ARKANSAS MATHEMATICS STANDARDS SUPPORT DOCUMENT

Grades K-2

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KINDERGARTEN MATH STANDARDS & NOTES

Number & Place Value

Counting & Number Foundations

Students know the number names and count sequence while exploring the relationships between numbers.

K.NPV.1: Count to 100 by ones and tens; count forward by ones from any given number up to 100.

Teacher Note:

- Learning focuses on forward rote counting. Understanding of quantity and place value is developed in other standards. The focus here is on counting and matching number names with the numeral. This begins with counting from one, but moves as students develop this skill, to counting from any given number. Students also count forward by tens.

Example:

- Counting collections is an excellent practice for this standard.

K.NPV.2: Count a set of objects up to 20 using one-to-one correspondence, demonstrating that the last number stated indicates the number of objects in the set regardless of the arrangement.

Teacher Note:

- Learning focuses on the counting principles: one-to-one correspondence, conservation, and cardinality.
    - One-to-one correspondence shows the understanding that each object is matched to a number word. The types of objects or size of the objects does not matter.
    - Conservation reflects the fact that no matter the arrangement of objects or the order they are counted in the total does not change.
    - Cardinality is the understanding that the last number stated when counting expresses the numerical quantity of the objects or answers “how many?”

Example:

- Counting collections builds all of these critical counting components.

K.NPV.3: Identify the position of objects in a set using ordinal numbers (first, second, third, etc.).

Teacher Note:

- Learning focuses on the ordinal position of objects. Objects are all counted as one unit, but the order the object is in determines its ordinal name.

Example:

- A student counts a collection of toy cars. Each car is counted as one unit. However, they can each be labeled by their position in the counting sequence such as first, second, third, etc.

Glossary: ordinal number

K.NPV.4: Identify quickly a number of items in a set from 0 to 10 without counting.

Teacher Note:

- Learning focuses on both perceptual and conceptual subitizing.
    - Perceptual subitizing is instantly seeing how many objects are in a group without counting. Usually, this can be done with up to five objects.
    - Conceptual subitizing is quickly seeing small groups of objects as parts of a whole. This is done with objects greater than five.
        - Conceptual subitizing leads to derived facts.

Example:

- Dot patterns and Quick Looks are great resources to support subitizing.

Glossary: subitize

Place Value

Students understand the base ten place value system.

K.NPV.5: Read, write, and represent whole numbers from 0 to 20.

Teacher Note:

- Learning focuses on reading and writing numbers in numeral form not word form. Students may represent whole numbers using a variety of objects.
- Due to the varied progression of fine motor and visual development, a reversal of numerals is anticipated for the majority of students. While reversals should be pointed out to students, the emphasis is on using numerals to represent quantities rather than the correct handwriting of the actual number itself.

Glossary: whole numbers

K.NPV.6: Show equivalent forms of whole numbers up to 20 as groups of tens and ones, using manipulatives and drawings.

Teacher Note:

- Learning focuses on composing and decomposing numbers by forming a group of ten ones and some more ones to set the foundation for place value understanding.

Glossary: equivalent, decompose, whole numbers

Comparison

Students use place value understanding to compare numbers.

K.NPV.7: Use matching and counting strategies to compare the number of objects in one group to the number of objects in

another group (0 to 10) using the terms greater than, less than, or equal.

Teacher Note:

- Learning focuses on developing comparison vocabulary and various comparison strategies using tangible objects. This allows students to progress to abstract comparisons with just number words or numerals.

Example:

- Begin with two sets that are obviously different to build vocabulary understanding of greater than and less than.
- Include sets that have the same number to explore equal or same as.
- Once vocabulary has been solidified, begin comparing sets closer in number but with different objects, such as two different colored counters, and then progress to sets of the same objects.

K.NPV.8: Compare two whole numbers, using the terms greater than, less than, or equal.

Teacher Note:

- Learning focuses on connecting the learning in K.NPV.7 to comparing the value of numbers (0 to 10) in abstract forms.
- The usage of the symbols for greater than/less than should not be introduced in this grade level. Appropriate terminology for this standard would be more than/ greater than, less than, or the same as/equal to.

Glossary: whole numbers

Computation & Algebraic Reasoning

Operations & Properties

Students perform operations using place value understanding and properties of operations.

K.CAR.1: Use objects, fingers, mental images, drawings, sounds, acting out situations, or verbal explanations to represent addition and subtraction from 0 to 10.

Teacher Note:

- Learning focuses on understanding of the action that takes place with addition and subtraction or putting together and taking away.
- This standard is focused on understanding the concept of addition and subtraction, rather than reading and solving addition and subtraction number sentences (equations). Therefore, before introducing symbols (+, -, =) and equations, kindergarteners require numerous experiences using addition and subtraction vocabulary in order to attach meaning to the various symbols.
- Sounds could include snaps, claps, and taps.

K.CAR.2: Use objects or drawings to decompose numbers less than or equal to 10 into pairs in more than one way, recording each decomposition.

Teacher Note:

- Learning focuses on building conceptual number sense within 10 by putting together and taking apart numbers in

different ways. This is done within 5 first, building to 10 and with concrete objects.

Glossary: decompose

K.CAR.3: Use a drawing or equation to find the number that makes 10 when added to a given number.

Teacher Note:

- Learning focuses on extending learning of K.CAR.2 by making the connection between concrete objects and abstract numbers in equations.

Glossary: equation

K.CAR.4: Use manipulatives and various strategies to fluently add and subtract within 10.

Teacher Note:

- Learning focuses on building the conceptual foundation for basic addition and subtraction facts (within 10) through a variety of experiences with concrete objects and drawings.
- For more information about fluency, see the Fluency Support Document.

Glossary: fluency

Problem Solving

Students solve real-world problems.

K.CAR.5: Solve real-world problems involving addition and subtraction within 10, using objects, drawings, or equations to represent the problem.

Teacher Note:

- Learning focuses on making connections between real life and abstract mathematical concepts. This should be done from the beginning and continued throughout the year to provide context for students to make sense of the mathematics.
- Learning focuses on the action and relationship in the context instead of using keywords.

Example:

- See the K-8 Glossary & Appendix for Addition and Subtraction Problem Types

Glossary: equation

Geometry & Measurement

Shapes

Students analyze attributes of shapes to develop generalizations about their properties.

K.GM.1: Describe the positions of objects and geometric shapes in the environment.

- Terms include: inside, outside, between, above, below, near, far, under, over, up, down, behind, in front of, next to, to the left of, and to the right of.

Teacher Note:

- Learning focuses on building vocabulary and spatial sense to allow students to describe and identify shapes and the environment around them.
- Shapes used should correspond with those listed in K.GM.2.

K.GM.2: Name shapes correctly regardless of their orientation or overall size.

- Shapes include: squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres.

Teacher Note:

- Learning focuses on identifying the shapes listed based on measurable attributes versus non-measurable attributes. Students should be exposed to all the shapes included along with the correct name for each.
- The intent is for students to be able to identify the shapes based on the appearance of measurable attributes instead of those that are non-measurable. In other words, know what a square, circle, triangle, etc. look like no matter what color, size, or orientation but based on the shape of the figure.

Example:

- Measurable attributes - number of sides and/or vertices (angles).
- Non-measurable attributes - color, size, orientation.

Glossary: cube, cone, cylinder, sphere

K.GM.3: Identify two-dimensional attributes of three-dimensional objects.

Teacher Note:

- Learning focuses on the understanding that three-dimensional objects are made out of two-dimensional shapes. The shapes used should be limited to those listed in K.GM.2.
- Correct academic terminology including faces, edges, and vertices should be used.

Glossary: attribute

K.GM.4: Analyze and sort a variety of two and three-dimensional shapes using informal language to describe their similarities, differences, and other attributes.

Teacher Note:

- Learning focuses on analyzing shapes for what they have in common and how they are different in order to help students identify general characteristics that make up the shapes around them.
- The following two and three-dimensional shapes are grade-level appropriate:
    - 2D shapes: squares, circles, triangles, rectangles, and hexagons
    - 3D shapes: cubes, cones, cylinders, and spheres

Glossary: attribute

K.GM.5: Compose and draw shapes found in the world using objects (e.g., straws, toothpicks, clay balls).

Teacher Note:

- Learning focuses on providing opportunities for students to construct objects from shapes they see around them, thus helping them be more aware of those shapes and their characteristics.

Measurement Concepts

Students develop understanding of measurement terms and concepts.

K.GM.6: Make direct comparisons of the length, capacity, weight, and temperature of objects, recognizing which object is shorter/longer, lighter/heavier, warmer/cooler, or holds more.

Teacher Note:

- Learning focuses on understanding of measurement terms not numerical values.
- Kindergarten students are not expected to measure objects with standard or non-standard units.

Example:

- When comparing two pencils of different lengths, students can identify which pencil is longer.

Glossary: length

Time & Money

Students explore time and money values and concepts.

K.GM.7: Understand concepts of time, recognizing that clocks and calendars are tools that measure time.

- Concepts of time include: morning, afternoon, evening, today, yesterday, tomorrow, day, week, month, and year.

Teacher Note:

- Learning focuses on general concepts that define how we measure time.

K.GM.8: Identify pennies and dimes by name and value.

Teacher Note:

- Learning focuses on identifying the two types of coins listed and knowing the value of each. Counting sets of these is not mentioned in this standard although it may be connected to counting by ones and tens in K.NPV.1.

Example:

- Counting collections is a way to address this standard.

Glossary: value

Data Analysis

Charts, Graphs, & Tables

Students organize and analyze data.

K.DA.1: Collect, sort, and organize data into two or three categories, using real-object graphs and picture graphs.

Teacher Note:

- Learning focuses on understanding what data is as well as how it can be collected and organized.

Glossary: data

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GRADE 1 MATH STANDARDS & NOTES

Number & Place Value

Counting

Students extend the counting sequence.

1.NPV.1: Count forward and back within 120 by ones and tens from any given whole number.

Teacher Note:

- Learning focuses on extending rote counting to 120 and building on place value patterns and understanding.

1.NPV.2: Skip count forward by multiples of fives within 120.

