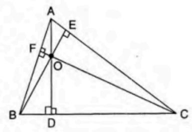


In the figure, DO, EO and FO are the perpendicular bisectors of the sides BC, CA and AB respectively.

∴ O is the **circumcentre** of the triangle and is equidistant from A, B and C.

i.e. OA = OB = OC = radius of the circumcircle.

4. Perpendiculars drawn from the vertices of a triangle to the opposite sides (*i.e.* altitudes), are concurrent and the point of concurrency is called **orthocentre** of the triangle.



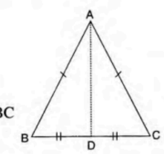
In the figure, AD, BE and CF are the altitudes corresponding to the sides BC, CA and AB respectively.

∴ Point O is the **orthocentre** of the triangle ABC.

respectively.

∴ Point O is the **orthocentre** of the triangle ABC.

5. In case of an isosceles triangle ABC, if AD is median, then it is bisector of angle A, perpendicular bisector of BC and altitude corresponding to BC

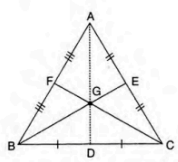


i.e. median AD = bisector of angle A

= perpendicular bisector of opposite side BC

= altitude corresponding to BC.

6. If ABC is an equilateral triangle, then



median AD = bisector of angle A

= perpendicular bisector of BC

= altitude corresponding to BC

median BE = bisector of angle B

= perpendicular bisector of AC

= altitude corresponding to AC

and the same is true for median CF also. Again, if G is centroid of equilateral triangle ABC then,

G = centroid of the Δ ABC = its incentre = its circumcentre = its orthocentre.

Theorem 5

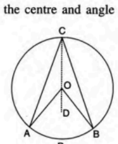
The angle at which an arc subtends an angle at the centre is double the angle it subtends at the circumference

The angle which, an arc of a circle subtends at the centre is double that which it subtends at any point on the remaining part of the circumference.

Given : A circle with centre O. Arc APB subtends angle AOB at the centre and angle ACB at point C on the remaining circumference.

To Prove : ∠AOB = 2∠ACB.

Construction : Join CO and produce it to a point D.



Proof :

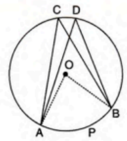
	Statement	***Reason***
	In Δ AOC;	
1.	OA = OC	Radii of the same circle.
2.	∴ ∠OAC = ∠OCA	Angles opposite to equal sides of a Δ are equal.
3.	∠AOD = ∠OAC + ∠OCA	Exterior angle of a Δ = sum of its interior opposite angles.
	= ∠OCA + ∠OCA	From (2) : ∠OAC = ∠OCA
	= 2∠OCA	
4.	Similarly, in Δ BOC, Ext. ∠BOD = 2∠OCB	
5.	**∠AOB** = ∠AOD + ∠BOD	
	= 2∠OCA + 2∠OCB	From (3) and (4)
	= 2 (∠OCA + ∠OCB)	
	= **2∠ACB**	

Hence Proved.

Theorem 6

Angles in from the same segment of a circle are equal

Angles in the same segment of a circle are equal.



Given : A circle with centre O. Angle ACB and angle ADB are in the same segment.

To Prove : ∠ACB = ∠ADB.

Construction : Join OA and OB.

Proof :

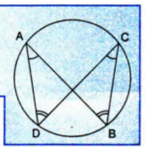
	Statement	***Reason***
1.	Arc APB subtends angle AOB at the centre and angle ACB at point C of the remaining circumference.	
	∴ ∠AOB = 2∠ACB	Angle at the centre is twice the angle at remaining circumference.
2.	Similarly, ∠AOB = 2∠ADB	" "
3.	∴ **∠ACB = ∠ADB**	From (1) and (2)

Hence Proved.

Similarly, in the adjoining figure :

(i) ∠DAB = ∠DCB [Angles in the same segment]

(ii) ∠ADC = ∠ABC [Angles in the same segment]

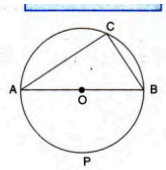


Theorem 7

Theorem 7

All angles of a semicircle are right angles

The angle in a semi-circle is a right angle.



Given : A circle with centre O. AB is a diameter and ACB is the angle of semi-circle.

