TANGLIN SECONDARY SCHOOL

PRELIMINARY EXAMINATION 2020
Secondary 4 Express

NAME _____

CLASS _____ **INDEX NO.** _____

ADDITIONAL MATHEMATICS 4047/01

Paper 1 **2 hours**

Candidates answer on the Question Paper.

READ THESE INSTRUCTIONS FIRST

Write your index number, class and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do **NOT** use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an approved scientific calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 80.
The total marks for this paper is **100**.
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For Examiner's Use	
Total	80

Calculator Model:

This document consists of **20** printed pages.

1. ALGEBRA

Quadratic Equation

For the quadratic equation $ax^2 + bx + c = 0$,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Binomial Theorem

$$(a+b)^n = a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{r}a^{n-r}b^r + \ldots + b^n,$$

where n is a positive integer and $\binom{n}{r} = \frac{n!}{(n-r)!r!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$.

2. TRIGONOMETRY

Identities

$$\sin^2 A + \cos^2 A = 1$$

$$\sec^2 A = 1 + \tan^2 A$$

$$\operatorname{cosec}^2 A = 1 + \cot^2 A$$

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1 - \tan^2 A}$$

Formulae for ΔABC

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$\text{Area of } \Delta = \tfrac{1}{2} bc \sin A$$

1 Given that θ is obtuse, and $\cos\theta = -\frac{12}{13}$, without using a calculator, find

(i) $\tan\theta$, [2]

(ii) $\sin\frac{\theta}{2}$, leaving your answer in surd form and explain why $\sin\frac{\theta}{2} > 0$. [3]

2 Find the set of values of the constant k for which the curve $y = (2k+3)x^2 - 4kx$ intersects the line $y = 3k - 1$ at two distinct points.

[3]

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3 Given that $y = \frac{1}{2}(e^{2x} + 5e^{-2x})$, and that $\frac{d^2y}{dx^2} - \frac{dy}{dx} = Ae^{2x} + Be^{-2x}$, find the value of each of the constants A and B. [4]

4 A spherical ice of radius r cm is melting in such a way that the volume, V cm^3, is decreasing at a constant rate of 5.13 cm^3/s. Assuming that the spherical ice retains its shape, calculate the radius, r when the radius is decreasing at $\frac{1}{5\pi}$ cm/s. [4]

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5 **(i)** An object is heated until it reaches a temperature of θ degrees Celsius. It is then allowed to cool. When it has been cooled for t minutes, its temperature, θ degrees Celsius, is given by $\theta = 10 + 82e^{-\frac{t}{3}}$. Find the value of θ when it just starts to cool. [1]

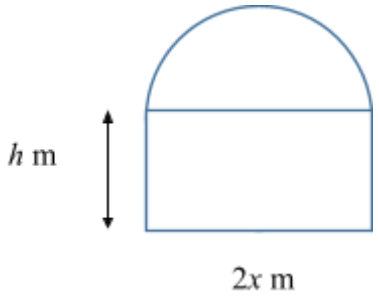
(ii) Find the value of θ when the object has been cooled for 9 minutes. [1]

(iii) Find the value of t when $\theta = 50$ °C. [3]

(iv) State the temperature (θ) of the object approaches after it is left to cool for a long period of time. [1]

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6 A window in the shape of a rectangle with a dimensions of $2x$ m by h m is surmounted by a semicircle of radius of x m. The window is being designed to let in the maximum amount of light. The glass used for the semi-circle is stained glass which lets in one unit of light per square metre. The rectangular part uses clear glass which lets in 2 units of light per square metre.



(a) If the perimeter of the whole window is 10 m, show that $h = 5 - \frac{(2+\pi)}{2}x$. [1]

(b) Hence, show that the amount of light, L, let in by the window is given by

$$L = 20x - \left(4 + \frac{3\pi}{2}\right)x^2.$$ [3]

6 **(c)** Find the value of x that must be used to allow the design to let in the maximum amount of light. Give the value of x correct to 3 significant figures. [4]

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7 **(a)** Evaluate the coefficient of x^3 in the binomial expansion of $\left(x^2 - \frac{5}{x}\right)^6$. [3]

(b) Given that the coefficient of x^3 in the expansion of $(ax+3)^5$ is 720, calculate the value of a.
[2]

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8 The equation of a curve is $y = \dfrac{2x^3}{3x-2}$.

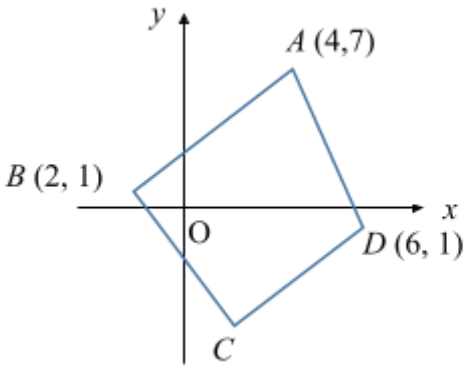
(i) Find $\dfrac{dy}{dx}$. [2]

(ii) Explain why y is a decreasing function for $x < 1$. [2]

8 **(iii)** Find the x-coordinates of the stationary points of the curve. [2]

(iv) Find the nature of each stationary point using the first derivative test. [3]

9 In the diagram AB is parallel to DC and angle $ABC = 90^o$. Given that the coordinates of A, B and D are (4,7), (−2, 1), and (6, −1) respectively,



(i) find the equations of BC and DC, [3]

9 (ii) find the coordinates of vertex C, [1]

(iii) find the equation of perpendicular bisector of AB, [2]

(iv) show and explain why the point (5,0) is equidistant from vertices A and B. [2]

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10 **(a)** Solve the simultaneous equations

$$27^{y}\left(9^{x}\right)=\frac{1}{9},$$

$$\log_4(2x+y-4)=1+\log_4(x-4).$$

[4]

10 **(b)** The volume of a rectangular block with a square base is $\left(10\sqrt{3}-2\sqrt{2}\right)$ cm^3. The length of each side of the base is $\left(\sqrt{3}-\sqrt{2}\right)$ cm. Express the height of the block in the form $\left(a\sqrt{2}+b\sqrt{3}\right)$, where a and b are integers. [4]

11 A particle P moves in a straight line so that its displacement, s metres, from a fixed point O, is given by $s = 9t + 3t^2 - t^3$, where t is the time taken in seconds after the start of the motion.

Calculate

(a) **(i)** the initial velocity and acceleration, [4]

(ii) the value of t when P is instantaneous at rest, [1]

(b) Explain clearly why the total distance travelled by P in the interval from $t = 0$ to $t = 4$ is not obtained by finding the value of s when $t = 4$,

[2]

(c) Find the total distance travelled by P in the interval from $t = 0$ to $t = 4$. [3]

12 It is given that $f(x) = 2\cos 2x$ and $g(x) = 2\sin\left(\frac{x}{3}\right) - 1$ where x is in degree measure.

(i) State the least and the greatest values of $f(x)$. [1]

(ii) State the least and the greatest values of $g(x)$. [2]

(iii) State the period of $f(x)$. [1]

(iv) State the period of $g(x)$. [1]

12 **(v)** Sketch, on the same axes, the graphs of $y = f(x)$ and $y = g(x)$ for $0^o \leq x \leq 270^o$. [4]

(vi) State the number of solutions of the equation $2\sin\left(\frac{x}{3}\right) - 2\cos 2x = 1$ for $0^o \leq x \leq 270^o$. [1]

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