Teacher Note:

- Learning focuses on rote counting by fives to extend place value pattern and understanding. This also connects with 1.GM.7, “counting collections of like coins including pennies, nickels, and dimes.”

Example:

- The phrase “by multiples of 5” means students can count by fives from 5, 10, 45, 75, etc. instead of having them count by fives from any given number such as 27.

Place Value

Students understand the base ten place value system.

1.NPV.3: Explain the place value of ones and tens in two-digit numbers, using concrete models, diagrams, numbers, or words.

Teacher Note:

- Learning focuses on making connections between the location of digits in a number and the value they represent in order to build place value understanding.
- In Kindergarten, everything was thought of as individual units, “ones”. In first grade, students are asked to unitize those ten individual ones as a whole unit: “one ten.”
- Students should understand -
    - 10 can be considered a bundle of ten ones - called a "ten.”
    - The numbers from 11 to 19 are composed of a ten and one, two, three, four, five, six, seven, eight, or nine ones.
    - The numbers 10, 20, 30, 40, 50, 60, 70, 80, 90 refer to one, two, three, four, five, six, seven, eight, or nine tens and 0 ones.

Glossary: place value

1.NPV.4: Read, write, and represent whole numbers up to 120, using concrete models or drawings, word form, base ten numerals, and expanded form.

Teacher Note:

- Learning focuses on connecting quantities to numerals and the written form of numbers in order to continue place value understanding.

Example:

- Using base-ten numerals (standard form or numerical form): 87
- Number name form (word form): eighty-seven
- Expanded form(s) including but not limited to:
  - $80 + 7 = 87$
  - $7 + 80 = 87$

Glossary: expanded form

1.NPV.5: Use concrete models or drawings to subtract multiples of 10 from multiples of 10 (within the range of 10-90), relate the strategy to a written expression or equation, and explain the reasoning used to solve.

Teacher Note:

- Learning focuses on patterns and understanding of ten in place value by using concrete models and connecting them to written equations.

Example:

- There are 70 students in the gym. 40 students leave. How many students are still in the gym?

Glossary: expression, equation

1.NPV.6: Use mental strategies to find 10 more or 10 less than a given two-digit number.

Teacher Note:

- Learning focuses on making connections with 1.NPV.5 and the use of concrete models to representations and then to mental strategies to subtract ten from tens. It is also an extension of 1.NPV.1.

Example:

- When the student is asked to solve 44 +10 or 44 - 10, they can solve it without counting by ones.

Comparison

Students use place value understanding to compare numbers.

1.NPV.7: Compare two two-digit numbers using symbols (<, =, >) based on the value of tens and ones in the given numbers.

Teacher Note:

- Learning focuses on applying place value understanding to compare numbers using comparison vocabulary and eventually symbols. Correct terminology should be used when comparing numbers and using symbols.

Fraction Foundations

Students build a conceptual understanding of fractions.

1.NPV.8: Partition circles and rectangles into two and four equal shares, describing the shares using the words halves, fourths, and quarters; understand that decomposing into more equal pieces creates smaller pieces.

Teacher Note:

- Learning focuses on fractional parts being equal shares or equal-sized portions of a whole and that they have special names. Also, understanding that the more fractional parts that are used to make the whole, the smaller the parts.
- Students understand that "the whole" is composed of two halves or four fourths.
- Students should use the terminology but are not required to recognize or write fraction notation (i.e., numerator over denominator, a/b).

Glossary: partition, decompose

Computation & Algebraic Reasoning

Operations & Properties

Students perform operations using place value understanding and properties of operations.

1.CAR.1: Add and subtract fluently within 10 with mastery by the end of first grade.

Teacher Note:

- Learning focuses on developing fluency from conceptual understanding, not rote memorization. Fluency includes three components - accuracy, efficiency, and flexibility.
- Students build on concrete experiences started in Kindergarten and move to use derived facts and eventually to master these basic facts.
- For more information about fluency, see the Fluency Support Document.

Glossary: fluency, mastery

1.CAR.2: Use computational fluency to add and subtract within 20 using manipulatives and/or a variety of strategies.

Teacher Note:

- Learning focuses on building fluency with addition and subtraction within 20 by providing opportunities for students to develop conceptual understanding. Fluency includes accuracy, efficiency, and flexibility.

Example:

- Possible Student Strategies: counting on, making ten, decomposing a number leading to ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums.
- Students are proficient with this standard if they solve using manipulatives or other strategies accurately, efficiently, and flexibly.
- For more information about fluency, see the Fluency Support Document.

Glossary: computational fluency, counting on, decompose, equivalent

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1.CAR.3: Apply properties of operations to add and subtract within 20.

Teacher Note:

- Learning focuses on the understanding and application of the properties when adding and subtracting. Teachers should use the correct terminology when referring to the properties, but students are not expected to use the terms.
- The understanding of the properties and how they work provides the foundation for derived facts and strategies needed for the flexibility and efficiency components of fluency.

Glossary: properties of operations, Associative Property of Addition, Commutative Property of Addition, Identity Property

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1.CAR.4: Use concrete models or drawings to add within 100, including a two-digit number and a one-digit number as well as a two-digit number and a multiple of ten; relate strategy used to a written expression or equation and explain reasoning.

Teacher Note:

- Learning focuses on developing strategies and making sense of adding one and two-digit numbers. As understanding progresses, connect expressions or equations to strategies used both vertically and horizontally.
- It is not the intent of the standard nor is it recommended for students to be exposed to the standard algorithm.

Example:

- There are 31 apples in the basket. Zeke put 7 more apples in the basket. How many apples are in the basket?
- There are 54 apples in the basket. Gavin put 20 more apples in the basket. How many apples are in the basket?
- For more information about fluency, see the Fluency Support Document

Glossary: expression, equation

1.CAR.5: Demonstrate the relationship between addition and subtraction by solving problems, using an inverse operation.

Teacher Note:

- Learning focuses on the relationship between addition and subtraction as stated in the standard. Understanding this relationship strengthens a student's ability to work flexibly with numbers and supports derived strategies such as counting up or counting back.
- Teachers should use the term "inverse," but students are not expected to do so.

Example:

- $12 + \square = 18 \rightarrow 18 - 12 = 6$

- $18 - \square = 12 \rightarrow 12 + \square = 18$

Glossary: inverse, counting back

Problem Solving

Students solve real-world problems.

1.CAR.6: Solve real-world problems involving addition and subtraction within 20.

- Problem types include: adding to, taking from, putting together, taking apart, and comparing with unknowns present throughout the addition and subtraction problem.

Teacher Note:

- Learning focuses on providing context for students to apply addition and subtraction strategies built in 1.CAR.2. This standard should be addressed before or in conjunction with 1.CAR.2 not after. Providing context for students supports conceptual understanding.
- Learning focuses on the action and relationship in the context instead of using keywords.
- Students may answer using a variety of strategies and representations from concrete to abstract.

Example:

- See Appendix for problem-type examples.

Glossary: conceptual understanding

1.CAR.7: Solve real-world problems involving addition of three whole numbers whose sum is less than or equal to 20.

Teacher Note:

- Learning focuses on providing opportunities for students to develop efficient strategies using the Associative Property of Addition.

Example:

- Justice has 6 apples. She picks 3 more apples. Caleb gives Justice 4 more apples. How many apples does Justice have in all?
  - Student A: $(6 + 4) + 3 = 13$
  - Student B: $(6 + 3) + (3 + 1) \rightarrow (9 + 1) + 3 = 13$

Glossary: whole numbers

Algebraic Concepts

Students develop and apply understanding of foundational algebraic concepts.

1.CAR.8: Apply understanding of the equal sign to determine if equations involving addition and subtraction are true or false.

Teacher Note:

- Learning focuses on the meaning of the equal sign in equations. The equal sign is a relational symbol, not an operational one. Whatever is on the left side of the equal sign must be equivalent to what is on the right.
- Use a balance to illustrate the equivalence needed on both sides of the equal sign.

Example:

- $14 = 7 + 7$ is true
- $15 = 10 + 3$ is false
- $8 = 8$ is true
- $12 = 8$ is false
- $9 + 1 = 4 + 5$ is false
- $4 + 1 = 3 + 2$ is true
- $8 + 4 = \square + 5$

Glossary: equation

1.CAR.9: Determine the unknown whole number in an addition or subtraction equation relating three whole numbers.

Teacher Note:

- Learning focuses on applying an understanding of the inverse relationship between addition and subtraction. This standard may be taught in conjunction with 1.CAR.5.

Example:

- $\square + 3 = 8$
- $3 + \square = 8$
- $11 - 8 = \square$

Glossary: equation

Geometry & Measurement

Shapes

Students analyze attributes of shapes to develop generalizations about their properties.

1.GM.1: Understand the difference between defining attributes (e.g., triangles are closed and three-sided shapes) and non-defining attributes (e.g., color, orientation, overall size), using that understanding to build and draw shapes that exhibit defining attributes.

Teacher Note:

- Learning focuses on recognizing the characteristics or attributes of shapes, specifically the ones that define the name of the shape. Correct academic terminology including faces, edges, and vertices should be used.
- Introduce the terminology of open and closed shapes.
- Shapes should include rectangles, squares, trapezoids, triangles, hexagons, half circles, and quarter circles.

Glossary: attribute

1.GM.2: Create a composite shape using two-dimensional or three-dimensional shapes.

- Two-dimensional shapes include: rectangles, squares, trapezoids, triangles, hexagons, half circles, and quarter circles.
- Three-dimensional shapes include: cubes, rectangular prisms, cones, and cylinders.

Teacher Note:

- Learning focuses on composing and decomposing shapes to build an understanding of the attributes of different shapes and to allow students to see them in a variety of perspectives and orientations.

Example:

- Students may solve shape puzzles, construct designs with specified shapes, or create and maintain a shape as a unit.

Glossary: composite shape, trapezoid, cube, rectangular prism, cone, cylinder

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Length & Width

Students investigate measurement with non-standard units.

1.GM.3: Express the length of an object as a whole number of units by laying multiple copies of a shorter object end-to-end, understanding that the length of one object is equal to the number of same-size units that span the object with no gaps or overlaps.

