

$$\begin{aligned}
& \int (\alpha_1^2 G_1^2 + \alpha_2^2 G_2^2 + 2\alpha_1 \alpha_2 \cos \psi G_1 G_2) dp \\
&= \int \alpha_1^2 G_1^2 + \int \alpha_2^2 G_2^2 + \int 2\alpha_1 \alpha_2 \cos \psi G_1 G_2 dp \\
&= \alpha_1^2 \int G_1^2 + \alpha_2^2 \int G_2^2 + 2\alpha_1 \alpha_2 \int \cos \psi G_1 G_2 dp \\
&= \alpha_1^2 + \alpha_2^2 + 2\alpha_1 \alpha_2 \int \cos \psi G_1 G_2 dp
\end{aligned}$$

$$\text{where } G = \frac{1}{(2\pi)^{D/4} |C_k|^{1/4}} \exp\left(-\frac{1}{4} (p - \mu_k)^T C_k^{-1} (p - \mu_k)\right)$$

I've had a trouble in integrating $\int \cos \psi G_1 G_2 dp$.

By studying https://en.wikipedia.org/wiki/Gaussian_integral that you link, I became aware of the Gaussian integral.

By now, I calculated below by refering https://en.wikipedia.org/wiki/Common_integrals_in_quantum_field_theory

$$\int G_k^2 = \int G_k dx \int G_k dy = \sqrt[2]{\frac{(2\pi)^d}{|C_k|}}$$

Thus

$$\int G_k = \sqrt[4]{\frac{(2\pi)^d}{|C_k|}}$$

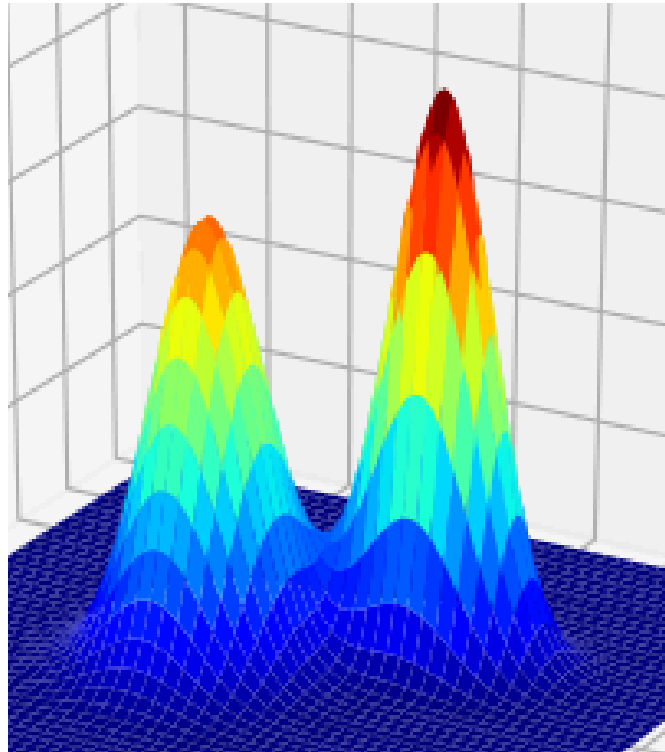
But the calculating $\int \cos \psi$ is tricky, as the initial values didn't present on the paper.

I'll spend this weekend on thinking about workaround.

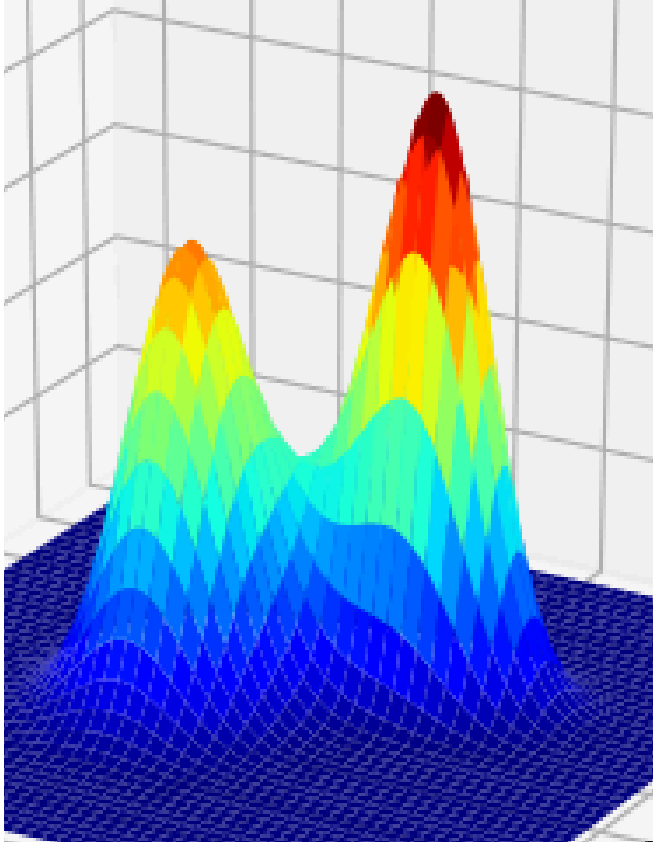
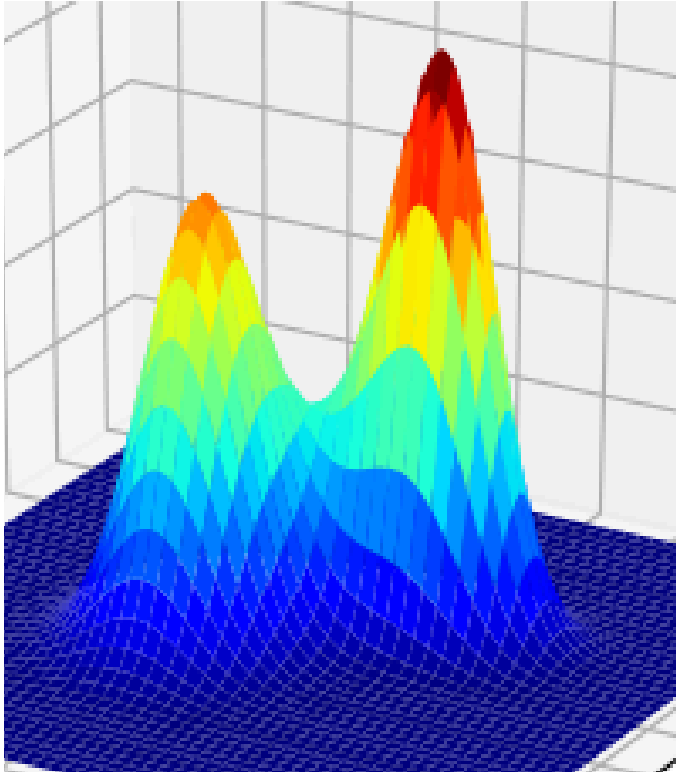
Sorry for the delay...

