Roll No…….. **Total No. of Printed Pages: 1**
Total No. of Questions: [09]

B. Tech. (Civil Engg.) (Semester – 3rd)
MATHEMATICS-III (TRANSFORM & DISCRETE MATHEMATICS)
Subject Code: BMATH4-301
Paper ID: 19110705

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(2 marks each)**

Q1. Attempt the following:

a) Evaluate $L\left\{\int_0^t \frac{\sin sin\, u}{u}\, du\right\}$

b) Prove that $T_n(x) - 2x\,T_{n-1}(x) + T_{n-2}(x) = 0$

c) Find $Z\left[(n+1)^2\right]$

d) Prove that $A \cap (B - C) \subset A - (B \cap C)$

e) Prove that if R is a symmetric relation, then $R \cap R^{-1} = R$.

f) Define order of an element of a group.

g) Minimize the expression $\overline{AB} + \overline{A} + AB$

h) Let X be a non-empty set, then prove that poset $(P(X), \subseteq)$ of all subsets of X is a lattice.

i) There are twelve students in a class. Find the number of ways that the twelve students take three different tests if four students are to take each test.

j) Prove that a connected multigraph is Eulerian iff every point of graph has even degree.

Section – B **(5 marks each)**

Q2. Apply Convolution theorem to find inverse Laplace transform of $\frac{s^2}{\left(s^2+a^2\right)^2}$

Q3. Let $f: A \to B$ *and* $g: B \to C$ are both one-one and onto, then $(gof)^{-1} = f^{-1} o g^{-1}$

Q4. Show that for a bounded distributive lattice, complement of an element is unique.

Q5. A non-directed graph G is connected if and only if G contains a spanning tree.

Q6. Show that a finite integral domain is a field.

Section – C **(10 marks each)**

Q7. Solve the initial value problem $y'' + 2y' + 5y = e^{-t} \sin sin\, t$, $y(0) = 0$, $y'(0) = 1$.

Q8. (a) Prove the validity of the arguments: "If man is a bachelor, he is unhappy. If a man is unhappy, he dies young. Therefore, bachelors die young".
(b) State and prove Lagrange's theorem.

Q9. For the recurrence relation $s_n - 6s_{n-1} + 8s_{n-2} = 0$ *for* $n \geq 2$ *and* $s_0 = 10$, $s_1 = 25$

(i) Find generating function
(ii) Find the sequence which satisfies it.