To Prove : ∠ACB = 90°.

Proof :

	Statement	***Reason***
1.	Arc APB subtends ∠AOB at the centre and ∠ACB at pt. C of remaining circumference.	
	∴ ∠AOB = 2∠ACB	Angle at the centre is twice the angle at remaining circumference.
2.	∠AOB = 180°	AOB is a straight line.
3.	2∠ACB = 180°	From (1) and (2)
	⇒ **∠ACB = 90°**	

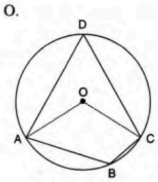
Hence Proved.

Theorem 8

Opposite angles of a cyclic quadrilateral add up to 180

Given : A quadrilateral ABCD inscribed in a circle with centre O.

To Prove : $\angle ABC + \angle ADC = 180°$
and, $\angle BAD + \angle BCD = 180°$.



Construction : Join OA and OC.

Proof :

Statement	Reason
1. Arc ABC subtends angle AOC at the centre and angle ADC at point D of the remaining circumference.	
$\therefore \angle AOC = 2\angle ADC$	Angle at the centre is twice the angle at remaining circumference.
$\Rightarrow \angle ADC = \frac{1}{2}\angle AOC$	
Similarly,	
2. $\angle ABC = \frac{1}{2}$ reflex $\angle AOC$	
3. $\angle ABC + \angle ADC$	
$= \frac{1}{2}$ (reflex $\angle AOC + \angle AOC$)	From (1) and (2)
$= \frac{1}{2} \times 360° = 180°$	Reflex $\angle AOC + \angle AOC = 360°$

Similarly, $\angle BAD + \angle BCD = 180°$ **Hence Proved.**

Theorem 9

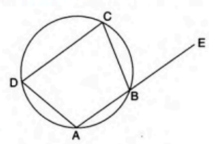
An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle

Theorem 9

The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.

Given : A cyclic quadrilateral ABCD whose side AB is produced to a point E.

To Prove : Ext. $\angle CBE = \angle ADC$



Proof :

Statement	Reason
1. $\angle ABC + \angle CBE = 180°$	$\angle ABC + \angle CBE = \angle ABE = 180°$
2. $\angle ABC + \angle ADC = 180°$	Opp. angles of a cyclic quadrilateral are supplementary.
$\therefore \angle ABC + \angle CBE = \angle ABC + \angle ADC$	From (1) and (2)
$\Rightarrow \angle CBE = \angle ADC$.	

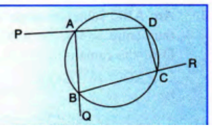
Hence Proved.

Similarly, in the adjoining figure :

(i) $\angle PAB = \angle BCD$,

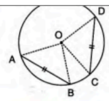
(ii) $\angle QBC = \angle ADC$,

(iii) $\angle RCD = \angle BAD$ and so on.



Observations related to circles

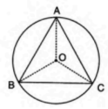
1. In a circle, equal chords subtend equal angles at the centre of the circle.



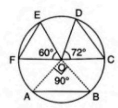
That is : if chord AB = chord CD
$\Rightarrow$ $\angle AOB = \angle COD$
Conversely, if $\angle AOB = \angle COD$
$\Rightarrow$ chord AB = CD

Clearly, O is the centre of the circle.

2. In the given figure, ABC is an equilateral triangle inscribed in a circle with centre O,
Since, AB = BC = AC
$\Rightarrow \angle AOB = \angle BOC = \angle AOC = \frac{360°}{3} = 120°$



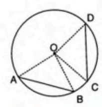
3. In the given figure, O is the centre of a circle. AB is the side of a square, CD is the side of a regular pentagon and EF is the side of a regular hexagon, then



$\angle AOB = \frac{360°}{4} = 90°$, $\angle COD = \frac{360°}{5} = 72°$

and $\angle EOF = \frac{360°}{6} = 60°$

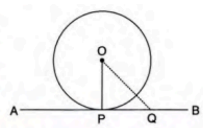
4. In the given figure, if
Chord AB : chord CD = 7 : 5,
$\angle AOB : \angle COD = 7 : 5$
And, if AB = 2CD $\Rightarrow \angle AOB = 2\angle COD$



Theorem 10

The tangent at any point of a circle is perpendicular to the radius at that point

The tangent at any point of a circle and the radius through this point are perpendicular to each other.