Classical GMM

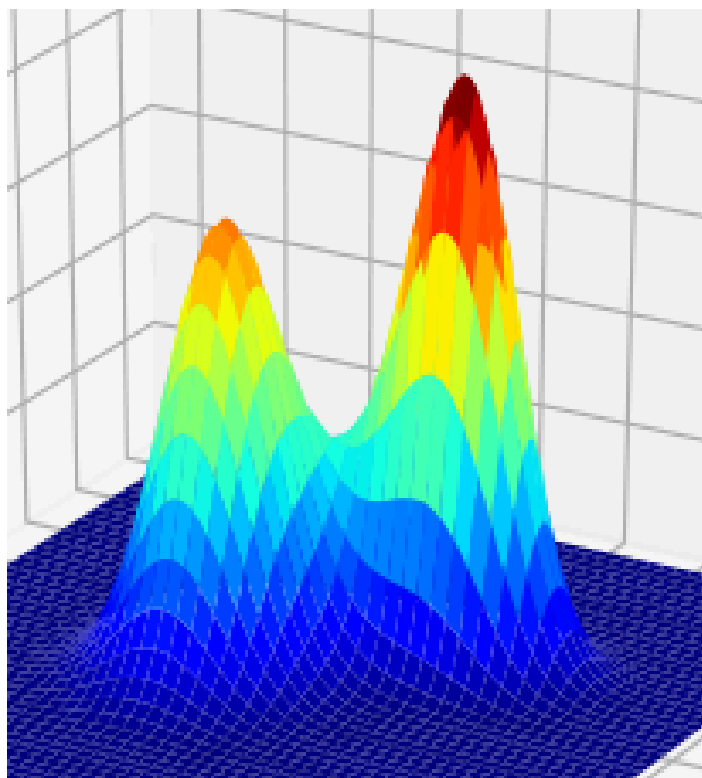
$\alpha_1 = 0.3$ $\alpha_2 = 0.22$



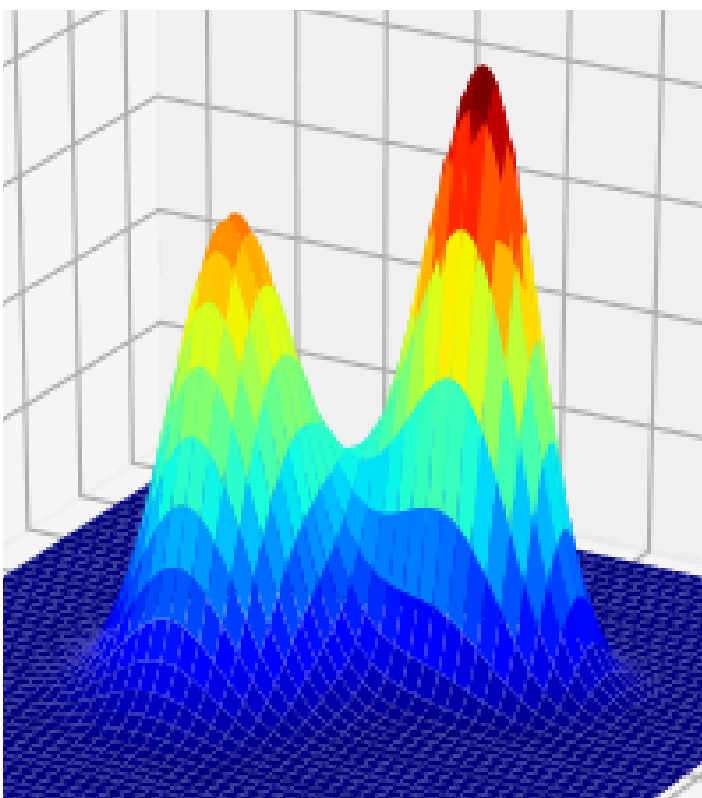
Quantum GMM

phi	alpha1 = 0.3 alpha2 = 0.22
0	 A 3D surface plot showing a bimodal distribution with two distinct peaks. The peaks are colored with a gradient from blue at the base to red at the top. The plot is set against a light gray grid background.
10	 A 3D surface plot showing a bimodal distribution, very similar to the one at phi=0. It features two peaks with a blue-to-red color gradient, set against a light gray grid background.