Teacher Note:

- Learning focuses on the concept of using a tool to measure objects starting with nonstandard units. Provide opportunities for students to estimate before measuring. Students should measure with units provided by the teacher such as snap cubes as well as with units they select themselves.

Glossary: length

1.GM.4: Order three objects by their length, indirectly comparing the lengths of two objects by using a third object.

Teacher Note:

- Learning focuses on using correct terminology when comparing the lengths of objects such as shorter, longer, taller, shortest, and tallest.

Example:

- Compare the length of objects to the length of a given marker. Find two objects that are longer than the marker and two that are shorter than the marker. Write a sentence for each comparison.

Glossary: length

Time & Money

Students explore time and money values and concepts.

1.GM.5: Tell and write time to the nearest hour and half hour using analog clocks; understand how to read hours and minutes using digital clocks.

Teacher Note:

- Learning focuses on introducing students to analog clocks and digital clocks.
- Students should be able to determine the hour and half hour on an analog clock. This will require students to understand that an hour is made of 60 minutes and that half of an hour is 30 minutes.
- Students should be able to correctly read the time, both the hour and minutes, on a digital clock.

1.GM.6: Identify coins by name and value, including penny, nickel, dime, and quarter.

Teacher Note:

- Learning focuses on recognizing pennies, nickels, dimes, and quarters by sight as well as knowing the value of each coin.

Glossary: value

1.GM.7: Count collections of like coins including pennies, nickels, and dimes to determine their total value up to 100 cents.

Teacher Note:

- Learning focuses on counting by ones, fives, and tens using coins with those values. This standard may be taught in conjunction with 1.NPV.1 and 1.NPV.2.
- Collections are limited to coins of the same value, not mixed collections.

Glossary: value

Data Analysis

Charts, Graphs, & Tables

Students organize and analyze data.

1.DA.1: Organize, represent, and interpret data with up to three categories (e.g., tally tables, picture graphs, bar graphs).

Teacher Note:

- Learning focuses on collecting and organizing data with multiple categories into different types of graphs.

Glossary: data

1.DA.2: Ask and answer questions about the total number represented such as how many in each category and how many more or less in one category compared to another.

Teacher Note:

- Learning focuses on answering questions using different types of graphs. This standard may be taught in conjunction with 1.DA.1.

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GRADE 2 MATH STANDARDS & NOTES

Number & Place Value

Counting

Students extend the counting sequence.

2.NPV.1: Count within 1,000 forwards and backwards by ones, tens, and hundreds from any given number.

Teacher Note:

- Learning focuses on flexibility with rote counting and solidifies patterns and understanding of place value.

Place Value

Students understand the base ten place value system.

2.NPV.2: Identify the value of hundreds, tens, and ones place in a three-digit number.

Teacher Note:

- Learning focuses on connecting the location of digits in a number and the value they represent in order to build place value understanding.
- Students should understand that 100 can be thought of as a group of ten tens — called a "hundred" and that the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine groups of 100

Glossary: value

2.NPV.3: Read, write, and represent whole numbers up to 1,000 using concrete models or drawings, number names, and a variety of expanded forms.

Teacher Note:

- Learning focuses on connecting quantities to numerals and the written form of numbers in order to continue place value understanding.
- Expanded form is a foundational skill for when students use place value strategies to add and subtract large numbers.

Example:

- Using base-ten numerals (standard form or numerical form): 347
- Number name form (word form): three-hundred forty-seven
- Expanded form(s) including but not limited to:
  - $300 + 40 + 7 = 347$
  - $40 + 7 + 300 = 347$
  - 12 tens + 5 ones = 120 + 5 = 125
  - 15 ones + 18 tens = 15 + 180 = 195

Glossary: expanded form, whole numbers

2.NPV.4: Mentally add 10 or 100 to a given number in the range of 100-900 and mentally subtract 10 or 100 from a given number in the range of 100-900.

Teacher Note:

- Learning focuses on solidifying place value patterns and understanding the tens and hundreds place.
- Provide opportunities for solving problems in which students work within the same hundred and then move to problems that cross the century once students have become comfortable adding and subtracting within the same hundred.

Comparison

Students use place value understanding to compare numbers.

2.NPV.5: Compare two three-digit numbers using symbols (<, =, >) based on the value of hundreds, tens, and ones in the given numbers.

Teacher Note:

- Learning focuses on applying place value understanding to compare numbers using comparison vocabulary and symbols. Teachers and students should use correct terminology.
- Students are expected to be able to compare three-digit numbers presented in various forms, therefore, this standard may be taught in conjunction with 2.NPV.3.

Fraction Foundations

Students build a conceptual understanding of fractions.

2.NPV.6: Partition circles and rectangles into two, three, or four equal shares, describing the shares using the words halves, thirds, and fourths (or quarters).

Teacher Note:

- Learning focuses on fractional parts being equal shares or equal-sized portions of a whole and having special names.
- Students should describe the whole as two halves, three thirds, four fourths.
- Students are expected to use the terminology but are not required to recognize or write fraction notation (i.e., numerator over denominator, a/b).

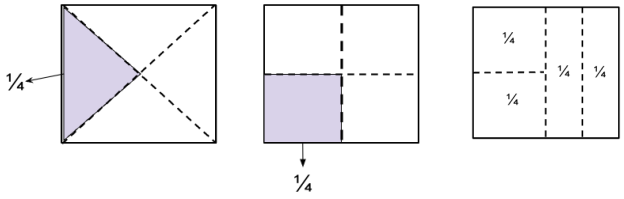
Glossary: partition

2.NPV.7: Recognize that equal shares of identical wholes need not have the same shape.

Teacher Note:

- Learning focuses on fractional parts being equal shares or equal-sized portions of a whole.
- Students should understand that equal shares of identical wholes may have different shapes and those shares or portions may not be symmetrical. The only requirement for equivalent fractions is that the shares or portions have an area that is equal.

Example:



Computation & Algebraic Reasoning

Operations & Properties

Students perform operations using place value understanding and properties of operations.

2.CAR.1: Use mental strategies to fluently add and subtract within 20 with mastery by the end of second grade.

Teacher Note:

- Learning focuses on fluency built from conceptual understanding, not rote memorization. Fluency includes three components - accuracy, efficiency, and flexibility.
- Students build on concrete experiences with numbers within 20 started in first grade and move to using derived facts and eventually to mastering these facts.
- For more information about fluency, see the Fluency Support Document.

Glossary: fluency, mastery

2.CAR.2: Use computational fluency to add and subtract within 100 using strategies based on place value, properties of operations, or the relationship between addition and subtraction.

Teacher Note:

- Learning focuses on developing strategies with addition and subtraction by providing opportunities for students to develop conceptual understanding. Fluency includes accuracy, efficiency, and flexibility.
- While many HQIM may introduce or teach the standard algorithm, fluency with multi-digit addition and subtraction as well as the standard algorithm is not expected until the end of fourth grade.
- For more information about fluency, see the Fluency Support Document.

Example:

- Possible Student Strategies: counting on/counting back, making tens, partial sums & differences, compensation, etc.

Glossary: computational fluency, place value, properties of operations, algorithm, standard algorithm

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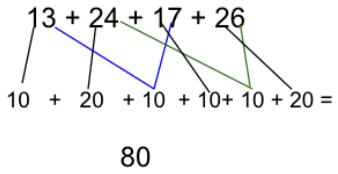
2.CAR.3: Add up to four two-digit numbers with sums not exceeding 100 using strategies based on place value and properties of operations.

Teacher Note:

- Learning focuses on extending the understanding in 2.CAR.2 and provides opportunities for students to use the properties of operations, specifically the Commutative Property of Addition, Associative Property of Addition, and the Identity Property.
- The intent of this standard is to extend conceptual understanding not for students to use the standard algorithm.

Example:

- 13 + 24 + 17 + 26 →(13 + 17) + (24 + 26) →30 + 50 = 80
- 

Glossary: place value, properties of operations

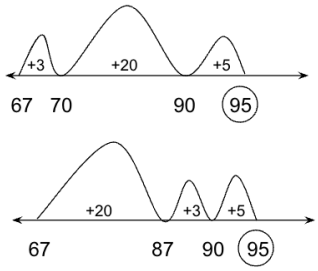
2.CAR.4: Use a number line to solve addition and subtraction problems within 100.

Teacher Note:

- Learning focuses on introducing students to the number line and using it to solve problems. Number lines may be numbered in a way that fits the problem and makes sense to the student (e.g., open number lines).

Example:

- 67 + 28



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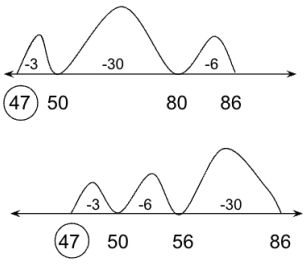

Glossary: number line ~~diagram~~

2.CAR.5: Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Teacher Note:

- Learning focuses on connecting repeated addition and equal groups to multiplication. This is foundational work for multiplication in third grade.

Glossary: rectangular array, equation, sum, addend

2.CAR.6: Use concrete models, drawings, or equations to solve addition and subtraction problems within 1000.

Teacher Note:

- Learning focuses on extending strategies and conceptual understanding to add and subtract multi-digit numbers. As understanding progresses, connect expressions or equations to strategies used both vertically and horizontally.
- While many HQIM may introduce or teach the standard algorithm, fluency with multi-digit addition and subtraction as well as the standard algorithm is not expected until the end of fourth grade.

Problem Solving

Students solve real-world problems.

2.CAR.7: Solve one and two-step real-world problems involving addition and subtraction within 100 in situations of adding to, taking from, putting together, taking apart, and comparing unknowns in all positions.

Teacher Note:

- Learning focuses on extending problem-solving experiences from one step to two steps.
- Learning focuses on the action and relationship in the context instead of using keywords.
- Students may answer using a variety of strategies and representations from concrete to abstract.

Example:

- See the Appendix for examples of each problem type.

Algebraic Concepts

Students develop and apply understanding of foundational algebraic concepts.

2.CAR.8: Determine whether a group of objects up to 20 has an odd or even number of members; write an equation to express an even number as a sum of two equal addends.