Given : A circle with centre O. AB is a tangent to the circle at point P and OP is the radius of the circle.

To Prove : OP ⊥ AB.

Construction : Take a point Q (other than P) on the tangent AB. Join OQ.

Remember :
Out of all the line segments drawn from a given point to a given line; the perpendicular is the shortest.

Proof :

Statement	*Reason*
1. OP < OQ	Since, each point of the tangent, other than point P, is outside the circle.
2. Similarly, it can be shown that out of all the line-segments which would be drawn from point O to the tangent line AB; OP is the shortest.	
3. ∴ **OP ⊥ AB**	The shortest line segment, drawn from a given point to a given line, is perpendicular to the line.

Hence Proved.

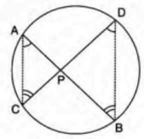
Remember :
1. No tangent can be drawn to a circle through a point inside the circle.
2. One and only one tangent can be drawn through a point on the circumference of the circle.
3. Only two tangents can be drawn to a circle through a point outside the circle.

Theorem 12

If two chords of a circle intersect internally or externally the product of the lengths of their segments will be equal

If two chords of a circle intersect internally or externally then the product of the lengths of their segments is equal.

Case I : *When chords intersect internally*



Given : Chords AB and CD of a circle intersect each other at point P inside the circle

To Prove : PA × PB = PC × PD

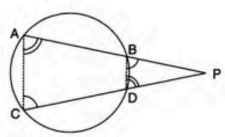
Construction : Join AC and BD.

Proof :

Statement	***Reason***
In Δ APC and Δ BPD,	
∠A = ∠D	Angles of the same segment
∠C = ∠B	"
⇒ Δ APC ~ Δ BPD	By A.A. Postulate
⇒ $\frac{PA}{PD} = \frac{PC}{PB}$	Corresponding sides of similar Δs are proportional
⇒ **PA × PB = PC × PD**	

Hence Proved.

Case II : *When chords intersect externally.*



Given : Chords AB and CD of a circle, when produced, intersect each other at point P outside the circle.

To Prove : PA × PB = PC × PD

Construction : Join AC and BD.

Proof :

Statement	***Reason***
In Δ PAC and Δ PDB,	
∠A = ∠PDB	Ext. angle of a cyclic quad. = Int. opp. angle
∠C = ∠PBD	" "
⇒ Δ PAC ~ Δ PDB	By A.A. Postulate
⇒ $\frac{PA}{PD} = \frac{PC}{PB}$	Corresponding sides of similar triangles are proportional
⇒ **PA × PB = PC × PD**	

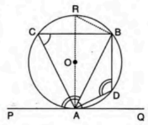
Hence Proved.

Theorem 13

The angle between a tangent and a chord is equal to the angle in the alternate segment

Theorem 13

The angle between a tangent and a chord through the point of contact is equal to an angle in the alternate segment.



Given : A circle with centre O. Tangent PQ touches the circle at point A. Through A, the point of contact, a chord AB is drawn.

To Prove : The angle between tangent PQ and chord AB through the point of contact A is equal to the angle in the alternate segment *i.e.* if C is point on major arc AB and D is a point on minor arc AB; then

$\angle BAQ = \angle ACB$

and, $\angle BAP = \angle ADB$

Construction : Draw the diameter AOR and join RB.

Proof :

Statement		Reason
In Δ ABR,		
$\angle ABR = 90°$		Angle of a semi-circle
$\Rightarrow \angle ARB + \angle RAB = 90°$	I	
$\angle OAQ = 90°$		Angle between the radius and the tangent
$\Rightarrow \angle RAB + \angle BAQ = 90°$	II	
$\therefore \angle ARB + \angle RAB = \angle RAB + \angle BAQ$		From I & II
$\Rightarrow \angle ARB = \angle BAQ$	III	
But, $\angle ARB = \angle ACB$	IV	Angles of the same segment
$\therefore$ **$\angle BAQ = \angle ACB$**		From III & IV

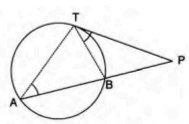
Hence Proved

Theorem 14

If a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.

