MODELS OF CONTROL SYSTEMS WITH PARAMETRIC AND ADDITIVE-PARAMETRIC FEEDBACK

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Abstract

For processes implemented in object management systems of various nature, quite often the input signal can be formalized as one of a limited set of some, so to speak, standard functions. In particular, such a function may correspond to an increasing exponential. The functioning of control systems for these processes should be ensured in such a way that the reaction of the system itself to such highly probable signals is adequate and successfully formed in accordance with the criterion of minimum error. One of the ways to solve this problem is to form a parametric or additive–parametric feedback circuit in the control system, as well as integrate components with parameters changing according to a certain law into the structure of the system. At the same time, the error should not depend on the magnitude of the parameters noted above and, in fact, is minimized.

The authors synthesized and substantiated models and algorithms for the functioning of control systems with parametric feedback by transmission coefficient, which make it possible to eliminate control errors and ensure the noise immunity of the system. It is also proposed to use other means that ensure the high efficiency of such systems that implement the management of objects and processes of various types.

Keywords: mathematical modeling, control systems, feedback, features of the functioning of systems, parametric feedback, optimization of systems.

I. Introduction

The fundamental foundations of the modern theory of optimal control were formed by a large body of researchers in the second half of the twentieth century. First of all, the efforts of researchers from various countries in this field were focused on the successful solution of problems correlating with the synthesis of control systems for objects of various nature, which in the process of their functioning in a more or less stable external environment did not undergo significant qualitative and quantitative changes [1-3].

Mathematical models reflecting the functioning of such control systems, as a rule, were synthesized by most authors on the basis of systems of differential equations. Over time, researchers began to pay attention to the development of theoretical foundations and practical recommendations for the design of control systems that would be able to adapt to significant changes in their operating conditions, including by correcting the structure of the system and the parameters of its individual components).

One of the basic concepts of the above-mentioned theory of control of dynamic systems is the concept of "feedback", which is used in the formalization of the description of the functioning of the so-called closed-type control systems. The main advantage of such systems over alternative systems of the "open" type is their ability to independently form a reaction (usually a compensation type reaction) to external disturbances. At the same time, effective algorithms for the functioning of feedback systems are either based on the principle of actually isolating the system from external disturbances, or on the principle of adaptation to dynamic conditions of an aggressive external environment.

The feedback principle is successfully applied by researchers and designers of systems and processes of various nature (technical, technological, informational, chemical-biological, economic, etc.) for their analysis and synthesis. In the most general case, the functioning of a feedback system can be understood as the formation of some impact of the results of the vital activity of the system on the nature of the implementation of this vital activity. The type of feedback (additive or parametric) determines a set of specific tasks that can be successfully solved in practice due to its formation in the structure of the system, as well as approaches to the synthesis and optimization of mathematical models for an adequate formal description of the functioning of the system. The synthesis of optimal control system models is still a very urgent task today [3-5].

II. Methodology and purpose of the study

The purpose of this article is the analysis, synthesis and scientific substantiation of mathematical models, as well as the analysis of the set of features of the functioning of control systems with parametric and additive–parametric feedback. Such systems are widely used in practice to ensure effective management of objects and processes of various nature. In particular, the authors pay appropriate attention to the issues of the accuracy of the functioning of such systems and their noise immunity.

The methodology of this scientific research is based on the theoretical provisions of technical cybernetics as the science of managing complex, hierarchical, multidimensional and adaptive systems, in particular, modern ideas in the field of automatic control theory.

III. Modeling of control systems with parametric feedback

Undoubtedly, a rather important point for the successful development and design of effective control systems for various objects and processes is to ensure the necessary level of accuracy of functioning, which, as a rule, determines their quality and, accordingly, the effectiveness of solving practical problems. In the current conditions, when any control system implements its functioning surrounded by a huge number of other technical and technological systems that exchange large flows of information with each other, its noise immunity is no less important. In turn, the noise immunity of the system determines its reliability.

As it was shown in previously published articles, the condition for accurate reproduction of exponential input effects in automated control systems is the equality of exposure time constants and an unstable aperiodic link, which is intentionally included in the feedback circuit. Since the value of the time constant of the exponential function corresponding to the input effect on the system may change during the operation of the system as a whole, it becomes necessary to adapt the value of the time constant of the unstable aperiodic link in such a way that the above equality condition is ensured. It is the fulfillment of such a condition that will minimize the magnitude of the dynamic error ε, in fact, reducing it to zero [6-7].

Thus, a peculiar parameter, depending on the value of which, in fact, optimization of the functioning of the control system as a whole, as well as the aperiodic link, in particular, is the gradient of the dynamic error.