Teacher Note:

- Learning focuses on understanding even and odd numbers first with concrete objects and drawings then relating this understanding to doubles and doubles + 1 addition facts.
- While noticing that even numbers end in 0, 2, 4, 6, and 8 is an interesting and useful observation, it should not be used as the definition of an even number.

Glossary: equation, addend

Geometry & Measurement

Shapes

Students analyze attributes of shapes to develop generalizations about their properties.

2.GM.1: Identify, describe, and draw two-dimensional shapes.

- Shapes include: triangles, regular pentagons, regular hexagons, and quadrilaterals (squares, rectangles, trapezoids, parallelograms, rhombuses)

Teacher Note:

- Learning focuses on the defining attributes of two-dimensional shapes. Specified attributes should be limited to the number and length of sides as well as the number of vertices. Other terms referring to the relationship between the shape's sides or the measure of the angles are not introduced or addressed at this grade level.
- Correct academic vocabulary including faces, edges, and vertices should be used.

Glossary: attribute, quadrilateral, trapezoid

2.GM.2: Identify and describe three-dimensional shapes based on the shape, number of faces, number of edges, and number of vertices.

- Shapes include: rectangular prisms, cubes, and square-based pyramids.

Teacher Note:

- Learning focuses on the defining attributes of three-dimensional shapes. Specified attributes should be limited to the shapes of faces, number of faces, number of edges, and number of vertices. Students are not expected to draw three-dimensional objects.
- Correct academic terminology including faces, edges, and vertices should be used.

Glossary: vertex, rectangular prism, cube

Length & Width

Students investigate measurement using rulers.

2.GM.3: Select appropriate measurement tools to estimate and measure the length of an object to the nearest whole inch or whole centimeters.

Teacher Note:

- Learning focuses on understanding standard measurement tools and how to use them to measure the length of objects. This is a student's first experience with standard measurement tools, building from nonstandard measurement in first grade.

- Students estimate in order to choose the appropriate tool to measure given objects and to determine the nearest whole unit.
- Units are limited to inches (customary) and centimeters (metric) only.

Glossary: length

2.GM.4: Demonstrate how the length of an object does not change, regardless of the units used to measure it, by measuring the length of an object twice; use two different length units, describing how the two measurements relate to the size of the chosen unit.

Teacher Note:

- Learning focuses on understanding that the measurement of an object will change depending on the unit used to measure even though the length of the object does not change.

Example:

- A student measures a pencil using centimeters and then measures the same pencil using inches. The measurements will be different numbers because the units are different sizes even though the pencil is the same length.

Glossary: length

2.GM.5: Measure to determine how much longer or shorter one object is than another, expressing the length difference in terms of a standard length whole unit.

Teacher Note:

- Learning focuses on providing students opportunities to select measurement tools and measure the length of objects as well as use correct comparison and measurement terminology.
- The word “standard” refers to a unit of measurement. Both the Customary and Metric systems can be used.

Example:

- A student measures two pencils using inches. One pencil measures 7 inches and the other pencil measures 5 inches. The student explains that one pencil is 2 inches longer or shorter than the other, using the comparison term “longer” or “shorter” correctly and including the units used to measure.

Glossary: length

2.GM.6: Solve real-world problems involving lengths of the same units, using addition and subtraction within 100.

Teacher Note:

- Learning focuses on applying understanding of length and addition and subtraction skills to solve problems in context. Focus is on understanding the action or relationship in the context and noting the units used.

Example:

- See the Appendix for examples of addition and subtraction problem types.

Glossary: length

Perimeter, Area, & Volume

Students explore the perimeter and area of shapes.

2.GM.7: Solve real-world and mathematical problems to find the perimeter of polygons.

Teacher Note:

- Learning focuses on extending understanding of length to find the perimeter of shapes in context.

Example:

- Polly drew a picture and is framing it with a ribbon. Her picture is rectangular. One side measures 9 inches and the other side measures 14 inches. How many inches of ribbon will she need?

Glossary: perimeter, polygons

2.GM.8: Partition a rectangle into rows and columns of same-size squares, counting the total number of squares to find the area.

Teacher Note:

- Learning focuses on introducing students to area as another attribute of shapes. This learning can be connected to 2.CAR.5.
- Students are only meant to explore the concept of area using concrete and pictorial representations not to calculate area using a formula.

Example:

- Students compare shapes to determine which is the largest. Ask them to explain why they chose each shape. This focuses students on the attributes of shapes that determine area.
- Students cover shapes (or tile them) with pattern blocks and discuss which shape required more pattern blocks. Use the same type of pattern blocks to use same units.

Glossary: partition, area

Time & Money

Students explore time and money values and concepts.

2.GM.9: Using an analog clock, tell and write time to the nearest five minutes using colon notation and indicate a.m. or p.m.

Teacher Note:

- Learning focuses on extending student understanding of analog clocks and recording time using colon notation.

Building on the ability to count by fives, students determine time to the nearest five minutes.

2.GM.10: Describe relationships of time.

- Times include: seconds in a minute; minutes in an hour; hours in a day; days in a week; and days, weeks, and months in a year.

Teacher Note:

- Learning focuses on understanding the structure of time measurement from seconds to months in a year.
- Students should know the time relationships in isolation and apply them in problem solving situations.

2.GM.11: Solve real-world problems involving addition and subtraction of time intervals in half hours or hours.

Teacher Note:

- Learning focuses on applying understanding of addition and subtraction to minutes and hours in context.

Example:

- Kevin arrived at the bus stop at 7:00 a.m. He waited 30 minutes for his bus to arrive. What time did his bus arrive?
- Emily had soccer practice at 4:00 p.m. The practice lasted 2 hours. What time did soccer practice end?

Glossary: interval

2.GM.12: Count collections of mixed coins and solve real-world problems involving quarters, dimes, nickels, and pennies within 99¢ and whole dollar amounts.

Teacher Note:

- Learning focuses on building on previous work with coins by counting mixed collections and solving contextual problems with money. Numbers should be 99¢ or less or contain whole number dollar amounts, not a combination of coins and dollar amounts. Decimals are not introduced until fourth grade.

Example:

- A student counts a cup of mixed coins equaling 52¢.
- Jacob has 12¢ in his bank. He finds a dime and three pennies under his bed. How much money does Jacob have altogether?
- Jacob received $20 from his friend for his birthday. Jacob also got $2 from the tooth fairy. How much money does Jacob have altogether?

Data Analysis

Charts, Graphs, & Tables

Students organize and analyze data.

2.DA.1: Use bar graphs, picture graphs, and line plots to organize and represent data, interpreting data with up to four

categories.

Teacher Note:

- Learning focuses on extending previous knowledge of graphs to include more categories and introduce line plots.
- Students may collect data or be given a data set in order to organize, represent, and interpret the data.

Glossary: line plot, data

2.DA.2: Ask and answer simple put together, take apart, and compare problems, using information presented in the bar graphs, picture graphs, and line plots.

Teacher Note:

- Learning focuses on answering questions using different types of graphs. This standard may be taught in conjunction with 2.DA.1.

Glossary: line plot

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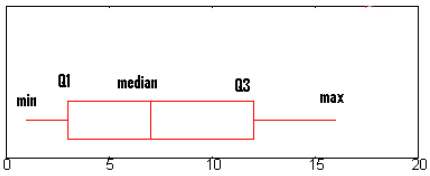
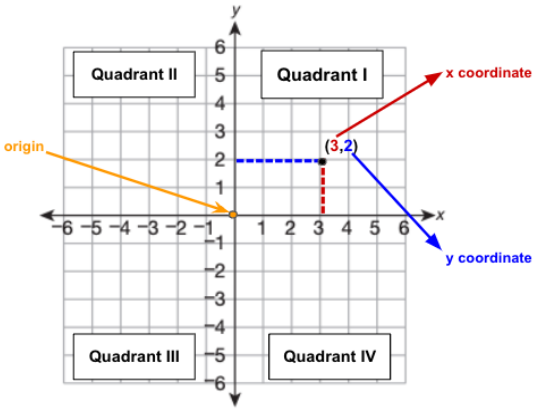
K - 8 GLOSSARY

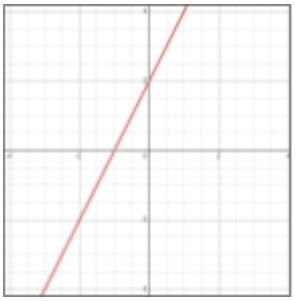
Absolute Value	A number's distance from 0 on the number line.
Addend	Any of the numbers added to find a sum.
Additive Comparison	Compare two amounts by asking how much more or less is one amount than the other.
Additive Inverse	A number that when added to another number gives a sum of 0. Example: $1 + (-1) = 0$.
Algorithm	An explicit step-by-step procedure for performing a mathematical computation or for solving a mathematical problem.
Area	The measure of the size of the interior of a figure, expressed in square units.
Association	The relationship between two variables.
Associative Property of Addition	A property of numbers that states that the sum of a set of numbers is the same, regardless of how the numbers are grouped. Example: (4 + 8) + 3 = 4 + (8 + 3)
Associative Property of Multiplication	A property of numbers that states that the product of a set of numbers is the same, regardless of how the numbers are grouped. Example: $(4 \bullet 8) \bullet 3 = 4 \bullet (8 \bullet 3)$
Attributes	Characteristics or properties of an object.
Axis	A vertical or horizontal number line, both of which are used to define a coordinate grid. The horizontal axis is the x-axis, and the vertical axis is the y-axis. The plural of axis is axes.
Benchmark Fraction	A common fraction used when comparing other fractions (e.g., $\frac{1}{2}$, $\frac{1}{4}$).
Bivariate Data	Data that has two variables.
Box Plot	A way to show the interquartile range, mean, and median of a data set. Also known as a box and whisker plot. 
Cardinality	The understanding that when you count items, the number word applied to the last object counted represents the total amount.
Circle	A two-dimensional figure for which all points are the same distance from its center. A circle is identified by its center point and radius.
Circumference	The perimeter of a circle, which is the distance around a circle.
Coefficient	A number or variable by which a variable is multiplied. Examples: $3x + 7$, 3 is the coefficient; $y = mx + b$, m is the coefficient.
Commutative Property of Addition	A property of numbers that states that the sum of two terms is unaffected by the order in which the terms are added; i.e., the sum remains the same. Example: $5 + 9 = 9 + 5$