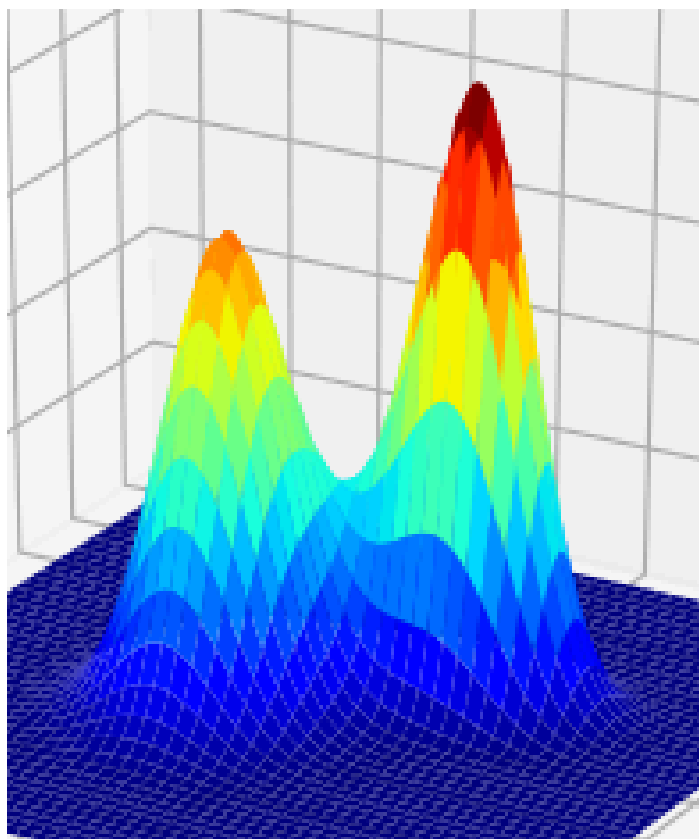
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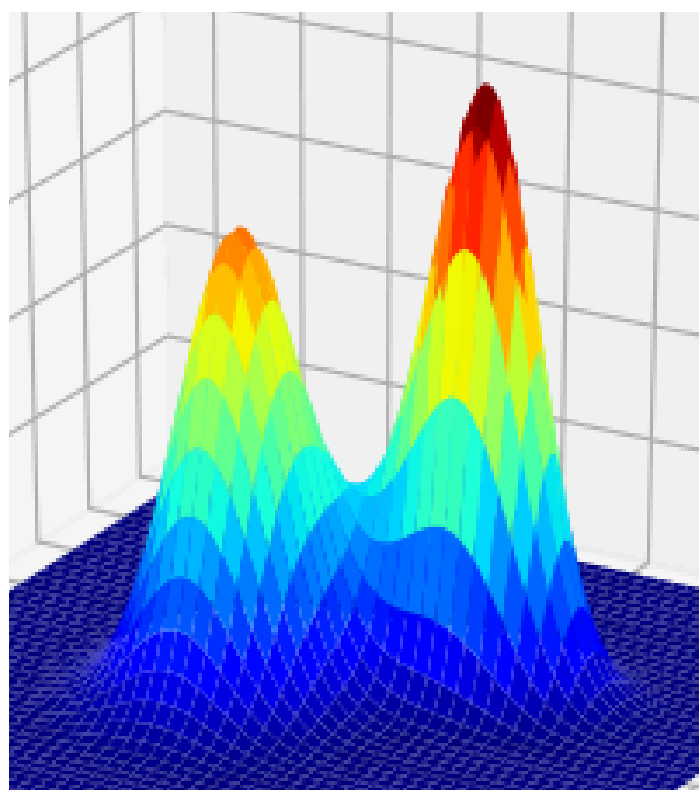
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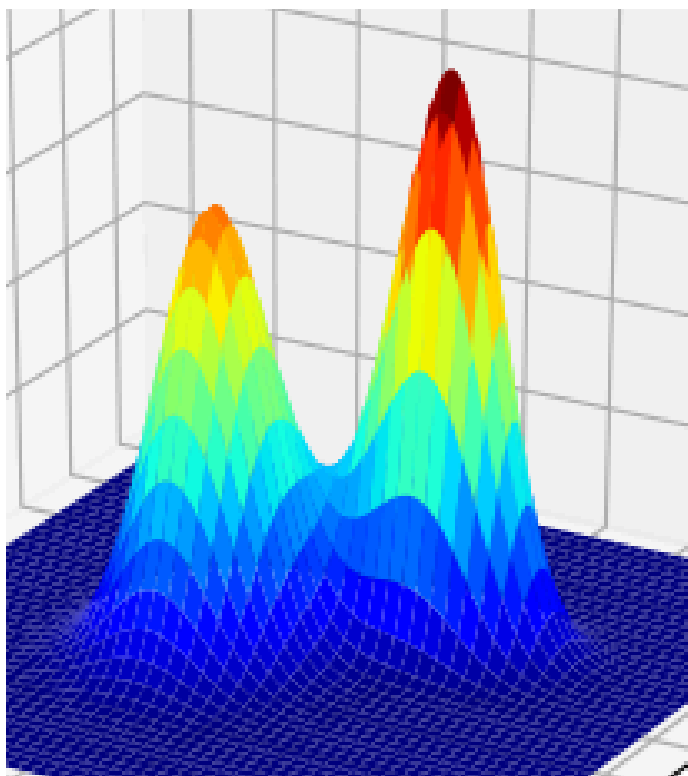
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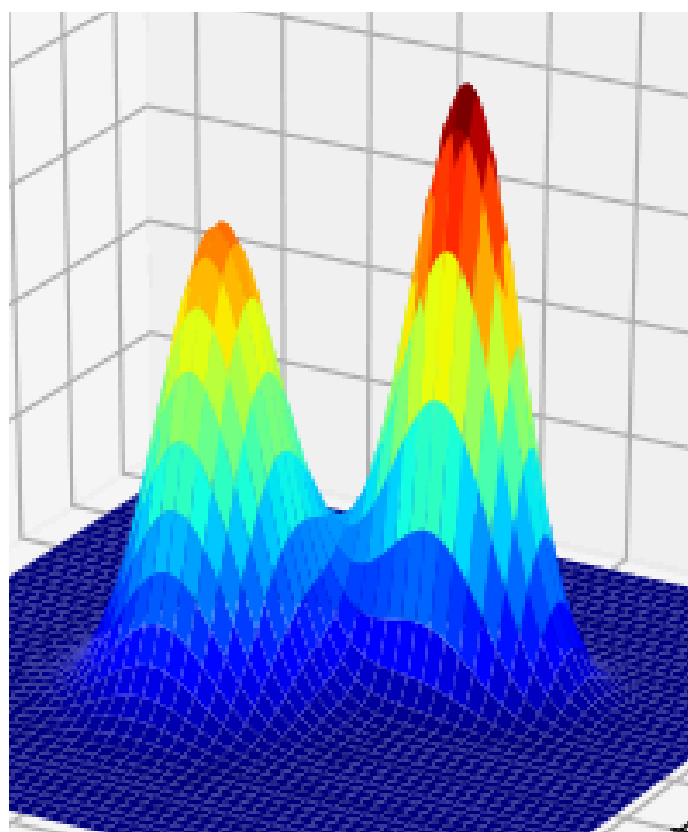
