1.

a. Expand $(3+x)^4$ fully.

$$\begin{aligned}(3+x)^4 &= 3^4 + 4\cdot 3^3\cdot x + 6\cdot 3^2\cdot x^2 + 4\cdot 3\cdot x^3 + x^4\\ &= 81 + 108x + 54x^2 + 12x^3 + x^4\end{aligned}$$

(2)

b. Hence find the exact value of 1003^4

$$\begin{aligned}1003^4 &= (3+x)^4\\ x &= 1000\\ 1003^4 &= 81 + 108,000 + 54,000,000 + 12,000,000,000 + 1,000,000,000,000\\ &= 1,012,054,108,081\end{aligned}$$

(2)

2.

a. Show that $(x-3)$ is a factor of $\mathrm{f}(x) \equiv 2x^3 - 5x^2 - 7x + 12$

$\mathrm{f}(3) = 2\cdot 3^3 - 5\cdot 3^2 - 7\cdot 3 + 12 = 0$

$\therefore (x-3)$ is a factor of $\mathrm{f}(x)$

(1)

b. Factorise $\mathrm{f}(x)$

$\mathrm{f}(x) = (x-3)\left(2x^2 + x - 4\right)$

(2)

c. Find the exact solutions of

i. $\mathrm{f}(x) = 0$

$$x = 3, \frac{-1 \pm \sqrt{33}}{4}$$

(1)

ii. $\mathrm{f}(x+2) = 0$

$$x = 1, \frac{-9 \pm \sqrt{33}}{4}$$

(1)

3.

$A = (-2, 6),\ B = (-1, -1)$ and $C = (5, 5)$

a. Find the gradient of line AB

$$m_{\mathrm{AB}} = \frac{6+1}{-2+1} = -7$$

(1)

b. Find the midpoint M of A and B

$$M = \left(\frac{-2-1}{2}, \frac{6-1}{2}\right) = \left(-\frac{3}{2}, \frac{5}{2}\right)$$

(1)

c. Find the equation of the perpendicular bisector of AB in the form of $y = mx + c$

$$y - \frac{5}{2} = \frac{1}{7}\left(x + \frac{3}{2}\right)$$
$$y = \frac{1}{7}x + \frac{19}{7}$$

(2)

d. Find the equation of the perpendicular bisector of BC

$$m_{\mathrm{BC}} = \frac{-1-5}{-1-5} = 1$$
$$\text{midpoint of BC} = \left(\frac{-1+5}{2}, \frac{-1+5}{2}\right) = (2, 2)$$

equation of $\perp$ bisector of BC

$$y - 2 = -1(x - 2)$$
$$y = -x + 4$$

(2)

e.

Find the intersection of the perpendicular bisector of AB and BC

$$\frac{1}{7}x + \frac{19}{7} = -x + 4$$
$$\frac{8}{7}x = \frac{9}{7}$$
$$x = \frac{9}{8}, \quad y = \frac{23}{8}$$

(2)

Given points A, B and C passing through the circle P,

f. Hence find the equation of the circle in the form of

$$(x-a)^2 + (y-b)^2 = r^2$$

where *a, b* and *r* constants to be found.

$$\text{radius of the circle} = \sqrt{\left(\frac{9}{8}+1\right)^2 + \left(\frac{23}{8}+1\right)^2} = \sqrt{\frac{625}{32}}$$

equation of the circle

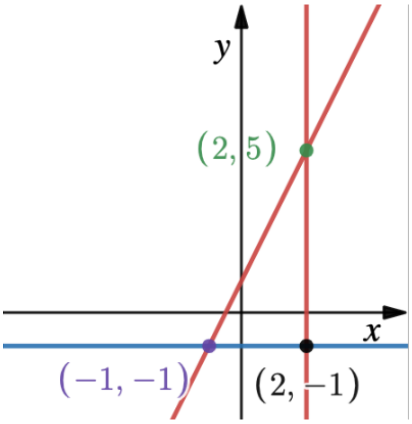
$$\left(x - \frac{9}{8}\right)^2 + \left(y - \frac{23}{8}\right)^2 = \left(\frac{25\sqrt{2}}{8}\right)^2$$

(3)

4.

a. Shade the region R for the inequalities below

$$y \leqslant 2x + 1$$
$$x \leqslant 2$$
$$y \geqslant -1$$



(3)

b. Find the area of region R.