Commutative Property of Multiplication	A property of numbers that states that the product of two factors is unaffected by the order in which the factors are multiplied, i.e., the product remains the same. Example: $5 \bullet 9 = 9 \bullet 5$
Complementary Angles	Two angles (adjacent or nonadjacent) whose sum is 90 degrees.
Complex Fraction	A fraction in which the numerator, denominator, or both are fractions themselves. Example: $\frac{\frac{1}{2}}{\frac{2}{3}}$ or $\frac{2}{\frac{3}{4}}$
Composite	A whole number with at least one whole number factor besides 1 and itself.
Composite Shape	A shape that can be divided into more than one of the basic shapes. Also called complex or compound shapes.
Computational Fluency	To have efficient and accurate methods for computing. Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand, and can explain these methods as well as produce accurate answers efficiently.
Conceptual Understanding	Students with conceptual understanding know more than isolated facts and methods. They understand why a mathematical idea is important and how it works. Conceptual understanding can be accomplished with and without manipulatives.
Cone	A three-dimensional figure that has a circular base and one vertex. A cone has two faces, the circular base and the lateral face.
Congruent Figures	Geometric figures that have the same size and the same shape. Congruent figures may have different orientations. Example: Congruent angles have the same degree measure. Congruent line segments have the same length.
Constant	A fixed value in an expression or equation. Example: 3 is the constant in the expression $4x + 3$.
Constant of Proportionality	The value of the ratio of two proportional quantities, equivalent to unit rate. Represented as *k* in the formula *y* = *kx*.
Conversion Factor	A number used to change one set of units to another by multiplying or dividing.

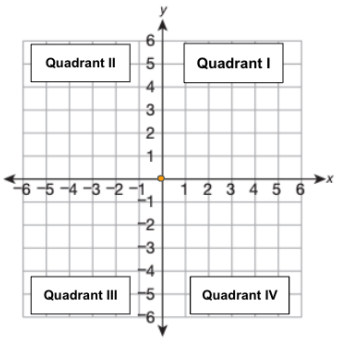
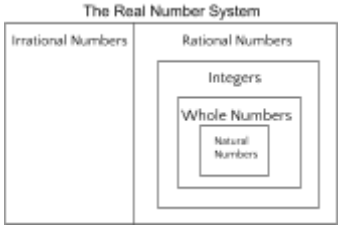
Coordinate Plane	A plane divided by perpendicular number lines creating four quadrants. The perpendicular number lines represent the axes, and where they intersect represents the origin (0,0). Points can be identified using coordinates (x, y) found within the quadrants (example below). 
Coordinates	An ordered pair of numbers that identifies a point on a grid, coordinate plane, or map written as (x, y).
Counting Back	A strategy for finding the difference using backward counting. Example: if a stack of books has 12 books and someone borrows 4 books to read, how many books are left? A student may start at 12 and count back for spaces or numbers saying 12…11, 10, 9, 8; there are 8 books left in the stack.
Counting On	A strategy for finding the number of objects in a group without having to count every member of the group. Example: if a stack of books has 8 books and 3 more books are added to the top, it is unnecessary to count the stack all over again. One can find the total by *counting on* pointing to the top book and saying "eight," following this with "nine, ten, eleven." There are eleven books now.
Cube	A three-dimensional figure with exactly six congruent, square faces.
Cylinder	A three-dimensional figure with two circular bases that are parallel and congruent. It has three faces, the two circular bases and the lateral face.
Data	Information collected and used to analyze a specific concept or situation.
Data Set	A set of numbers collected to be examined, considered, and used to help decision-making.
Decompose	Breaking a quantity or shape into smaller quantities/units/shapes in order to assist computation.
Denominator	The number in a fraction written below the fraction line that divides the number above the fraction line.
Dependent Variable	The output variable in a function; the variable whose value depends on the input or independent variable.
Difference	The distance between two values; result of a subtraction problem.
Dilation	A nonrigid transformation that enlarges or reduces a geometric figure by a scale factor relative to a point.
Distributive Property	When a single-term expression is being multiplied by a sum or difference, the single-term expression can be multiplied by each term before finding the sum or difference. Example: $3(7 + 5) = 3 \bullet 7 + 3 \bullet 5$ $w(5 - 2) = 5w - 2w$

Dividend	A number that is being divided by another number (divisor).
Divisor	The number by which another number is being divided.
Domain	The set of input (x) values for a function.
Double Number Line	Two number lines used when quantities have different units to easily see there are numerous pairs of numbers in the same ratio.
Edge	The line segment where a base and a lateral face, a base and lateral surface(s), two lateral faces, or two lateral surfaces of a three-dimensional figure intersect.
Equation	A statement that has one number or expression equal to another number or expression, such as $8 + 3 = 11$ or $2x - 3 = 7$.
Equivalent	Equal in value.
Evaluate	Calculate or find the value of a mathematical expression.
Expanded form	A multi-digit number is expressed in expanded form when it is written as a sum of the single-digit multiples of powers of ten. Example: $643 = 600 + 40 + 3$ $(3 \bullet 100,000) + (4 \bullet 10,000) + (7 \bullet 1,000) + (2 \bullet 100) + (6 \bullet 10) + (5 \bullet 1) = 347,265;$ $(3 \bullet 100) + (4 \bullet 10) + (7 \bullet 1) + (2 \bullet 0.1) + (6 \bullet 0.01) + (5 \bullet 0.001) = 347.265$ $(3 \bullet 100) + (4 \bullet 10) + (7 \bullet 1) + (2 \bullet \frac{1}{10}) + (6 \bullet \frac{1}{100}) + (5 \bullet \frac{1}{1000}) = 347.265$
Expanded Form (Exponents)	Expressing exponential expressions using multiplication without an exponent. Example: $x^5 = x \bullet x \bullet x \bullet x \bullet x$
Experimental Probability	The ratio of the number of times an event occurs to the total number of trials or times the activity is performed.
Exponent	A symbol that is written above and to the right of a number, *p* in exponential notation a^p, to show how many times the number is to be multiplied by itself.
Expression	A mathematical phrase consisting of numbers, variables, and operations. Algebraic expressions contain variables, constants, and algebraic operations. Numerical expressions contain only numbers and algebraic operations.
Fluency	There are three types of *fluency*. All of them require students to be accurate, efficient, and flexible. The types are defined as follows: Basic fact fluency - Operations with single-digit numbers. Students add, subtract, multiply, and divide accurately, efficiently, and flexibly with single-digit numbers. Computational fluency - Computation with the four operations. Students are able to accurately, efficiently, and flexibly compute problems using the four operations across all number types (whole numbers, fractions, etc.) without regard for magnitude of the numbers. Procedural fluency - Includes more than the four operations. "Ability to transfer procedures to different problems and contexts; to build or modify procedures from other procedures, and to recognize when one strategy or procedure is more appropriate to apply than another." (NCTM 2014, 2020)
Factor	1. A number or quantity that when multiplied with another number or quantity, produces a given number or expression. Example: 5 and 2 are factors of 10. 2. To break down into the terms that multiply to make the quantity to be factored.

Fraction	A number expressible in the form $\frac{a}{b}$ where *a* is a whole number and *b* is a whole number. The word fraction in these standards(K-5) always refers to a non-negative number. This includes all forms of fractions: fractions less than one, fractions greater than one (improper fractions), and mixed numbers.
Function	A rule or relationship in which there is exactly one output value for each input value.
Function notation	$f(x)$ is a way to represent a function, named *f*, where the input is represented by *x* and the output (y-value) is represented by $f(x)$. Example: $f(x) = 3x$ *f*(*x*) = 3*x* is the same as $y = 3x$.
Graph (Verb)	To show or plot information on a number line or coordinate plane.
Greatest Common Factor	The greatest whole number that divides two numbers.
Hierarchy	A system of ranking or organizing things.
Histogram	A graph used to represent the frequency distribution of data points of one variable.
Identity Property	The property asserts that when adding an identity (0) or multiplying an identity (1) with a number (n), the end result will be n. Example: Additive $\rightarrow n + 0 = n; 0 + n = n$ Multiplicative $\rightarrow n \bullet 1 = n; 1 \bullet n = n$
Independent Variable	A variable whose values do not depend on changes in other variables.
Inequality	A numerical sentence containing one of the symbols: >, <, ≥, ≤, or ≠ to indicate the relationship between two quantities. Example: $8 - 2 > 6 \div 3; 7v \leq 49; 5 \neq 2 + 2$
Inference (Statistical)	Deriving logical conclusions about a statistical population based on samples.
Integers	Natural numbers, their opposites, and zero. Integers can be positive or negative and do not contain fractions or decimals.
Interquartile Range	A measure of variation in a set of numerical data; the interquartile range is the distance between the first and third quartiles of the data set.
Interval	Includes all the numbers that come between two particular numbers.
Inverse (Operation)	An operation that is the opposite of, or undoes, another operation. Addition and subtraction are inverse operations as are multiplication and division.
Irrational Number	A number that cannot be expressed as a fraction $\frac{p}{q}$ for any integers *p* and *q*; irrational numbers have decimal expansions that neither terminate nor become periodic.
Least Common Multiple	The smallest positive whole number that is exactly divisible by each member of a given set of positive whole numbers.
Length	The distance from one end of an object to the other end.
Like Terms	Terms in an equation or expression whose variables are raised to the same power. Constants are also considered like terms. Example: $7x$ and $2x$ are like terms and $4y^2$ and $3y^2$ are like terms.
Likely Events	A chance event with a probability between 0.5 and 1; the closer the probability is to being 1, the more likely the event is to occur.
Line Plot	A method of visually displaying a distribution of data values where each data value is shown as an X or mark above a number line. Also known as a dot plot.