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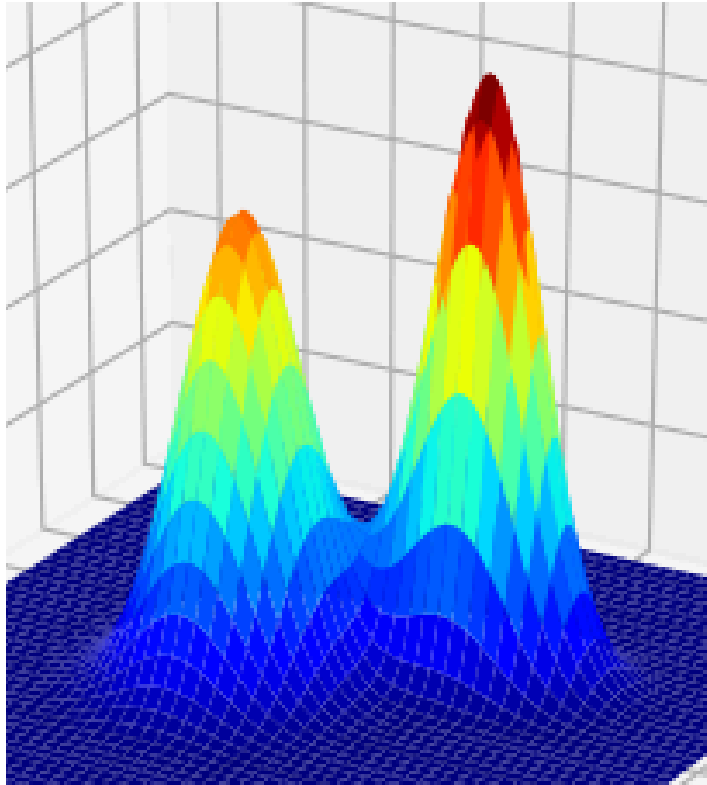
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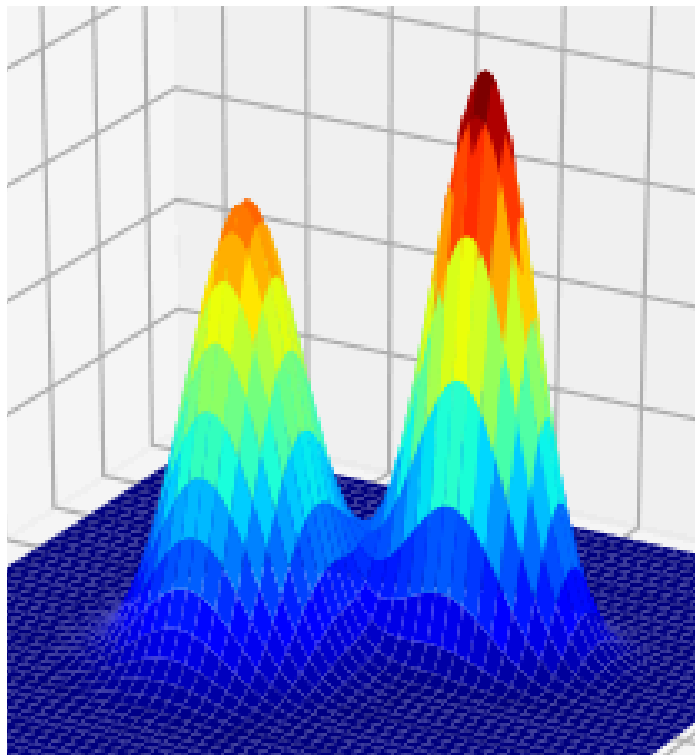
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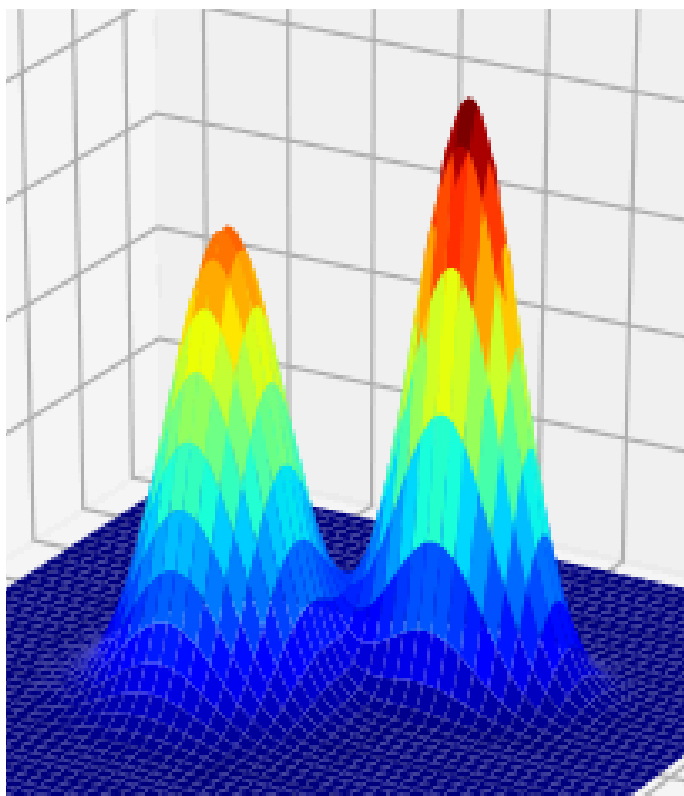