Theorem 14

If a chord and a tangent intersect externally, then the product of the lengths of the segments of the chord is equal to the square of the length of the tangent from the point of contact to the point of intersection.



Given : Chord AB and tangent TP of a circle intersect each other at point P outside the circle.

To Prove : $PA \times PB = PT^2$

Construction : Join TA and TB.

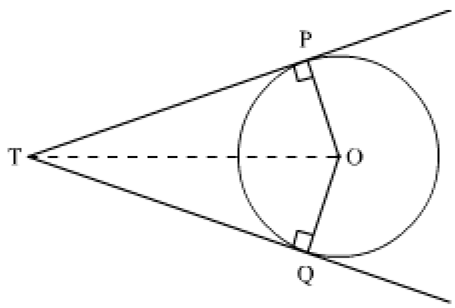
Proof :

Statement	***Reason***
In Δ PAT and Δ PTB,	
$\angle PTB = \angle A$	Angle in the alternate segment
$\angle P = \angle P$	Common
$\Rightarrow \Delta PAT \sim \Delta PTB$	By A.A. Postulate
$\Rightarrow \frac{PA}{PT} = \frac{PT}{PB}$	Corresponding sides of similar Δs are proportional
$\Rightarrow$ **$PA \times PB = PT^2$**	

Hence Proved.

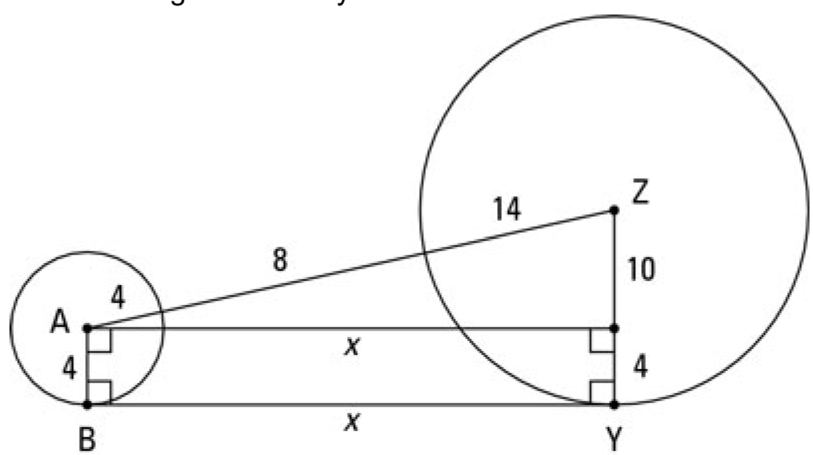
Theorem 15

There are two and only two tangents that can be drawn from a point outside the circle. Both these tangents are of equal length.