Linear Equation	An algebraic equation in which the variables are of the first degree (raised only to the first power). The graph of such an equation is a straight line. 
Linear Expression	An algebraic statement where each term is either a constant or a variable raised to the first power.
Mass	The amount of matter in an object. Often measured by the amount of material it contains which causes it to have weight. However, mass is not to be confused with weight. Weight is determined by the force of gravity on an object while mass is not. For example, a watermelon on Jupiter would have a greater weight than one on Earth because Jupiter's gravity is stronger than Earth's. The mass of the watermelon would be the same on both planets.
Mastery	Knowledge and skill that allows you to do, use, or understand something consistently well.
Mean	A measure of center in a set of numerical data, computed by adding the values in a list and then dividing by the number of values in the list.
Mean Absolute Deviation	A measure of variation in a set of numerical data, computed by adding the distances between each data value and the mean, then dividing by the number of data values.
Measurement Division	Division situation where the number of groups is unknown.
Measures of Center	Statistical measures that are intended to provide numerical representations of the center of a set of numerical data, also called measures of central tendency.
Measures of Variation	Statistical measures that are intended to provide numerical representations of the variability of a set of numerical data.
Median	A measure of center in a set of numerical data; the median of a list of values is the value appearing at the center of a sorted version of the list – or the mean of the two central values if the list contains an even number of values.
Mode	A measure of center in a set of numerical data; the most common value in a list of values.
Multiple	The result of multiplying a number by an integer. Example: 12 is a multiple of 4 because $4 \bullet 3 = 12$.
Multiplicative Comparison	Compare two amounts by asking how many times larger or smaller is one amount than the other.
Multiplicative Inverse	A number that, when multiplied by the given number, gives 1 as the product. By multiplicative inverse definition, it is the reciprocal of a number. Example: $\frac{1}{5} \bullet 5 = \frac{5}{5} = 1$; therefore 5 is the multiplicative inverse of $\frac{1}{5}$.
Natural Numbers	Counting numbers 1, 2, 3, 4, 5, 6…
Nested Grouping Symbols	More than one set of grouping symbols within a math problem. Example: $3[2 - 4(3^2 - 2^3)] = (-6)$
Net	Two-dimensional representation of a three-dimensional figure that can be folded up into a three-dimensional figure.
Number Line	A graph that represents the real numbers as ordered points on a line. A number line may be either horizontal (left and right) or vertical (up and down). Starting at zero, the positive numbers progress to the right (or up) and the negative numbers progress to the left (or down).

Numerator	The number in a fraction that is above the fraction line and that is divided by the number below the fraction line.
Order of Operations	A set of rules that define which procedures to perform first in order to evaluate a given expression.
Ordered Pair	A set of two numbers named in an order that matters; represented by (x, y) such that the first number, x, represents the x-coordinate and the second number, y, represents the y -coordinate when the ordered pair is graphed on the coordinate plane; each point on the coordinate plane has a unique ordered pair associated with it.
Ordinal Number	A number defining a position in a series.
Origin	The point in a Cartesian coordinate system where axes intersect.
Outlier	An observation or data point that lies an unusual distance from other values in the data.
Parallel Lines	Two or more distinct lines in the same plane that never intersect, these lines are always equidistant. In the coordinate plane, non-vertical parallel lines have equal slopes.
Partition	Divide or separate into smaller shapes or quantities.
Partitive Division	Division situation where the size of the groups is unknown.
Percent	A number expressed in relation to 100; represented by the symbol %.
Perimeter	The distance around a closed figure.
Perpendicular	Intersecting lines or planes that form right angles.
Place Value	The value of the place of a digit in a numeral; the relative worth of each number that is determined by its position.
Plane Figure	Two-dimensional shapes.
Plot	To place a point(s) on a coordinate plane.
Polygons	A closed two-dimensional figure made up of straight sides.
Prime	A whole number greater than 1 with only two factors. The factors of a prime number are 1 and the number itself.
Prime Factorization	A method of writing a composite number as a product of its prime factors.
Prism	A three-dimensional (solid) figure that has two congruent and parallel faces that are polygons called bases. The remaining faces, called lateral faces, are parallelograms (often rectangles). Prisms are named by the shape of their bases.
Probability	A number between 0 and 1 used to quantify likelihood for processes that have uncertain outcomes (tossing a coin, selecting a person at random from a group of people, tossing a ball at a target, or testing for a medical condition).
Product	The resulting quantity when two or more factors are multiplied. (The answer to a multiplication problem.)
Properties of Operations	Rules that apply to the operations with numbers. (See Table 1 below.)
Proportional	When two quantities have the same ratios.
Pyramid	A three-dimensional figure whose base is a polygon and the lateral faces are triangles that share a common vertex.
Pythagorean Theorem	The mathematical relationship stating that in any right triangle the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the two legs ($a^2 + b^2 = c^2$).

Quadrant	One of the four sections of a coordinate plane separated by horizontal and vertical axes; they are numbered I, II, III, and IV, counterclockwise from the upper right. 
Quadrilaterals	A polygon with four sides and four angles.
Quotient	The resulting quantity when one quantity (dividend) is divided by another quantity (divisor). The answer to a division problem.
Random Sample	A sample obtained by a selection from a population, in which each element of the population has an equal chance of being selected.
Range	1. The difference between the lowest and highest values from a set of data. 2. The set of output (*y*) values for a function.
Rate	A ratio that relates quantities of different units. Example: miles per hour, price per pound, students per class, heartbeats per minute.
Rate of Change/Slope	The ratio of the vertical change compared to the horizontal change between two points on a coordinate plane. Slope is often expressed as $\frac{rise}{run}$ or $\frac{change\ in\ y}{change\ in\ x}$.
Ratio	A comparison of two quantities, *r* and *s,* which can be written: ● $\frac{r}{s}$, where *r* is the numerator and *s* is the denominator ● *r* : *s* ● *r* to *s*
Rational Number	A number that can be written as a fraction of two integers $\frac{a}{b}$, where $b \neq 0$. Rational numbers can be integers, fractions, and repeating and terminating decimals.
Ray	A part of a line that has one endpoint and continues infinitely in one direction or on one side of that point. A ray is identified by two points: first its endpoint and then another unique point on the ray.
Real Number System	The real number system is the set of numbers containing all of the rational numbers and all of the irrational numbers. Example: 
Rectangular Array	A set of quantities arranged in rows and columns.

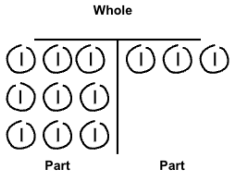
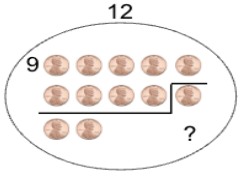
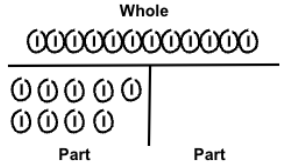
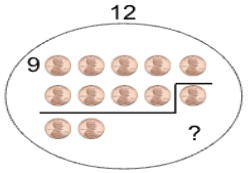
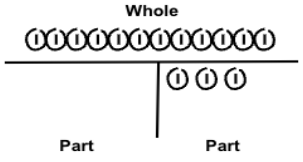
Rectangular Prism	A three-dimensional figure whose six faces are rectangles.
Reflection	A transformation in which every point and its images are on opposite sides and the same distance from a fixed line.
Relative Frequency	The ratio of the observed frequency of some outcome to the total outcomes of a random experiment.
Repeating Decimal	A decimal form of a rational number that, after a certain point, one digit or a set of digits repeat themselves an infinite number of times. Repeating digits are designated with an ellipsis or a bar above them. Example: 0.3333... , $0.\overline{3}$, or 4.00757575…, $4.00\overline{75}$
Rotation	A transformation in which each point is moved by the same angle measure in the same direction along a circular path about a fixed point.
Sample Space	The set of all possible outcomes for a probability experiment. Sample spaces can be displayed as diagrams, lists, and tables.
Scale Drawing	A drawing with dimensions at a specific ratio relative to the actual size of the object.
Scale Factor	The ratio between the corresponding measurements of an object and a representation of that object. A whole number scale factor represents an enlargement, and a fractional scale factor represents a reduction.
Scatter Plot	A two-variable data display where points are plotted to show the relationship (correlation) between two variables.
Scientific Notation	A form of writing a number as the product of a power of 10 and a decimal number such that the absolute value of the decimal number is greater than or equal to one and less than ten.
Similar	Two figures are similar if and only if all corresponding angles are congruent and lengths of all corresponding sides are proportional.
Slope/Rate of Change	The ratio of the vertical change compared to the horizontal change between two points on a coordinate plane. Slope is often expressed as $\frac{rise}{run}$ or $\frac{change\ in\ y}{change\ in\ x}$.
Solution	Any value(s) that make an equation, inequality, or open sentence true.
Sphere	A three-dimensional figure that consists of a set of points in space that are equidistant from a fixed point called the center.
Square Root	The square root of a number is the factor that we can multiply by itself to get that number. The symbol for square root is $\sqrt{\ }$. Finding the square root of a number is the opposite of squaring a number. For example $\sqrt{25} = \pm 5$, since $5 \bullet 5 = 25$ and $(-5) \bullet (-5) = 25$.
Standard Algorithm	One of the conventional algorithms used in the U.S. based on place value and properties of operations for addition, subtraction, multiplication, and division.
Statistical Question	A question that anticipates variability in the data.
Subitize	Quickly see how many objects are in a group without counting. There are two types of subitizing: perceptual and conceptual. Perceptual - instantly see how many objects without counting, usually done with up to 5 objects. Conceptual - quickly seeing small groups of objects as parts of a whole, usually done with more than 5 objects.
Substitution	1. Use of a numerical value to replace a variable. 2. A strategy for solving systems of equations that include solving for one variable and using that solution to find the other variable.
Sum	The resulting quantity when two or more addends are combined. The answer to an addition problem.
Supplementary Angles	Two angles(adjacent or nonadjacent) for which the sum of their measures is 180°.
Surface Area	The sum of the areas of all the faces of a three-dimensional (solid) figure or object.