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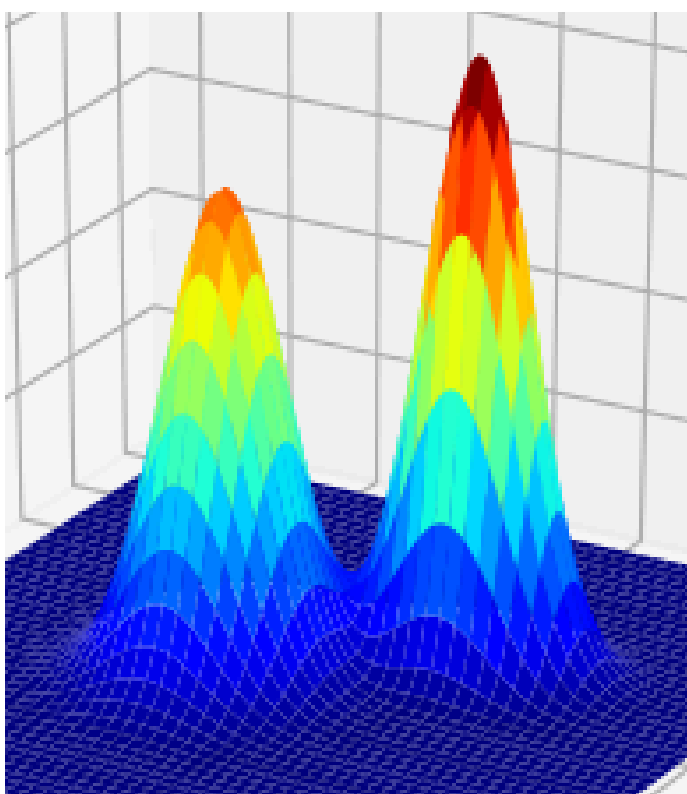
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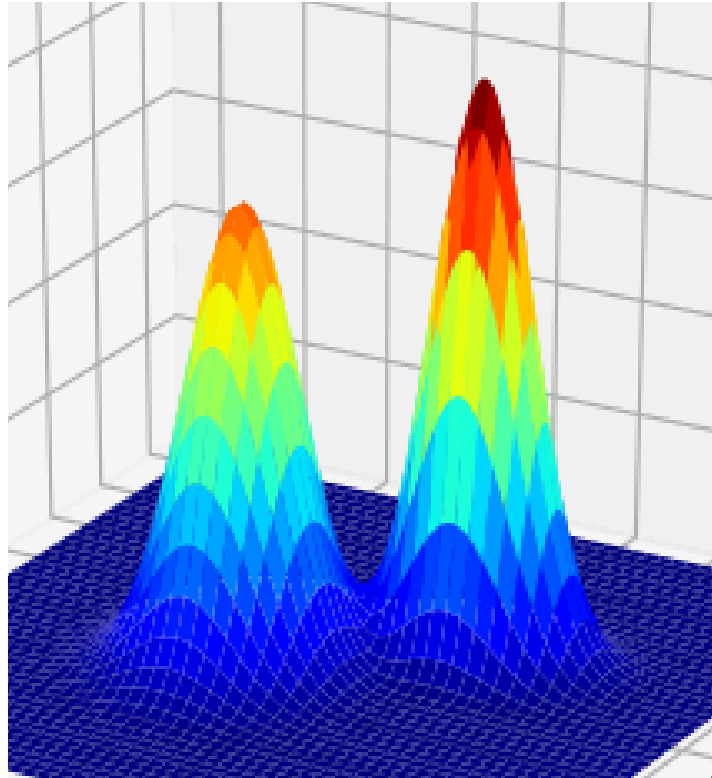
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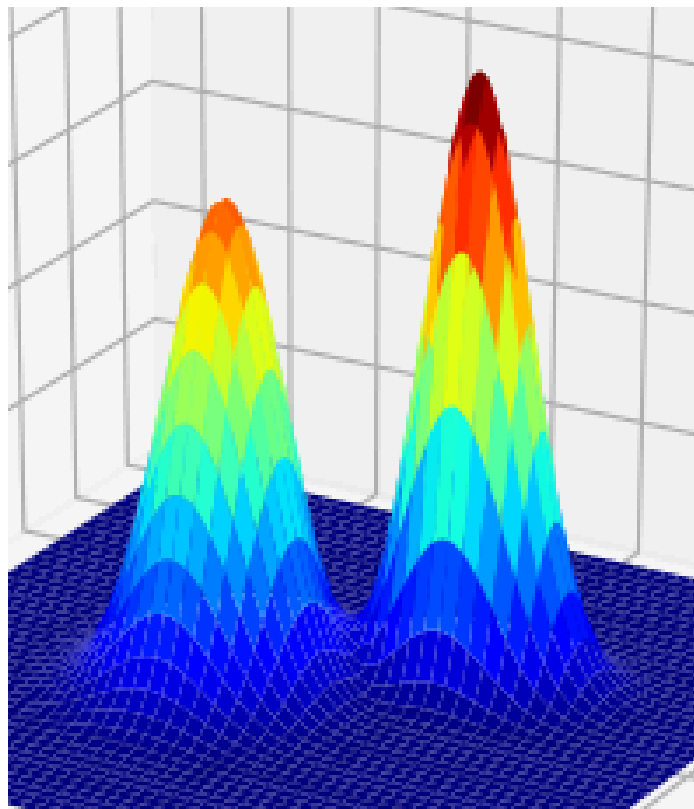