Theorem 16

Finding the common tangent:
Draw the diagram this way

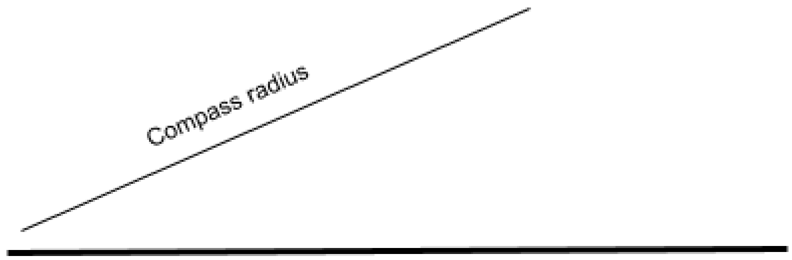


Construction

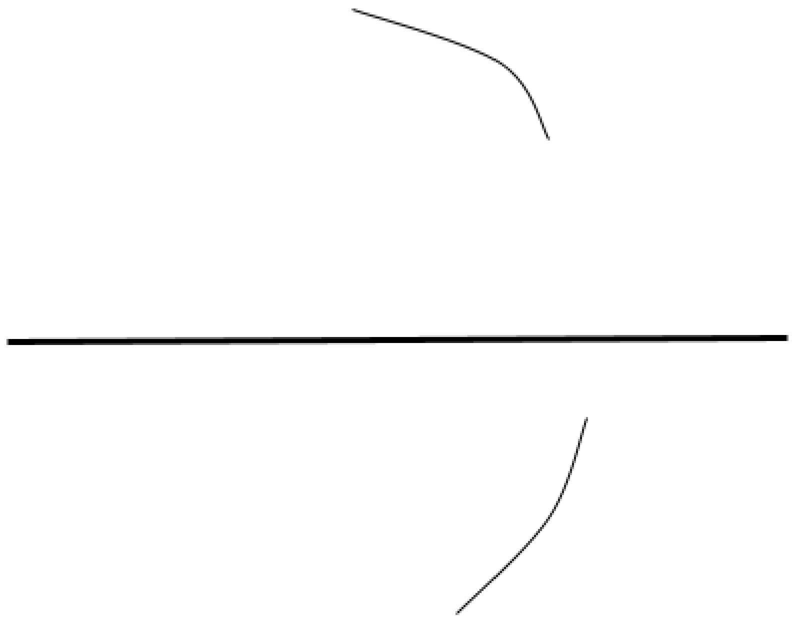
Perpendicular

Step 1: Draw a line.

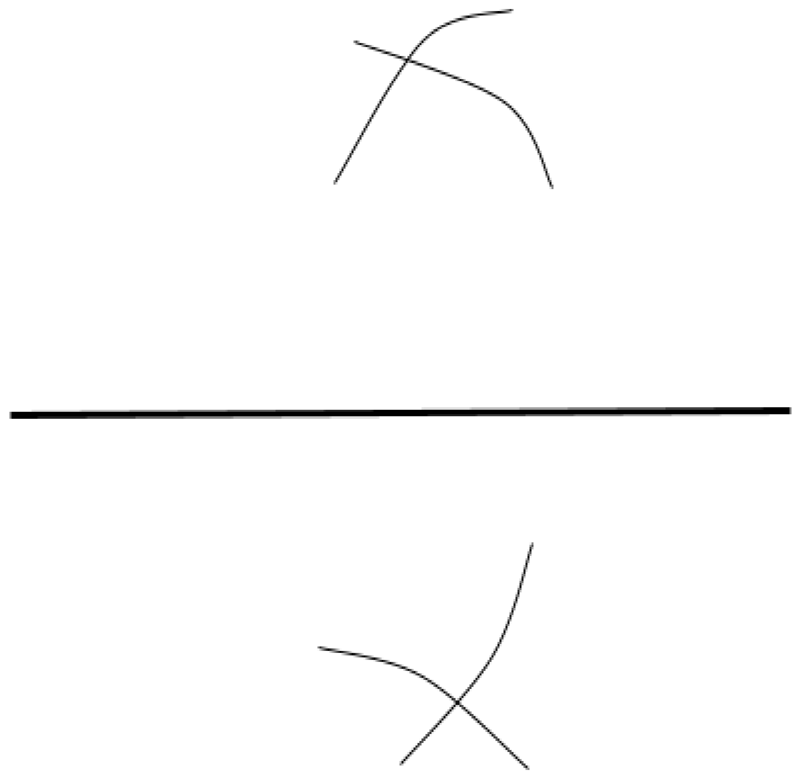
Step 2: Take your compass needle on one end point of the line and stretch it a little farther than the midpoint of your line