$$\text{area of R} = \frac{1}{2}(3)(6) = 9\,\text{units}^2$$

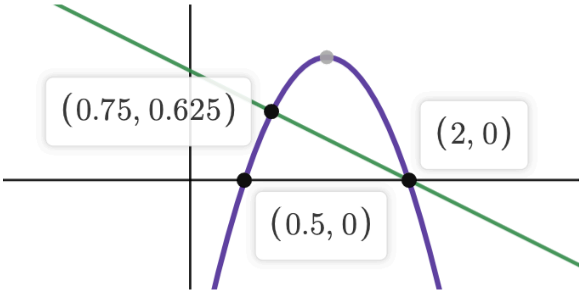
(1)

5.

$$\text{f}(x) = 1 - \frac{x}{2}$$

$$\text{g}(x) = 5x - 2 - 2x^2$$

a. Sketch on the same axes for $y = \text{f}(x)$ and $y = \text{g}(x)$



(5)

b. Hence solve the inequality for $\text{f}(x) \leqslant \text{g}(x)$

$$\frac{3}{4} \leqslant x \leqslant 2$$

(1)

6.

"$3^n + 2$ is a prime number for all positive integers n"

Disprove this statement.

$3^5 + 2 = 245 = 5 \times 49$

At least when $n = 5$ is not true.

(2)

7.

$$\frac{9^{x-2}}{3} = \frac{81^y}{27^x}$$

Find y in terms of x

$$\begin{aligned} 3^{2(x-2)-1} &= 3^{4y-3x} \\ 2(x-2)-1 &= 4y-3x \\ 2x-5 &= 4y-3x \\ y &= \frac{5}{4}x - \frac{5}{4} \end{aligned}$$

(3)

8.

$$\mathrm{f}(x) = \left(2 - \frac{x}{k}\right)^8$$

a. Find the coefficient of x^3 in terms of k

$$\binom{n}{r} a^{n-r} \cdot b^r$$

$$\begin{aligned} \text{required term} &= \binom{8}{3} 2^{8-3} \cdot \left(-\frac{x}{k}\right)^3 \\ &= 56 \cdot 32\left(-\frac{x^3}{k^3}\right) \\ \text{coefficient of } x^3 &= -\frac{1792}{k^3} \end{aligned}$$

(2)

$$\mathrm{g}(x) = (2+3x)\left(2 - \frac{x}{k}\right)^8$$

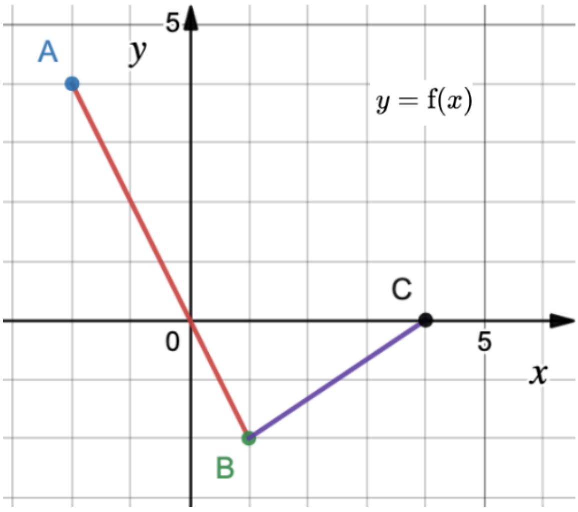
Given $k = 1$

b. Find the the value of the coefficient of x^2 of $\mathrm{g}(x)$

$$\begin{aligned} \mathrm{g}(x) &= (2+3x)\left(2 - \frac{x}{k}\right)^8 \\ &\approx (2+3x)\left[2^8 + 8 \times 2^7 \times (-x) + 28 \times 2^6 \times (-x)^2\right] \\ &\approx (2+3x)\left(256 - 1024x + 1792x^2\right) \\ \text{coefficient of } x^2 &= 2 \times 1792 - 3 \times 1024 \\ &= 512 \end{aligned}$$

(4)

9.



The graph of $y = \mathrm{f}(x)$ consists of 2 line segments between $A(-2,4)$ and $B(1,-2)$ and between B and $C(4,0)$

a. Find the length of AB, AC and BC

$$AB = \sqrt{(-2-1)^2+(4+2)^2} = \sqrt{45} = 3\sqrt{5}$$
$$AC = \sqrt{(-2-4)^2+(4-0)^2} = \sqrt{52} = 2\sqrt{13}$$
$$BC = \sqrt{(1-4)^2+(-2-0)^2} = \sqrt{13}$$

(3)

b. Find $\angle BAC$ correct to 1 decimal place

$$BC^2 = AB^2 + AC^2 - 2(AB)(AC)\cos\angle BAC$$
$$13 = 45 + 52 - 2\left(3\sqrt{5}\right)\left(2\sqrt{13}\right)\cos\angle BAC$$
$$\cos\angle BAC = \frac{7}{\sqrt{65}}$$
$$\angle BAC = 29.7^\circ \text{ (1dp)}$$