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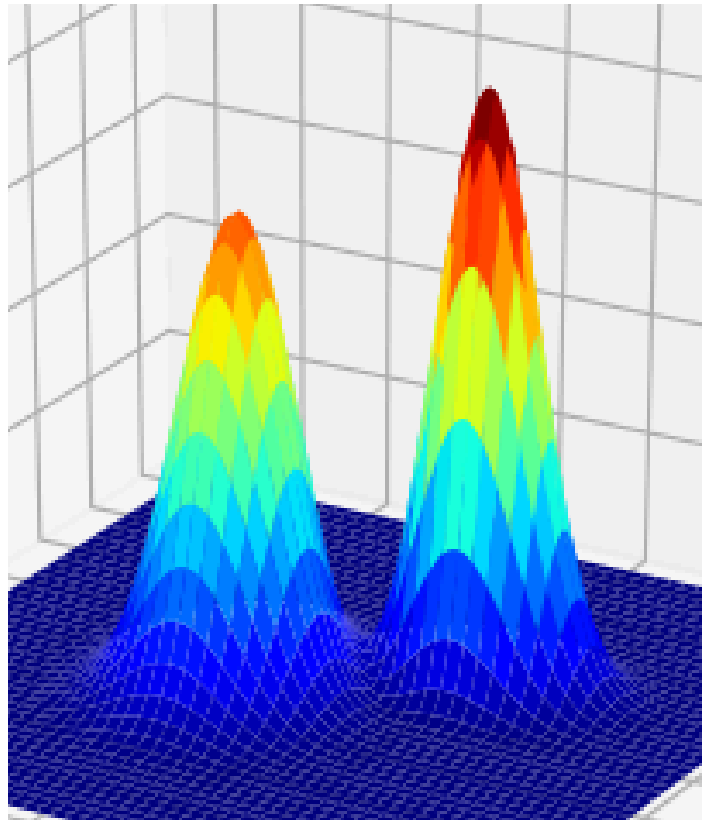
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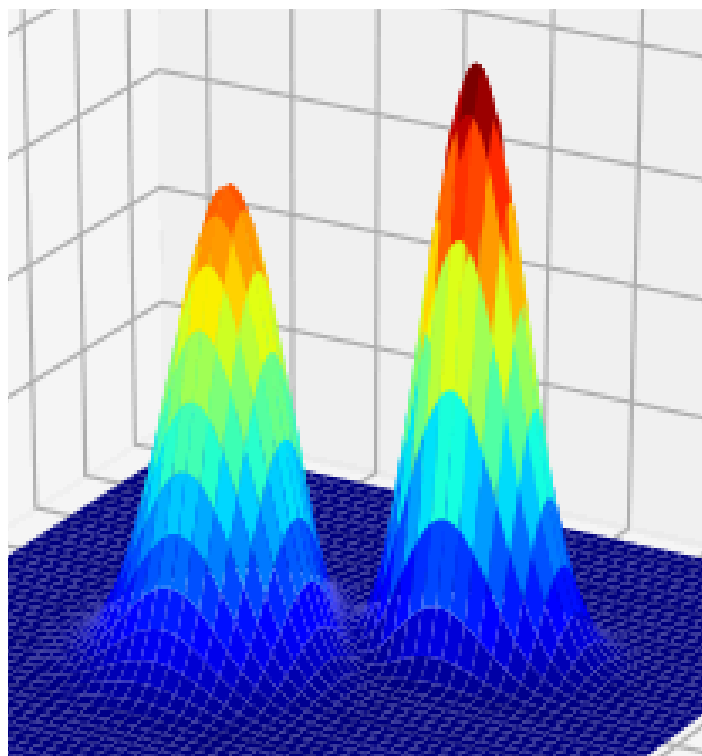
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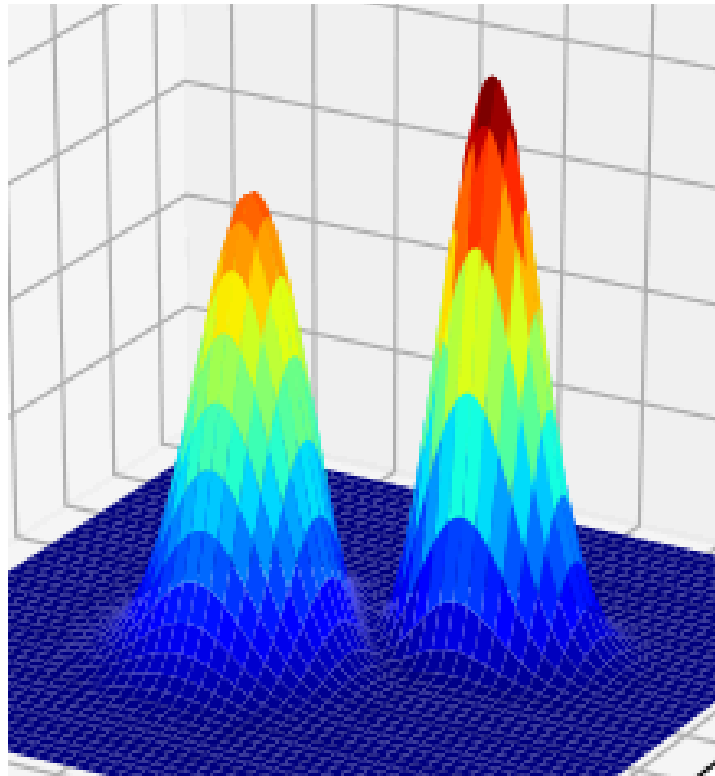