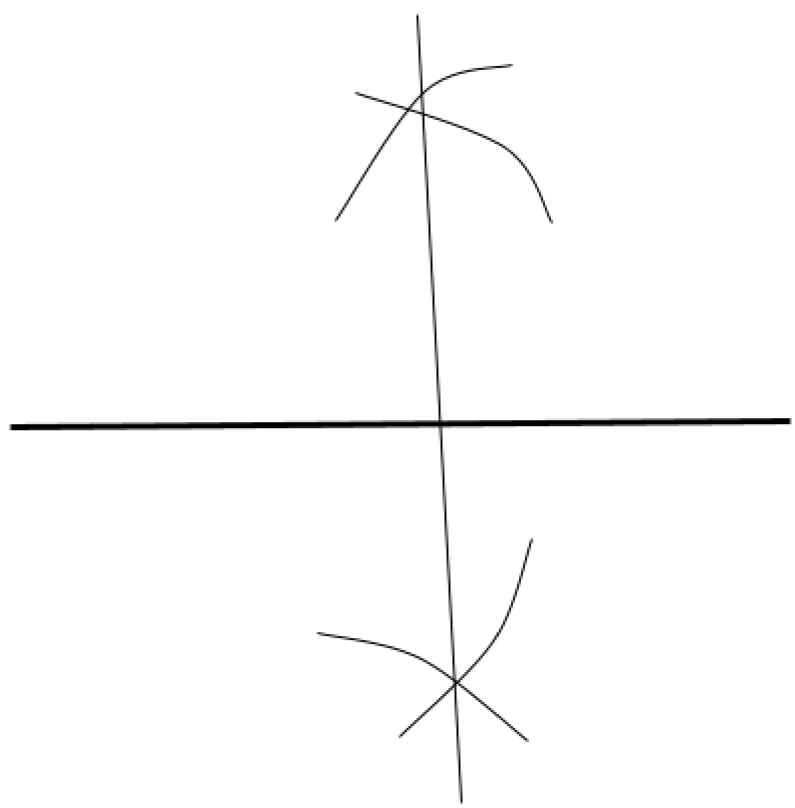
Step 3: Draw two arcs on either side of the line



Step 4: Take the compass needle to the other point keeping the same radius and then repeat step 3.



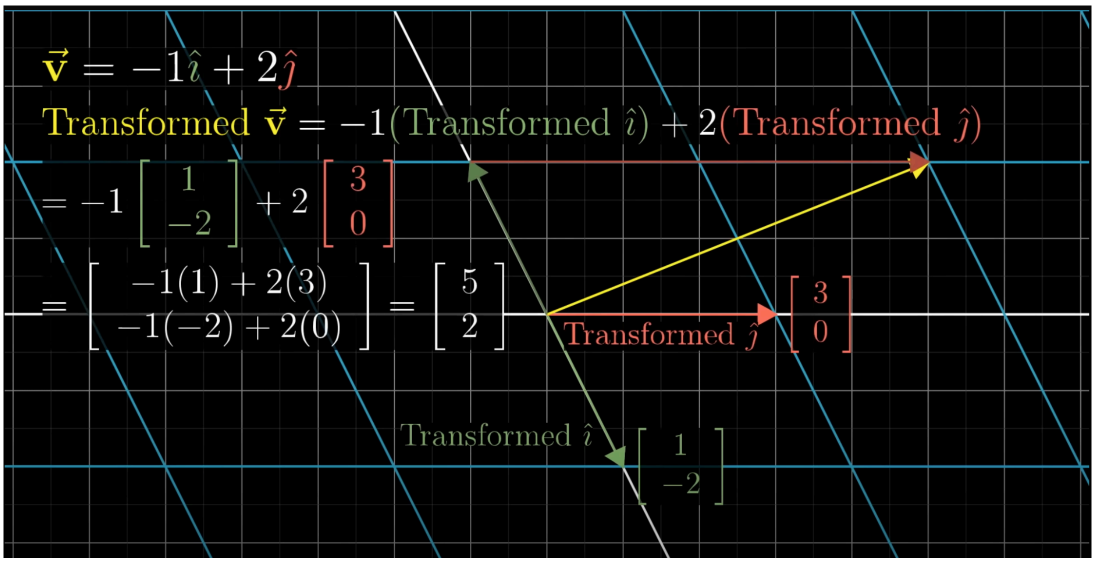
Step 5: Draw a line b/w the intersections