In previously published works [8-9], the authors of this article substantiated the thesis that the aperiodic link using a parametric feedback circuit is characterized by parameters that functionally depend on the amplitude and sign of the input signal, in particular, in our case, the signal corresponding to the error $\pm$ ε. In particular, the formal functioning of such a circuit can be represented by a set of the following equations.

For the direct circuit of the control system block diagram (1):

$$T_1 \frac{du}{dt} - u = k_n u_1 \tag{1}$$

where $u(t) -$ is the output signal for this component of the control system; $u_1(t) -$ is the input signal for the aperiodic component of the control system; $k_n -$ is the transmission coefficient of the so–called "short circuit link"; $T_1 -$ is the time constant of the aperiodic component of the control system.

For the closure link, which, in the case of additive feedback, is a comparator (2):

$$u_1 = \left(k_m + k_1 x_1\right)\varepsilon \tag{2}$$

where $k_m -$ is the transmission coefficient of the closed control system; $k_1 -$ is the transmission coefficient of the open control system; $x_1 -$ is the output signal of the parametric feedback circuit; $\varepsilon -$ is the error signal.

For the feedback circuit of the control system (3):

$$x_1 = k_0 u \tag{3}$$

Then the following equation (4) will be characteristic of the control system as a whole, which describes its functioning from a formal point of view

$$T_{ekv} \frac{du}{dt} - u = k_{ekv}\varepsilon \tag{4}$$

where $T_{ekv} = T_1/(1 + k_1 k_0 k_n \varepsilon$,

$$k_{ekv} = k_m k_n/(1 + k_1 k_0 k_n \varepsilon \tag{5}$$

For $\varepsilon = 0$, the equation of the transfer function of the quasi-static link of the control system with parametric feedback, when the condition of adaptation of the time constant is fulfilled, must correspond to (4). In order to ensure the fulfillment of this condition, according to the authors, it is necessary to accept $k_m = 1$.

Depending on the magnitude and sign of the error ε the T_{ekv} parameter will also change. By changing the parameters k_1 and k_0, it is possible to ensure an optimal ratio between the variations of ε and T_{ekv}. Simultaneously with the change in the time constant T_{ekv}, a proportional change in the transmission coefficient k_{kv} is realized, the value of which (5) was defined by us somewhat higher in the text as a parameter of equation (4). In turn, the fulfillment of such a condition will ensure very small changes in the values of the roots of the characteristic equation during the adaptation process of the functioning of the entire system. Consequently, the quality indicators of the transition process implemented in such a management system will actually be constant. Moreover, the effective signal bandwidth will also be unchanged, and the characteristics of the control system as a whole will be optimal.

The implementation of an approach to optimizing the functioning of the control system by adapting the parameters of a quasi-static component forcibly integrated into the system allows, in fact, to transform it from a dynamic to a static system, since additional information about the values of the parameters can be obtained only if the condition $\varepsilon \neq 0$ is met. And in order to implement astatic adaptation, it is additionally necessary to have an integrating link in the structure of the quasi-static component.

In any case, the adaptation of self-adjusting automatic control systems, the operation of which ensures constant quality indicators of transients, regardless of the type and nature of parametric effects, is realized due to the presence of a parametric feedback circuit in its structure. In some cases, in addition to the parametric feedback circuit, an additional circuit is used that provides a direct parametric connection of the input signal with the parameters of the direct circuit. In general, systems have proven themselves well, in the structure of which there is a combination of the following components: a means of determining process quality parameters; a means of forming an adaptation algorithm (self-tuning); some specific executive device.

Parametric feedback is also used in the design of control systems with the implementation of the identification of control objects. But, in any case, the structure of adaptive (self-adjusting) control systems contains elements that provide combined additive-parametric feedback. At the same time, the main circuit is providing additive feedback, and the auxiliary one is providing parametric feedback.

If the parameters of the control object vary quite slowly compared to the variations of the processes in the system itself, then the analysis of the additive and parametric feedback circuits can be implemented quite simply by separating them. If the speeds of the above processes are sufficiently close to each other, in this case the self-adjusting control system is a complex nonlinear system, the analysis of the functioning of which requires the use of some specific methods, which were described by researchers in a number of publications at the time [7-9].