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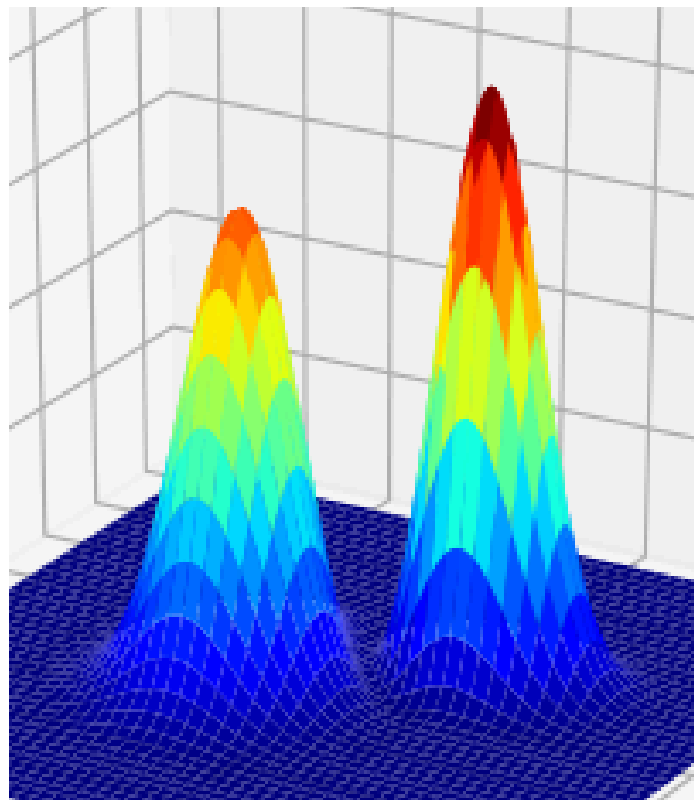
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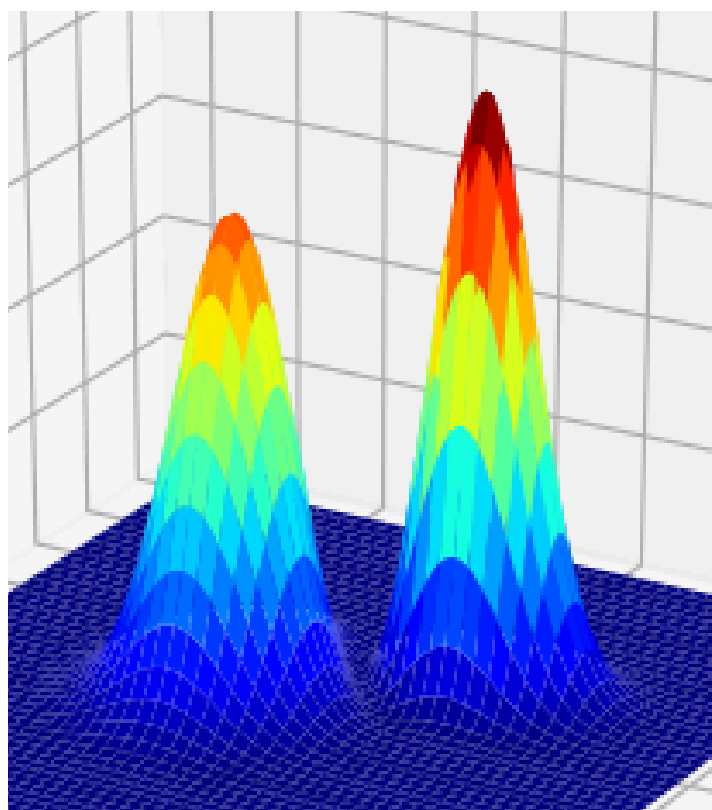
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170