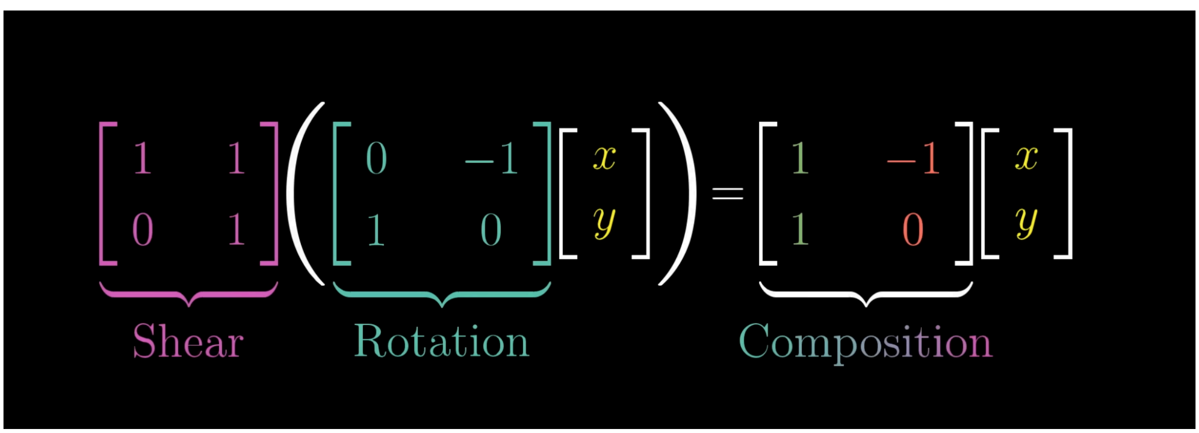


Angular Bisector

- Draw an arc b/w the two lines
- Keep the compass needle on the intersections that the og arc makes with the angle's lines
- Mark one small arc from each point
- Connect the origin of the angle and the point where the two small arcs intersect

Matrice Multiplication Intution



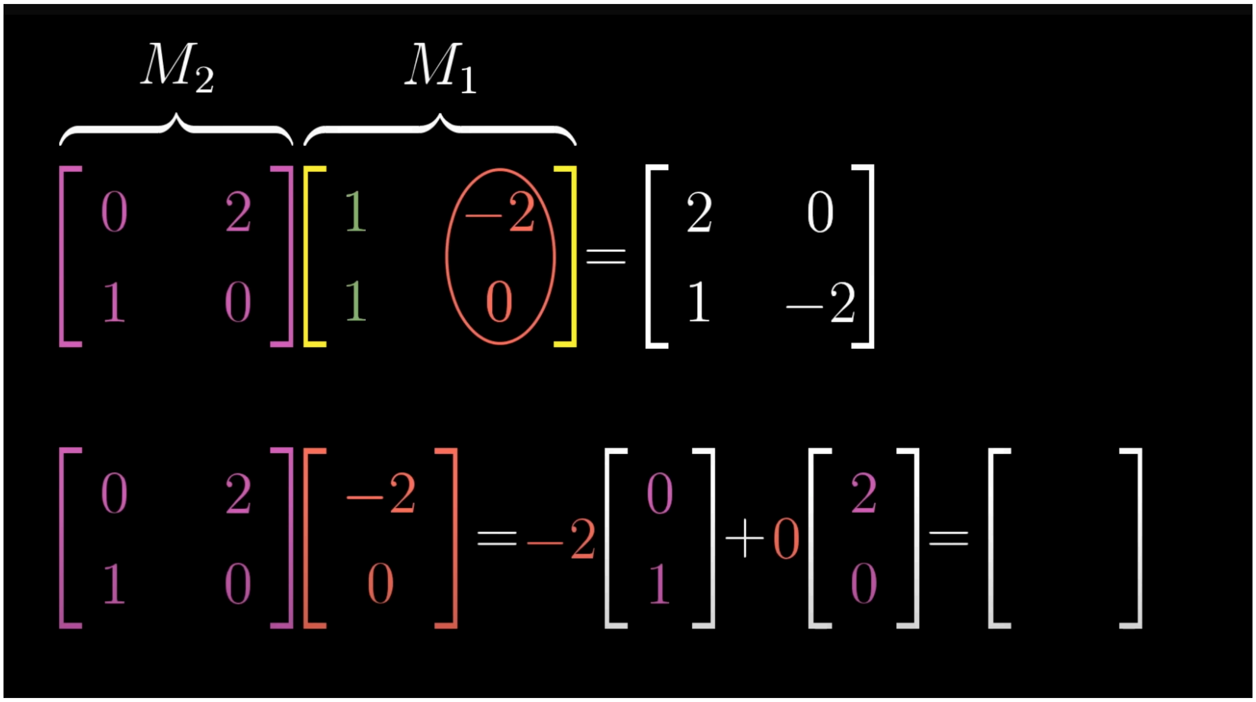


Read right to left

$$\underbrace{\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}}_{\text{Shear}} \underbrace{\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Rotation}} = \underbrace{\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}}_{\text{Composition}}$$

$$\overbrace{\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix}}^{M_2} \overbrace{\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}}^{M_1} = \begin{bmatrix} ? & ? \\ ? & ? \end{bmatrix}$$

$$\begin{bmatrix} 0 & 2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 1 \begin{bmatrix} 2 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$



