Function Transformations 1

Version 101.1

At any time, you can reset the toy to its original state with the button.

- Feel free to explore on your own, but then
- Reset to follow this guided exploration.

Section 1: Lines of slope 1

At the top of the drawing pad, you should see a slider showing “Page=1”. Below that, find two equations for a line named L_1, which is graphed nearby. These equations are in Point-Slope form and Slope-Intercept form.

1) Move A to (1,0). What are the two equations for line L_1? Write them down.

   Equation (1): Equation (2):

   Are these equivalent? How do you know?

2) Move A to (2,1), and write down the new equations.

   Equation (1): Equation (2):

   How did the equations change? Why did one change but not the other?

3) Predict what the two equations will become when you move A to (3,4), and record your predictions below.

   Predicted (1): Predicted (2):

   Then move A to (3,4).

   Actual (1): Actual (2):

   How did you do?

4) Move A to (-2,1). Now what are the equations?

Equation (1): Equation (2):

5) Predict what the two equations will become when you move A to (5,-1), and record your prediction.

Predicted (1): Predicted (2):

Then move A to (5,-1).

Actual (1): Actual (2):

How did you do?

6) Repeat #5 with two other points of your own choosing.

First point you chose:

Predicted (1): Predicted (2):

Actual (1): Actual (2):

Second point you chose:

Predicted (1): Predicted (2):

Actual (1): Actual (2):

Reflect on your progress so far. If everything is going smoothly, continue to the next section. Otherwise, consider talking to a classmate, a tutor, or your instructor. Do you have any questions or comments so far?

Section 2: Lines of other slopes

Move A back to (0,0), and then move the dot on the Page slider to Page=2. Now let's work with line L_2. You can control the slope of line L_2 with the Slope slider.

7) Move the Slope slider to different values. How does the line look when Slope is positive? What about when Slope is negative?

8) Notice how the lines look when Slope is -5, 0, and 5. Are two of these more alike than the other? If you think so: which two, and why?

9) Move Slope back to 2, and make sure A is still at (0,0). Write down the two equations for line L_2.

Equation (1): Equation (2):

10) Move A to (1,2), and write down the new equations.

Equation (1): Equation (2):

Can you explain why only one of these is different from your answer to #9?

11) Find another point on line L_2, besides (0,0) and (1,2). Write down your point. Predict how the two equations will look when you move A to that point.

Point:

Predicted (1): Predicted (2):

Now move A to the point you chose.

Actual (1): Actual (2):

How did you do? If you like, do that again.

Point:

Predicted (1): Predicted (2):

Now move A to the point you chose.

Actual (1): Actual (2):

12) Move A to (1,2), and move Slope to 5. Write down the two equations.

Equation (1): Equation (2):

13) Here's a list of numbers you might see in #12: 1, -1, 2, -2, 3, -3, 5.
Some of these numbers have geometric significance–they give you good information about the graph. Which ones, and what do they tell you?

14) Compare your answers to #10 and #12. Can you predict how Equation (1) will look when Slope is -3?

Predicted (1): Actual (1):

If you got that right, you're doing well. Please solve for y in that equation, and simplify.

Does your answer match Equation (2)?

15) If you know A and Slope, which is the easier equation to predict: (1) or (2)?

16) Predict what Equation (1) will be when A is (4,-1) and Slope is 1.5. Solve it for y and simplify to create a prediction for Equation (2).

Predicted (1): Predicted (2):
How did you do?

17) Do a few more like this on your own:

- Choose a point and slope, but don't move there yet.
- Predict Equation (1).
- Use your predicted Equation (1) to predict Equation (2).
- Move A and Slope to the values you chose for them, and check your predictions.

If you can do three in a row correctly, you're ready to continue to the next section. If you get two wrong, check in with the instructor.

Section 3: Parabolas

Move A back to (0,0), and then move the dot on the Page slider to Page=3.

18) What are the equations you see for parabola p?

Equation (1):

Equation (2):

Equation (3):

19) Move A to (0,1). What are the new equations? Simplify each of them.

Equation (1):	Simplified:
Equation (2):	Simplified:
Equation (3):	Simplified:

20) Now move A to (0,2). What are the new equations? Simplify each of them.

Equation (1):	Simplified:
Equation (2):	Simplified:
Equation (3):	Simplified:

21) Do you see the pattern? Predict what the three equations will become when you move A to (0,5).

Pre (1): Pre (2): Pre (3):

Now move A to (0,5). How did you do?

