CHAPTER 5

DAY 15 – LOGARITHMIC DIFFERENTIATION

In this section, we will differentiate the functions of the form $y = f(x) = [u(x)]^{v(x)}$. We differentiate such functions by taking logarithm on both sides and then applying the properties of logarithm.

Properties of log function

1. $\log \log pq = \log \log p + \log \log q$

2. $\log \log \frac{p}{q} = \log \log p - \log \log q$

3. $\log \log p^n = n \log p$

Note:

- $e^{\log \log x} = \log \log e^x$

- $\frac{d}{dx}(a^x) = a^x \log \log a$

Consider the function $y = \frac{(x+1)(x+2)^2}{(x+3)(x+4)}$. We differentiate the function as follows:

Taking log on both sides, we have

$$\log \log y = \log \log \left(\frac{(x+1)(x+2)^2}{(x+3)(x+4)} \right)$$

$$\log \log y = \log \log \left((x+1)(x+2)^2 \right) - log((x+3)(x+4))$$

$$\log \log y = \log \log (x+1) + \log \log (x+2)^2 - [\log \log (x+3) + \log \log (x+4)]$$

$$\log \log y = \log \log (x+1) + 2\log \log (x+2) - \log \log (x+3) - \log \log (x+4)$$

Differentiating with respect to x on both sides, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(x+1)} + \frac{2}{(x+2)} - \frac{1}{(x+3)} - \frac{1}{(x-4)}$$

$$\frac{dy}{dx} = y\left[\frac{1}{(x+1)} + \frac{2}{(x+2)} - \frac{1}{(x+3)} - \frac{1}{(x-4)} \right]$$

$$\frac{dy}{dx} = \left(\frac{(x+1)(x+2)^2}{(x+3)(x+4)} \right)\left[\frac{1}{(x+1)} + \frac{2}{(x+2)} - \frac{1}{(x+3)} - \frac{1}{(x-4)} \right]$$

Questions

Find the derivative of following functions with respect to x

1. $\dfrac{(x-2)}{(x+4)}$

2. $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

3. $(x-1)(x+2)^3$

4. $\log\log x^{\cos\cos x}$

5. x^x

6. $x^{\sin\sin x} + x)^{\cos\cos x}$

7. If $e^{y-x} = x^y$, prove that $\dfrac{dy}{dx} = \dfrac{\log\log x}{[\log\log ex\,]^2}$

8. Find $\dfrac{dy}{dx}$ for the following functions

    a. $x^y + y^x = 1$

    b. $x)^y = (\cos\cos y)^x$

    c. $xy = e^{x-y}$

If the question is $x^y = y^x$, then we can write $x^y - y^x = 0$ and proceed as (a)

More questions must be practiced.