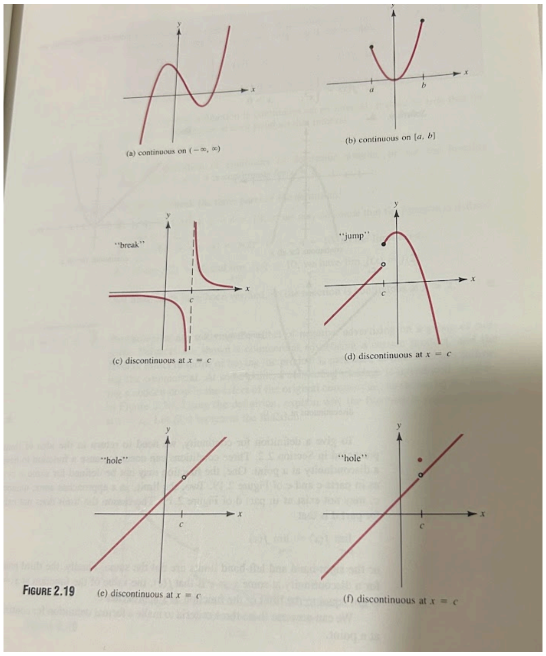
Calculus

Limits

- Functions need not necessarily mean just whole number values.
- For Example, there could be a function like:

$$f(x) = \frac{x-1}{x-1}$$

- If we substitute x = 1 over here, we get an undefined value (0 divided by 0)
- However, you could also cancel both values out and get 1, as long as $x \neq 1$
- Meaning that on a graph, that point is undefined. It's a hole.
- A limit is what a value is as it approaches a certain value, as long as it is NOT that value.
- We write this like:
- $\lim_{x \to 1} f(x) = 1$
- So the limit [or the value of the function f(x)] approaches 1 as x approaches 1
- There are various types of funny and weird graphs with gaps in them (see below)



- A limit is what the value of f(x) seems to approach as the value of x tends to go towards a certain point, from forwards or backwards
-

Dec 13, 2024

Maxima and Minima

Resources:

Finding Local Maximum and Minimum Values of a Function - Relative Extrema

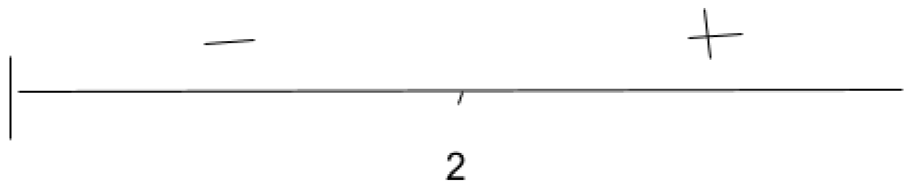
Second Derivative Test

ChatGPT: <insert links here>

Learnings

- The inverse of a function will be a reflection of that function along the long y = x line (x^2 and $\sqrt[2]{x}$ or a^x and $log\ x$
- This is because y and x are interchanged
- Local maxima and minima are the highest and lowest points on the graph that we can see / limit we are confined to
- How do you find this?
    - Take the function and take its derivative
    - Taking Derivatives using power rule:
        - Derivative of x^n is nx^{n-1}
        - If it is 2x it will become 2
        - Derivative of a constant is 0

- Set the first order derivative $f'(x) = 0$
- You will get a few x values
- Now: we can put this on a number line, check how the sign of the value changes
    - For example my x value was 2 and when i substitute x = 1 the values are going negative and x = 3 values are positive: this means the x value is for minima



- Alternatively we can use the second order derivative (derivative of the derivative) to do this. It will tell us how the function moves on either side of the point.
- We get the second derivative and substitute the x value we got into it
    - If this value is negative, then the x value is for maxima (since it decreases on either side of the function)
    - If this value is positive, then the x value is for minima (since it increases on both sides)
    - *(negative slope means the graph is going downwards, positive means it's going upwards)*.