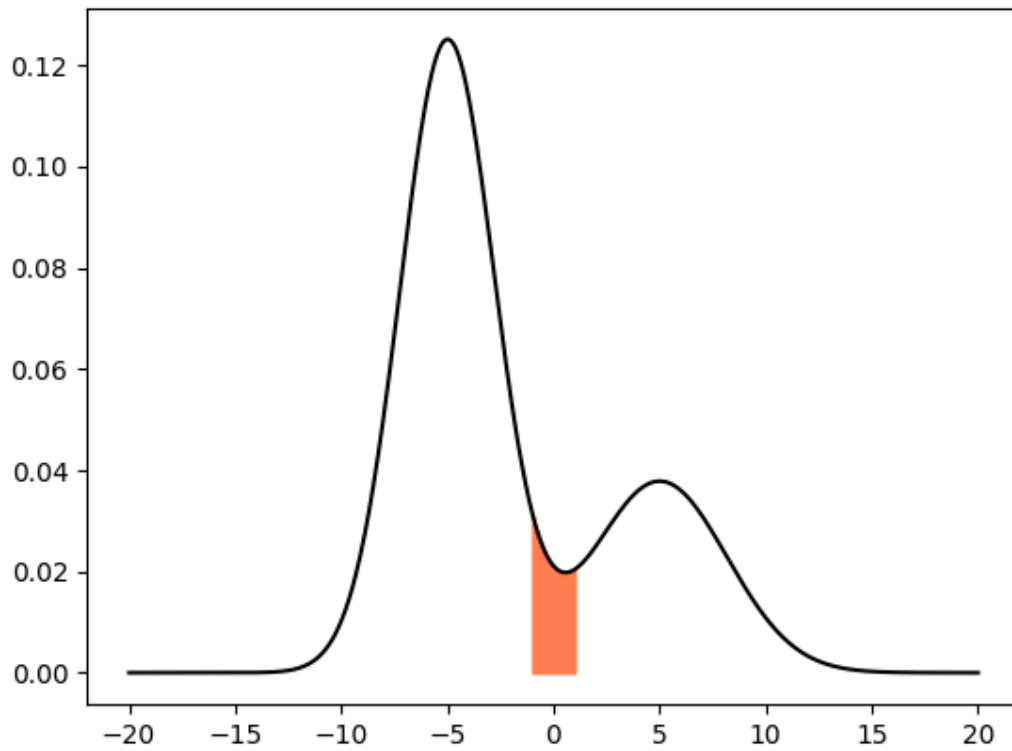


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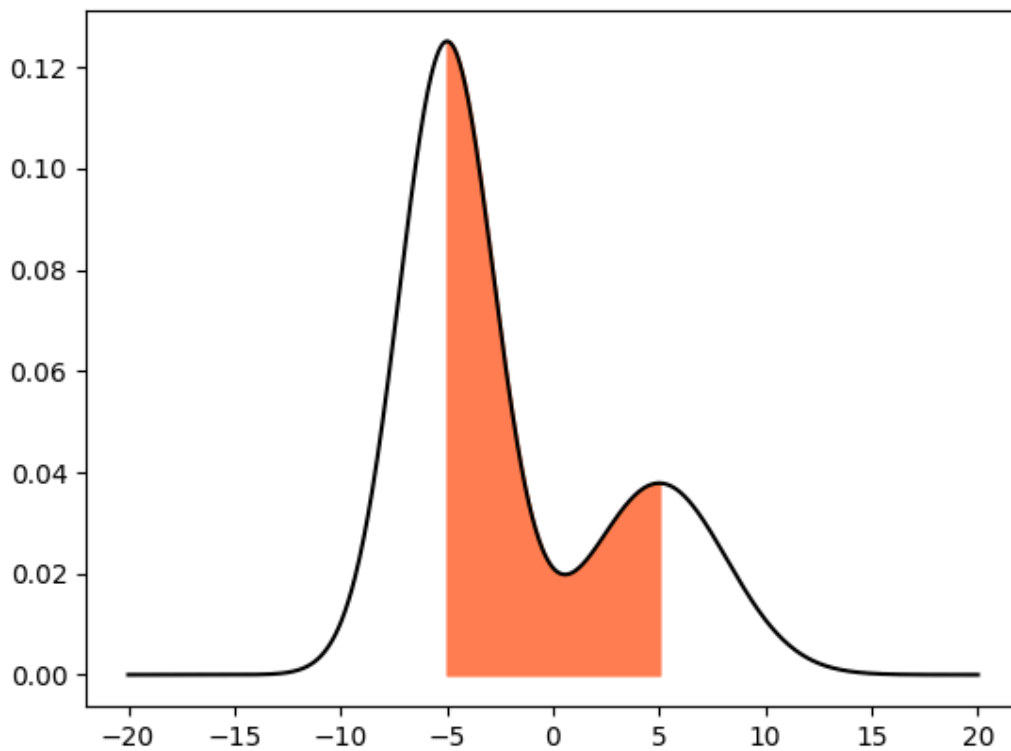


Integral of QGMM probability space

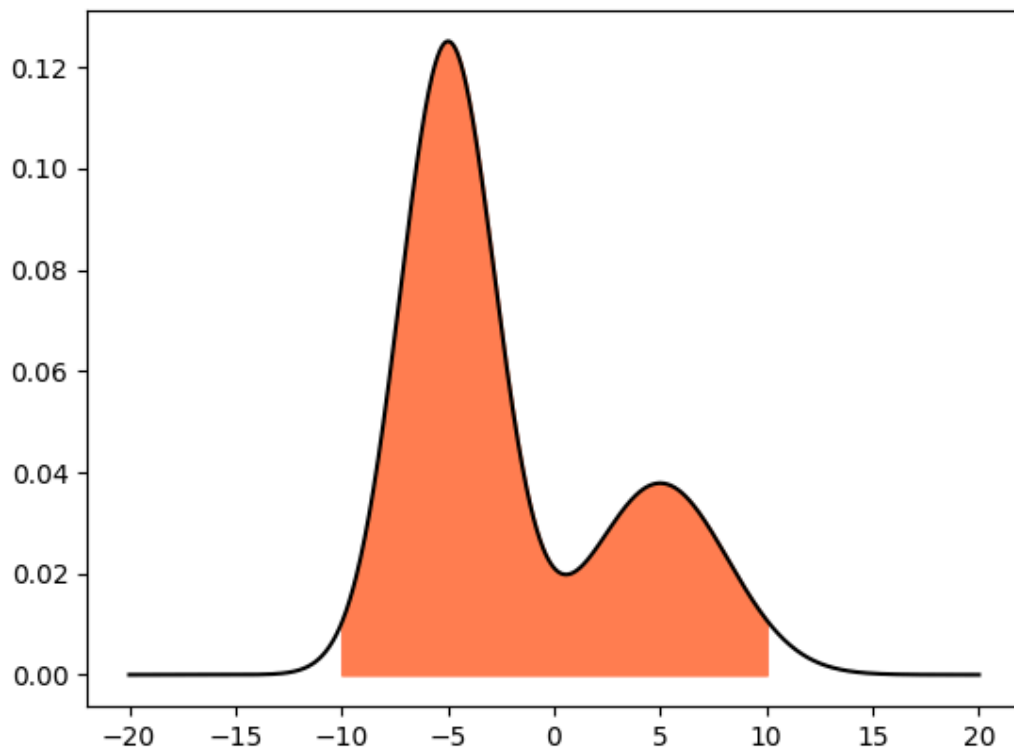
mean = 0, covariance = 1, probability = 0.04544



mean = 0, covariance = 5, probability = 0.4988



mean = 0, covariance = 10, probability = 0.974



mean = 0, covariance = 20, probability = 0.99