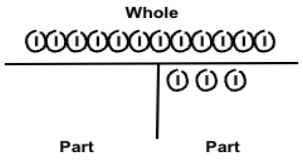
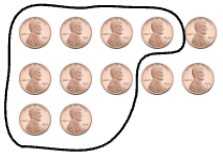
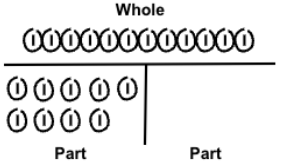
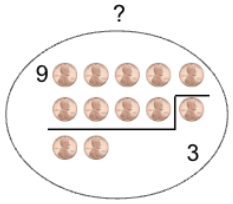
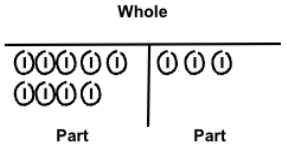
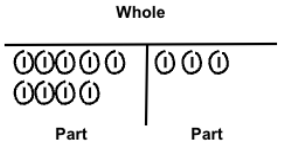
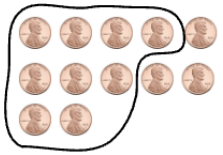
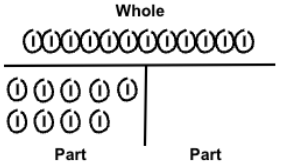
Tape Diagram	A rectangular model that looks like a segment of tape used to illustrate number relationships. Also known as a strip diagram, bar model, fraction strip, or length model.
Term	Terms are constants, variables, or the product or quotient of constant(s) and variable(s).
Terminating Decimal	The decimal form of a rational number that ends in 0. Example: $\frac{1}{2} = 0.50$
Theoretical Probability	The ratio of the number of favorable outcomes to the number of possible outcomes.
Three-Dimensional	An object that has three dimensions such as height, width, and depth.
Transformation	A rule that assigns to each point of a figure another point in the plane.
Translation	A transformation where each point is moved in the same direction and the same distance.
Trapezoid	A quadrilateral with *at least* one pair of parallel sides.
Tree Diagram	A diagram that shows the possible outcomes of an event by means of a connected, branching graph.
Triangular Prism	A three-dimensional figure made up of two triangular bases and three rectangular sides or faces.
Two-Dimensional	A flat figure or shape that has two dimensions such as length and width.
Unit Fraction	A fraction where the numerator is 1 and the denominator is a positive integer.
Unit Rate	A comparison of two measurements in which one of the terms has a value of 1.
Unlikely Event	A chance event with a probability between 0 and 0.5; the closer the probability is to being 0, the less likely the event is to occur.
Value	Numerical worth or amount.
Variable	1. A symbol used to represent an unknown or undetermined value in an expression or equation. 2. A quantity that may change within the context of a mathematical problem.
Variability	A measure of the dispersion or spread of data.
Vertex (Plural: Vertices)	A point where two or more line segments meet.
Visual Fraction Model	A graphic representation of fractions, such as a tape diagram, number line diagram, or an area model.
Volume	The amount of space contained in a three-dimensional figure; measured in cubic units.
Whole Numbers	A set of numbers containing all natural numbers and 0 (0, 1, 2, 3…).
X-Coordinate	The first number in an ordered pair representing the point's distance from the y-axis.
X-Intercept	A point where a graph of an equation intersects the x-axis.
Y-Coordinate	The second number in an ordered pair representing the point's distance from the x-axis.
Y-Intercept	A point where a graph of an equation intersects the y-axis.

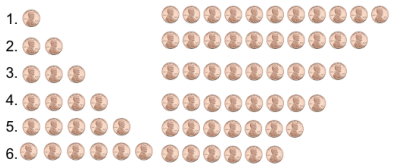
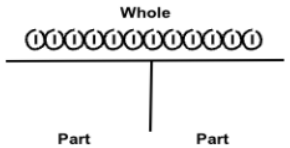
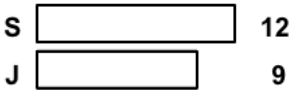
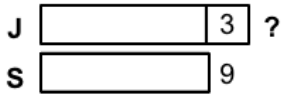
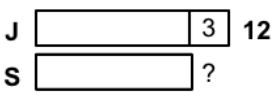
APPENDIX

Problem Types

Addition & Subtraction

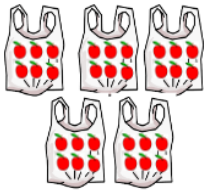
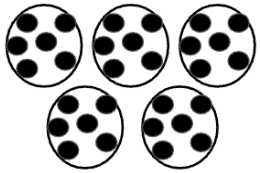
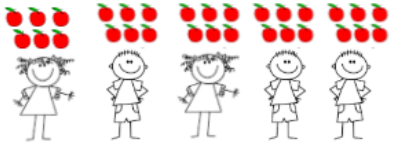
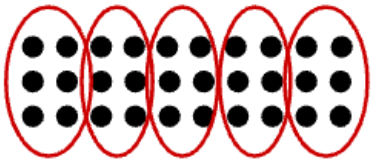
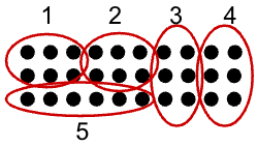
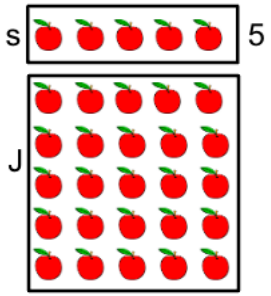
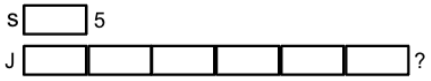
		Problem	Concrete	Representational	Abstract/ Equation(s)
Join	**Result Unknown**	Sarah had 9 pennies. Jacob gave her 3 more. How many pennies does Sarah have altogether?			$9 + 3 = \square$
	Change Unknown	Sarah had 9 pennies. Jacob gave her some more. Now Sarah has 12 pennies. How many pennies did Jacob give her?			$9 + \square = 12$
	Start Unknown	Sarah had some pennies. Jacob gave her 3 more pennies. Now Sarah has 12 pennies. How many did Sarah have in the beginning?			$\square + 3 = 12$

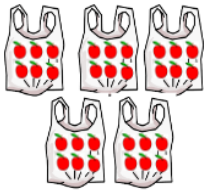
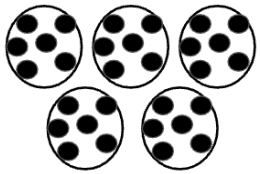
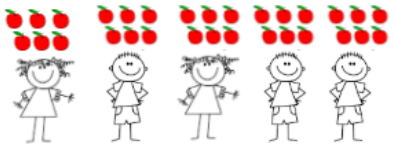
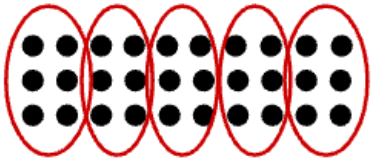
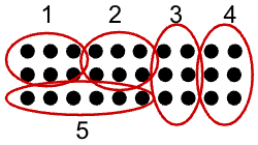
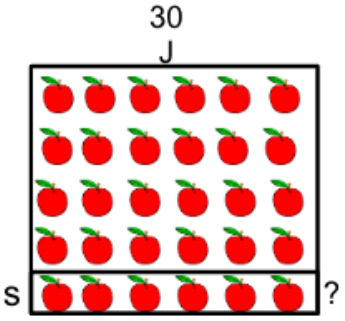
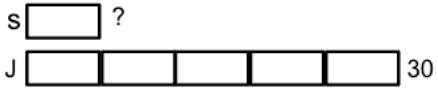
Separate	**Result Unknown**	Sarah had 12 pennies. She gave 3 pennies to Jacob. How many pennies does Sarah have now?			$12 - 3 = \square$
	Change Unknown	Sarah had 12 pennies. She gave some to Jacob. Now Sarah has 9 pennies. How many pennies did she give Jacob?			$12 - \square = 9$
	Start Unknown	Sarah had some pennies. She gave 3 to Jacob. Now Sarah has 9 pennies. How many pennies did she start with?			$\square - 3 = 9$
Part-Part-Whole	**Whole Unknown**	Jacob has 3 pennies and Sarah has 9 pennies. How many pennies do they have altogether?			$9 + 3 = \square$
	Part Unknown	Jacob and Sarah have 12 pennies in a jar. Sarah put in 9 pennies. How many pennies did Jacob put in the jar?			$12 = 9 + \square$

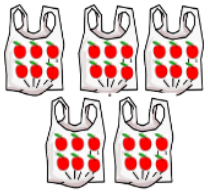
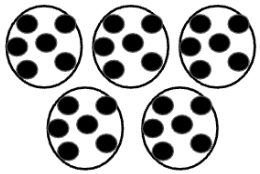
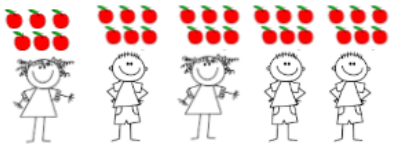
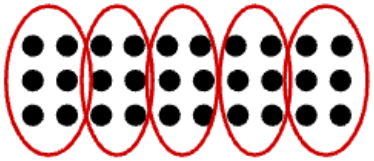
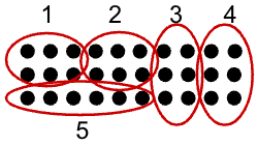
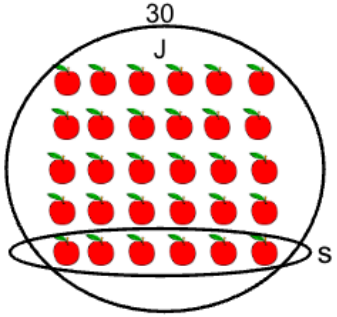
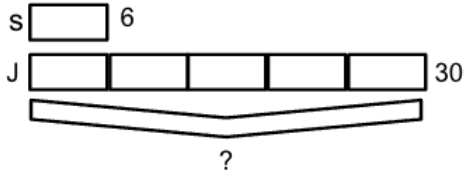
	Both Parts Unknown	There are 12 pennies in the jar. They want to put the pennies into two containers. How many pennies can they put into each container?			$1 + 11 = 12$ $2 + 10 = 12$ $3 + 9 = 12$ $4 + 8 = 12$ $5 + 7 = 12$ $6 + 6 = 12$
Compare	**Difference Unknown**	Sarah has 12 pennies and Jacob has 9 pennies. How many more pennies does Sarah have than Jacob? Sarah has 12 pennies and Jacob has 9 pennies. How many fewer pennies does Jacob have?			$12 - \square = 9$ $9 + \square = 12$
	Larger Unknown	Jacob has 3 more pennies than Sarah. Sarah has 9 pennies. How many pennies does Jacob have? Sarah has 3 fewer pennies than Jacob. Sarah has 9 pennies. How many pennies does Jacob have?			$9 + 3 = \square$ $3 + 9 = \square$
	Smaller Unknown	Jacob has 3 more pennies than Sarah. Jacob has 12 pennies. How many pennies does Sarah have?			$12 - 3 = \square$ $3 + \square = 12$

		Sarah has 3 fewer pennies than Jacob. Jacob has 12 pennies. How many pennies does Sarah have?			