IV. Adaptive properties of parametric feedback systems in circuits providing self-tuning

Next, we will consider a number of problems that may arise in control systems with a large variation in the transmission coefficient of the control object $k(t)$. In this case, the equation of the control object takes the following form (6).

$$\sum_{i=0}^{n} a_i y^i = k(t) \sum_{j=0}^{m} b_j u^j \tag{6}$$

Further, let's assume that there is a possibility of some compensation for variations in the transmission coefficient $k(t)$ due to the corresponding variations in the transmission coefficient of the regulator $k_r(t)$ itself. If $u = \varepsilon k_r(t)$, and $\varepsilon = x - y$, then the equation of the feedback control system, taking into account (6), will take the following form (7):

$$\sum_{i=0}^{n} a_i y^i = k(t) \sum_{j=0}^{m} b_j \left(k_r(t)(x - y)\right)^j \tag{7}$$

By revealing the right side of equation (7) according to the Leibniz formula and simultaneously implementing the rearrangement of the components of the formula, the following fact can be established. The coefficients on the left side of this equation (7) for derivatives up to the order of m will functionally depend on the transmission coefficient of the regulator $k_r(t)$ as well as on its derivatives.

Therefore, only with sufficiently slow changes in the values of this coefficient $k_r(t)$ and, accordingly, sufficiently slow changes in the values of the coefficient $k(t)$, it is possible to transform (7) into (8):

$$\sum_{i=m+1}^{n} a_i y^i + \sum_{j=0}^{m} (a_i + k(t)k_r b_j)y^j = k(t)k_r(t) \sum_{j=0}^{m} b_j x^j \quad (8)$$

It follows from equation (8) that in order to ensure the condition of independence of the properties of the control system from the values of the transmission coefficient $k(t)$, it is necessary that, if possible, the following equality (9) be fulfilled:

$$k(t)k_r(t) = const \quad (9)$$

And, accordingly, the fulfillment of equality (9) can be ensured only if there is a parametric feedback circuit in the control system. However, the more precisely the ratio (9) is fulfilled, the more unacceptable are the conditions for ensuring effective transmission of information in the system as a whole. Therefore, as a rule, control subsystems and adaptation subsystems are separated in such systems.

A number of problems that arise in this case can be stopped by exciting some natural oscillations in the main circuit of the system, used as a signal for the implementation of self-adjustment (self-adaptation). But, at the same time, such fluctuations have practically no effect on the implementation of the basic management regime. As some of the features, it is necessary to note a number of specific requirements for the parameters of self-oscillations in the main circuit of the system.

The first requirement is the constancy of the oscillation frequency, which makes it possible to stabilize other self-oscillation parameters. The second requirement is to exceed the self-oscillation frequency of the maximum frequency of the spectrum of the input signal acting on the system. This condition is determined by the need to ensure a sufficient level of noise immunity of the system and to prevent distortion of the input signal.

The following types of signals can also be used to configure control systems with parametric feedback by monitoring the passage of certain testing influences: various types of harmonic signals, the so-called "white noise", as well as periodically repeating sequences of pulses of various shapes (rectangular, triangular, trapezoidal, etc.). To form such "test" signals, it is necessary to have an appropriate external source is available, the main requirement for the practical implementation of which is the stability of the parameters of the signal generated by it. In some cases, it is very problematic to ensure such stability.

If, for the implementation of testing of the system, its own oscillations are used, excited in the main circuit of the control system, as we indicated above, then such a problem is automatically solved. In this case, with respect to the control signal x(t), the control system behaves as if there is no parametric feedback circuit in its structure. This is due to the fact that the control signal does not really penetrate the self-tuning (adaptation) circuit of the system. It is stopped by a special filter. And the transition process in such a system differs from the transition process in a system in which there is no self-adaptation chain, only by the presence of an additional harmonic signal of very small amplitude.

An essential feature of self-adjusting systems with a so-called limit cycle and parametric or additive parametric feedback is their significant performance, structural simplicity and high level of reliability of operation. This allows, in particular, minimizing hardware and software tools to ensure successful management of a wide range of facilities, including those for which the operator's direct participation in the implementation of management actions is very difficult.

V. Conclusions

The material presented in this article is based on a set of fundamental approaches within the framework of technical cybernetics, and, therefore, can be used for the successful synthesis of control systems for complex technical and technological objects and processes. However, the practical application of the above ideas is not limited to technical or technological fields. This is due to the fact that the feedback principle, in this case parametric feedback, is a powerful and universal tool for the successful implementation of effective management on a scientific basis of a set of processes in any other industries (economics, biology, etc.) In all the above cases, the objects of management are complex, multidimensional, hierarchical systems with the property of adaptation [9-10].

Successful achievement of the set of goals for the functioning of such multidimensional systems is impossible without the formation and implementation of numerous feedback loops (both positive and negative, both additive and parametric). Today, this is an indisputable fact. In this article, we have considered individual cases of the implementation of adaptive management, focused on ensuring compliance with the criterion of minimum errors that occur during the operation of complex objects and systems. The use of additional components that, in a certain sense, stop errors that occur during the operation of systems, in practice turns out to be very productive. In the future, in a number of their publications, the authors of this article plan to specify a set of practical tools for implementing such control using systems with parametric and additive-parametric feedback.

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