(3)

c. Find the exact value of the area of $\triangle ABC$

$$\sin x = \sqrt{1-\cos^2 x} = \frac{4}{\sqrt{65}}$$
$$\text{area of } \triangle ABC = \frac{1}{2}(AB)(AC)\sin\angle BAC$$
$$= \frac{1}{2}\left(3\sqrt{5}\right)\left(2\sqrt{13}\right)\frac{4}{\sqrt{65}}$$
$$= 12\,\text{units}^2$$

(3)

d. Hence find the shortest distance between from point B to line AC

$$\text{shortest distance} = \frac{24}{3\sqrt{5}} = \frac{8\sqrt{5}}{5}$$

(1)

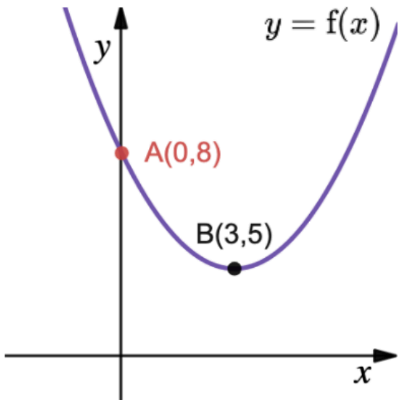
10.

Find the range of values of k such that the line $y = x + k$ cuts the circle $(x+2)^2 + y^2 = 8$ at two distinct points.

$$\begin{aligned}
(x+2)^2 + (x+k)^2 &= 8 \\
x^2 + 4x + 4 + x^2 + 2kx + k^2 - 8 &= 0 \\
2x^2 + (4+2k)x + \left(k^2 - 4\right) &= 0 \\
(4+2k)^2 - 4(2)\left(k^2-4\right) &> 0 \\
16 + 16k + 4k^2 - 8k^2 + 32 &> 0 \\
-4k^2 + 16k + 48 &> 0 \\
k^2 - 4k - 12 &< 0 \\
(k-6)(k+2) &< 0 \\
-2 < k &< 6
\end{aligned}$$

(4)

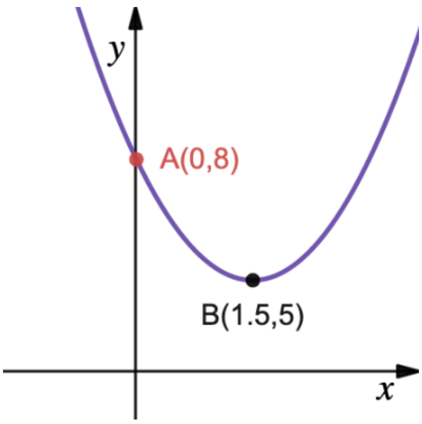
11.



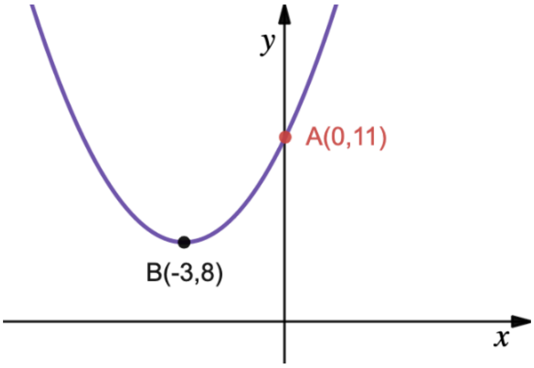
The graph of $y = \mathrm{f}(x)$ is shown in fig 3. It cuts the y axis at $A(0,8)$ and has a minimum point at $B(3,5)$

a. Sketch on separate axes the graphs of

i. $y = \mathrm{f}(2x)$



ii. $y = 3 + \mathrm{f}(-x)$



giving the coordinates of the points to which A and B are transformed.

(6)

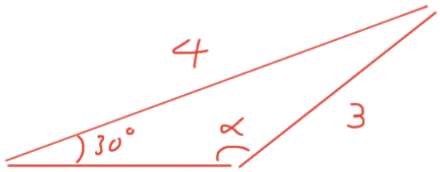
The graph of $y = a + \mathrm{f}(x + b)$ has a minimum point at the origin.

b. Find the values of *a* and *b*.

$a = -5,\ b = 3$

(2)

12.



Given α is an obtuse angle, find α in degrees correct to 1 decimal place

$$\frac{3}{\sin 30} = \frac{4}{\sin \alpha}$$
$$\sin \alpha = \frac{4 \sin 30}{3}$$
$$\alpha = 41.8$$
$$\text{obtuse angle} = 180 - 41.8 = 138.2^\circ \text{ (1dp)}$$

(4)
