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M.Sc. (Mathematics) (Semester-1st)
DIFFERENTIAL EQUATIONS
Subject Code: MMAT1104
Paper ID: [19220504]

Time: 03 Hours **Maximum Marks: 60**

Instruction for candidates:

1. Section A is compulsory. It carries 16 marks. It consists of 4 questions of 4 marks each.
2. Section B consist of 4 questions of 8 marks each. The student has to attempt any 3 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A **(4 marks each)**

Q1. Find the Green's function of differential equation $\frac{d^2y}{dx^2}+y=x,\ y(0)=1,\ y'(1)=0$.

Q2. Explain the stability and Lyapunov functions with examples.

Q3. Find the general solution of the linear partial differential equation $z(xp-yq)=y^2-x^2$

Q4. Solve $ys+p=\cos(x+y)-y\sin(x+y)$

Section – B **(8 marks each)**

Q5. Find the characteristics values and characteristics functions of the Sturm- Liouville Problem

$$\frac{d}{dx}\left[x\frac{dy}{dx}\right]+\frac{\lambda}{x}y=0,\ \ y(1)=0,\ \ \ y\left(e^{\pi}\right)=0$$

Q6. State and prove Poincare-Bendixson Theorem.

Q7. Find the complete integral of $xp+3yq=2\left(z-x^2p^2\right)$

Q8. Reduce the equation to the canonical form $\frac{\partial^2 z}{\partial x^2}+x^2\frac{\partial^2 z}{\partial y^2}=0$

Section – C **(10 marks each)**

Q9. Check the existence and uniqueness of solution of initial value problem

$\frac{dy}{dx}=y^2, y(1)=-1$. Hence solve it.

Q10. (a) Find the critical point of the system of differential equations

$\frac{dy}{dx}=-2x+3y,\ \frac{dy}{dx}=-x+y$. Also determine the nature and stability of critical point.

(b) Solve by Jacobi's Method $p_1x_1+p_2x_2=p_3^2$ (5+5)

Q11. What do you mean by origin of partial differential equation of second order? Classify the equation $y^2\frac{\partial^2 z}{\partial x^2}-2xy\frac{\partial^2 z}{\partial x\partial y}+x^2\frac{\partial^2 z}{\partial y^2}=\frac{y^2}{x}\frac{\partial z}{\partial x}+\frac{x^2}{y}\frac{\partial z}{\partial y}$. Hence Solve it.