Take a break

Clear your mind. Once you're back…

22) Move A to (0,0). Glance back at #18; are those still correct?

Now move A to (1,0) and record the equations.

Eq'n (1): Eq'n (2): Eq'n (3):

23) How could you use Equation (1) or (2) to find Equation (3)? Make a short note to yourself here.

24) Predict how the three equations will look when you move A to (2,0). You may need to do some algebra to get Equation (3). (Is your note to self on #23 helpful?)

Pre (1): Pre (2): Pre (3):

Now move A to (2,0). How did you do?

25) Predict how the three equations will look when you move A to (-3,0).

Pre (1): Pre (2): Pre (3):

Now move A to (-3,0). How did you do?

If you are having trouble, this is a good time to get help from a neighbor or the teacher. Otherwise, try it two more times, using points you choose on the x-axis.

26) Move A to (2,1). What are the three equations now?

Eq’n (1): Eq’n (2): Eq’n (3):

27) Describe how you would turn #26 Equation (2) into #26 Equation (3).

28) Quick reverse-psychology check: Where do you think you should put A in order to make Equation (1) say $y - 2 = (x - 3)^2$? How about $y - 2 = (x + 4)^2$?

29) Using #26 as a hint, predict what Equation (1) will be when you move A to (2,-1). From this, compute predictions for Equation (2) and Equation (3).

Pre (1): Pre (2): Pre (3):

Then move A to (2,-1) to see how you did.

Er. . . when did Equation (4) show up?!

Public Service Announcement

Do you often wonder WHY OH WHY are there so many equations?
Have you ever noticed that handy people usually own more than one tool?
Not a coincidence!
Different equations are good for different things!

Here in Section 3…

- Equations (1) and (2) can both be called the **vertex form** of the equation for a parabola. Especially (2), but they’re very nearly the same.
- Equation (3) can be called the **standard form** or **expanded form**.
- Equation (4) can be called the **factored form**.

Terminology may vary.

Algebraic skills like factoring, distributing, and completing the square help you change the form of an equation to make it tell you different things. It’s like being able to turn your hammer into a crescent wrench and then a screwdriver.

30) Move A around. Compare two numbers:

- The rightmost number in Equation (1), a.k.a. the leftmost in equation (2)
- The leftmost number in Equation (3), a.k.a. the *linear coefficient*.

How are these two numbers related?

31) Wherever a graph touches the x-axis, we call that point an x-intercept.

- Sometimes we mean the whole ordered pair, like (3,0), and
- Sometimes we just mean its x-coordinate, like 3. (The y-coordinate is always 0.)

Move A around. Watch Equation (4) and the x-intercepts. What do you see?

32) Challenge! Suppose we want the parabola to have x-intercepts at 3 and 7.

What would Equation (4) look like?

Predicted Equation (4):

Multiply that out to see what Equation (3) would be.

Predicted Equation (3):

Based on your observations from #30, what will be the x-coordinate of A?

Predicted x-coordinate of A:

Can you think of a way to predict the y-coordinate of A?

Predicted y-coordinate of A:

Move A to see if your predictions were right.

This activity can be extended for students who know or are learning completing the square. For example, imagine any reasonable Equation (3). Complete the square to find Equation (2), and use this to predict where A should be. Move A there and see if you got the Equation (3) you wanted!

Section 4: Free play

Page 4 allows you to translate any function you like. Play with it this way:

- Move A back to (0,0), and then move the dot on the Page slider to Page=4.
- Enter any function of your choosing in the field marked “f(x) =”. Your function will appear in green as the “parent function”.
- Move A. As you do, a copy of your parent function will be moved appropriately. The transformed function will show up in orange, with the name g, and its formula will appear below the f(x) input field.

This whole concept—moving a graph left, right, up or down without distortion, and seeing how its equation changes—is called *translation*. My next GeoGebra guided exploration, FunctionTransformations2, addresses another kind of transformation you might use on a function.

If this exercise interests you and you’d like to work with me to make more like it, please contact me at
brad@humboldt.edu.

Links

The original copy of *this file* is at
https://docs.google.com/document/d/1agy2kC0zg_msTYXnxdximDCL1wZuZ8pJie4WJOnKaGI
and the virtual manipulative (“toy”) is at https://www.geogebra.org/m/Z6DtQYq8.

Discussion

If you have used this activity and want to share opinions about it, here’s the place. Please use the Google Doc Comment feature to leave a comment here.