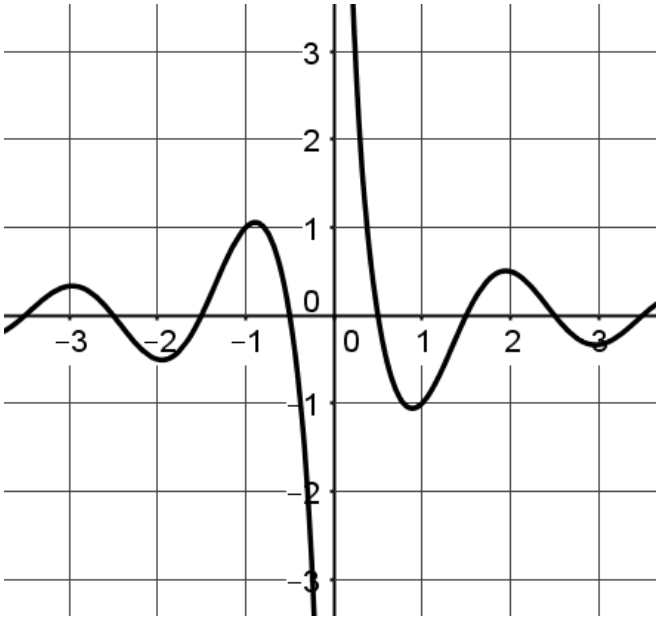
Unit 5 Test Ver 2 Key Name _____ Date _____ Per __

1. The graph of $y = \frac{cos(\pi x)}{x}$ is shown below. Determine the equations of the tangent lines at (a) x = 1 (b) x = -2. Verify your answers by sketching the tangent lines.



$$\frac{dy}{dx} = \frac{x(-sin(\pi x)\pi) - cos(\pi x)}{x^2}$$

$$\frac{dy}{dx} = \frac{-\pi x sin(\pi x) - cos(\pi x)}{x^2}$$

a)
Let x = 1

$$\frac{dy}{dx} = \frac{-\pi 1 sin(\pi 1) - cos(\pi 1)}{1^2}$$

$$\frac{dy}{dx} = \frac{0+1}{1} = 1$$ therefore m = 1

y = mx + b still need an x and a y to find b

$$y = \frac{cos(\pi 1)}{1} = \frac{-1}{1} = -1$$ therefore (1,-1)

-1 = 1(1) + b
b = -2

y = x - 2

b)

$$\frac{dy}{dx} = \frac{-\pi(-2)sin(\pi(-2)) - cos(\pi(-2))}{(-2)^2}$$

$$\frac{dy}{dx} = \frac{2\pi sin(-2\pi) - cos(-2\pi)}{4}$$

$$\frac{dy}{dx} = \frac{0-1}{4} = -\frac{1}{4}$$

y = mx + b still need an x and a y to find b

$y = \frac{cos(\pi(-2))}{-2} = \frac{1}{-2} = -\frac{1}{2}$ therefore (-2, -½)

-1/2 = (-1/4)(-2) + b
-0.5 = 0.5 + b
b = -1

y = -(1/4)x-1

2. A balloon guy uses a machine to pump helium into spherical balloons. The machine pumps helium at a rate of 60 cubic inches per second (in^3/sec). How fast is the radius of the balloon increasing when the radius is (a) 1 inch (b) 2 inches?

$$V = \frac{4}{3}\pi r^3$$

$$\frac{d}{dt}V = \frac{d}{dt}\frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

We want to know how fast the radius is changing, aka (dr/dt), so solve for (dr/dt)

$$\frac{\frac{dV}{dt}}{4\pi r^2} = \frac{dr}{dt}$$

a) Let r = 1

$$\frac{60}{4\pi 1^2} = \frac{dr}{dt}$$

(dr/dt) = 4.775 in/sec

b)

$$\frac{60}{4\pi 2^2} = \frac{dr}{dt}$$

(dr/dt) = 1.194 in/sec

3. Before Mr. Rose starts making notes, he might drink some energy drinks. If Mr. Rose has no energy drinks before making notes, Mr. Rose can make 2 pages of notes before he gets bored and stops. During this time, 10% of what he makes is useless. Fortunately, for every energy drinks he drinks, he has the motivation to make 2 more pages of notes. Unfortunately, every energy drinks increases the amount of uselessness he makes by 30%.
    a. How many pages of useful notes will Mr. Rose produce if he drinks 2 energy drinks?
    b. How many energy drinks should Mr. Rose drink to produce the maximum amount of useful notes?
    c. How many pages of useful notes would this allow Mr. Rose to make?

a)
W = PU
W = (2 + 2x)(0.9 - 0.3x)

Let x = 2

W = (6)(0.3) = 1.8 pages of useful notes

b)

W = PU

W: Pages of useful work
P: Pages of work done P = 2 + x
U: Usefulness fraction U = 90% - 20%x but we don't use % in math so U = 0.9 - 0.2x
x: Energy drinks drank before making notes

W = (2 + 2x)(0.9 - 0.3x)

$W = 1.8 - 0.6x + 1.8x - 0.6x^2$
$W = -0.6x^2 + 1.2x + 1.8$

Let's maximize

$$\frac{d}{dx}W = \frac{d}{dx}[-0.6x^2 + 1.2x + 1.8]$$

$$\frac{dW}{dx} = -1.2x + 1.2$$

Let (dW/dx) = 0

$0 = -1.2x + 1.2$
$1.2x = 1.2$
$x = 1$ energy drink

c)
Let $x = 1$
$W = (2 + 2)(0.9 - 0.3) = (4)(0.6) = 2.4$ pages of useful work

Slope Field
B