

**M.Sc. (Mathematics) (Semester-1<sup>st</sup>)**  
**DIFFERENTIAL EQUATIONS**  
**Subject Code: MMAT1104**  
**Paper ID: [19220504]**

**Time: 03 Hours**

**Maximum Marks: 60**

**Instruction for candidates:**

1. Section A is compulsory. It carries 16 marks. It consists of 4 questions of 4 marks each.
2. Section B consist of 4 questions of 8 marks each. The student has to attempt any 3 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

**Section – A**

**(4 marks each)**

- Q1. Find the Green's function of differential equation  $\frac{d^2y}{dx^2} + y = x, y(0) = 1, y'(1) = 0$ .
- Q2. Explain the stability and Lyapunov functions with examples.
- Q3. Find the general solution of the linear partial differential equation  $z(xp - yq) = y^2 - x^2$
- Q4. Solve  $ys + p = \cos(x + y) - y \sin(x + y)$

**Section – B**

**(8 marks each)**

- Q5. Find the characteristics values and characteristics functions of the Sturm- Liouville Problem
- $\frac{d}{dx} \left[ x \frac{dy}{dx} \right] + \frac{\lambda}{x} y = 0, y(1) = 0, y(e^\pi) = 0$
- Q6. State and prove Poincare-Bendixson Theorem.
- Q7. Find the complete integral of  $xp + 3yq = 2(z - x^2 p^2)$
- Q8. Reduce the equation to the canonical form  $\frac{\partial^2 z}{\partial x^2} + x^2 \frac{\partial^2 z}{\partial y^2} = 0$

**Section – C**

**(10 marks each)**

- Q9. Check the existence and uniqueness of solution of initial value problem

$\frac{dy}{dx} = y^2, y(1) = -1$ . Hence solve it.

- Q10. (a) Find the critical point of the system of differential equations

$\frac{dy}{dx} = -2x + 3y, \frac{dy}{dx} = -x + y$ . Also determine the nature and stability of critical point.

(b) Solve by Jacobi's Method  $p_1 x_1 + p_2 x_2 = p_3^2$  (5+5)

Q11. What do you mean by origin of partial differential equation of second order? Classify the

equation  $y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x} \frac{\partial z}{\partial x} + \frac{x^2}{y} \frac{\partial z}{\partial y}$  . Hence Solve it.