Multiplication & Division

		Problem	Concrete	Representation	Abstract/ Equation(s)
Equal Groups	**Whole Unknown (Multiplication)**	Jacob has 5 bags of apples. There are 6 apples in each bag. How many apples does Jacob have altogether?			$5 \cdot 6 = \square$
	Size of Groups Unknown (Partitive Division)	Jacob has 30 apples. He wants to share them equally among his five friends. How many apples will each friend receive?			$5 \cdot \square = 30$ $30 \div 5 = \square$
	Number of Groups Unknown (Measurement Division)	Jacob has 30 apples. He put them into bags with 6 apples in each bag. How many bags did Jacob use?			$\square \cdot 6 = 30$ $30 \div 6 = \square$
Comparison	**Product Unknown (Multiplication)**	Sarah picked 5 apples. Jacob picked 6 times as many apples as Sarah. How many apples did Jacob pick?			$5 \cdot 6 = \square$

		Problem	Concrete	Representation	Abstract/ Equation(s)
Equal Groups	**Whole Unknown (Multiplication)**	Jacob has 5 bags of apples. There are 6 apples in each bag. How many apples does Jacob have altogether?			$5 \bullet 6 = \square$
	Size of Groups Unknown (Partitive Division)	Jacob has 30 apples. He wants to share them equally among his five friends. How many apples will each friend receive?			$5 \bullet \square = 30$ $30 \div 5 = \square$
	Number of Groups Unknown (Measurement Division)	Jacob has 30 apples. He put them into bags with 6 apples in each bag. How many bags did Jacob use?			$\square \bullet 6 = 30$ $30 \div 6 = \square$
	Set Size Unknown (Partitive Division)	Jacob picked 30 apples. He picked 5 times as many apples as Sarah. How many apples did Sarah pick?			$5 \bullet \square = 30$ $30 \div 5 = \square$

		Problem	Concrete	Representation	Abstract/ Equation(s)
Equal Groups	**Whole Unknown (Multiplication)**	Jacob has 5 bags of apples. There are 6 apples in each bag. How many apples does Jacob have altogether?			$5 \cdot 6 = \square$
	Size of Groups Unknown (Partitive Division)	Jacob has 30 apples. He wants to share them equally among his five friends. How many apples will each friend receive?			$5 \cdot \square = 30$ $30 \div 5 = \square$
	Number of Groups Unknown (Measurement Division)	Jacob has 30 apples. He put them into bags with 6 apples in each bag. How many bags did Jacob use?			$\square \cdot 6 = 30$ $30 \div 6 = \square$
	Multiplier Unknown (Measurement Division)	Jacob picked 30 apples. Sarah only picked 6 apples. How many times as many apples did Jacob pick than Sarah?			$\square \cdot 6 = 30$ $30 \div 6 = \square$

Scaffolding Addition and Subtraction

When introducing grade-level addition and subtraction problem types, some students will need additional assistance with engaging in the problem. Considering students in the classroom progress at different rates, educators should use meaningful scaffolds that support grade-level content. Changing the number size is one strategy for helping students. When implementing these strategies, the scaffold aims to implement just-in-time support that compels student thinking toward grade-level content. Remember that teaching skills in context rather than isolation are practical and effective scaffolds for students.

At the end of fourth grade, Arkansas students are expected to be fluent with multi-digit addition and subtraction and the standard algorithm.

Addition		
Grade Level	**Description**	**Example**
K	1 digit + 1 digit	3 + 7
1, 2	1 digit + 1 digit	8 + 7
1	2 digit + 1 digit (no regrouping)	21 + 7
1	2 digit + a multiple of 10	34 + 20
2	2 digit + 1 digit (regrouping)	38 + 6
2	2 digit + 2 digit (no regrouping)	43 + 24
2	2 digit + 2 digit (regrouping)	37 + 28
2	3 digit + 1 digit (no regrouping)	351 + 4
2	3 digit + 1 digit (regrouping)	426 + 8
2	3 digit + 2 digit (no regrouping)	523 + 43

2	3 digit + 2 digit (regrouping)	728 + 76
2, 3	3 digit + 3 digit (no regrouping)	284 + 112
2, 3	3 digit + 3 digit (regroup ones to tens)	338 + 247
2, 3	3 digit + 3 digit (regroup tens to hundreds)	592 + 235
2, 3	3 digit + 3 digit (regroup ones to tens and tens to hundreds)	456 + 275
3	3 digit + 3 digit (regrouping with hundreds)	571 + 718
3	3 digit + 3 digit (regroup all places)	684 + 548
4	4 digit + 3 digit	1,245 + 322
4	4 digit + 4 digit	2,856 + 4,479
4	5 digit + 4 digit	24,732 + 5,201
4	5 digit + 5 digit	56,902 + 38,392
4	6 digit + 5 digit	823,651 + 62,745
4	6 digit + 6 digit	562,884 + 319,023

Subtraction		
Grade Level	**Description**	**Example**
K	1 digit - 1 digit	7 - 3

1	multiple of 10 - multiple of 10	30 - 20
1	2 digit - 10	54 - 10
1, 2	2 digit - 1 digit (within 20; no regrouping)	15 - 3
1, 2	2 digit - 1 digit (within 20; regrouping)	12 - 8
2	2 digit - 1 digit (greater than 20; no regrouping)	38 - 6
2	2 digit - 1 digit (greater than 20; regrouping)	42 - 7
2	2 digit - 2 digit (no regrouping)	43 - 21
2	2 digit - 2 digit (regrouping)	87 - 38
2	3 digit - 1 digit (no regrouping)	357 - 4
2	3 digit - 1 digit (regrouping)	422 - 8
2	3 digit - 2 digit (no regrouping)	583 - 43
2	3 digit - 2 digit (regrouping)	724 - 76
2, 3	3 digit - 3 digit (no regrouping)	284 - 112
2, 3	3 digit - 3 digit (regroup tens to ones)	332 - 217
2, 3	3 digit - 3 digit (regroup hundreds to tens)	532 - 290
2, 3	3 digit - 3 digit (regroup tens to ones and hundreds to tens)	451 - 275
4	4 digit - 3 digit	1,245 - 322

4	4 digit - 4 digit	4,856 - 2,479
4	5 digit - 4 digit	24,732 - 5,201
4	5 digit - 5 digit	56,902 - 38,392
4	6 digit - 5 digit	823,651 - 62,745
4	6 digit - 6 digit	562,884 - 319,023

Table 1: Properties of Operations	
Associative property of addition	$(a + b) + c = a + (b + c)$
Commutative property of addition	$a + b = b + a$
Additive identity property of 0	$a + 0 = 0 + a = a$
Existence of additive inverses	For every a, there exists –a so that $a + (-a - a) = (-a - a) + aa = 0$.
Associative property of multiplication	$(a \bullet b) \bullet c = a \bullet (b \bullet c)$
Commutative property of multiplication	$a \bullet b = b \bullet a$
Multiplicative identity property of 1	$a \bullet 1 = 1 \bullet a = a$
Existence of multiplication inverses	For every $a \neq 0$, there exists $\frac{1}{a}$ so that $a \bullet \frac{1}{a} = \frac{1}{a} \bullet a = 1$.
Distributive property of multiplication over addition	$a \bullet (b + c) = a \bullet b + a \bullet c$

Table 2: Properties of Equality	
Reflexive property of equality	$a = a$
Symmetric property of equality	If $a = b$, then $b = a$.
Transitive property of equality	If $a = b$, and $b = c$, then $a = c$.
Addition property of equality	If $a = b$, then $a + c = b + c$.
Subtraction property of equality	If $a = b$, then $a - c = b - c$.
Multiplication property of equality	If $a = b$, then $a \bullet c = b \bullet c$.
Division property of equality	If $a = b$ and $c \neq 0$, then $a \div c = b \div c$.
Substitution property of equality	If $a = b$, then b may be substituted for a in any expression containing a.

Table 3: Properties of Inequality
Exactly one of the following is true: $a < b$, $a = b$, $a > b$.
If $a > b$ and $b > c$, then $a > c$.
If $a > b$, $b < a$.
If $a > b$, then $a + c > b + c$.
If $a > b$, then $a - c > b - c$.
If $a > b$ and $c > 0$, then $a \cdot c > b \cdot c$.
If $a > b$ and $c < 0$, then $a \cdot c < b \cdot c$.
If $a > b$ and $c > 0$, then $a \div c > b \div c$.
If $a > b$ and $c < 0$, then $a \div c < b \div c$.

Table 4: Inequalities	
>	Greater than
<	Less Than
=	equal
≈	approximately
≠	Not equal to
≤	Less than or equal to
≥	Greater than or equal to
Graphing inequalities	
○	Open circle for non-inclusion symbols: >, <, or ≠
⚫	Closed circle for inclusion symbols: ≤, ≥, or =

Table 5: Exponent Rules				
Product Rule	$a^m \cdot a^n = a^{m+n}$		Zero Exponent Rule	$a^0 = 1$
Quotient Rule	$a^m \div a^n = a^{m-n}$		Identity Exponent Rule	$a^1 = a$
Power of a Power Rule	$(a^m)^n = a^{mn}$		Negative Exponent Rule	$a^{-m} = \frac{1}{a^m}$
Power of a Product Rule	$(ab)^m = a^m b^m$			
Power of Quotient Rule	$(\frac{a}{b})^m = \frac{a^m}{b^m}$			