CHAPTER 5

DAY 15 – LOGARITHMIC DIFFERENTIATION

In this section, we will differentiate the functions of the form $y = f(x) = [u(x)]^{v(x)}$. We differentiate such functions by taking logarithm on both sides and then applying the properties of logarithm.

Properties of log function

1. $\log log\, pq = \log log\, p + \log log\, q$
2. $\log log\, \frac{p}{q} = \log log\, p - \log log\, q$
3. $\log log\, p^n = n log\, p$

Note:

- $e^{\log log\, x} = \log log\, e^x$
- $\frac{d}{dx}(a^x) = a^x \log log\, a$

Consider the function $y = \frac{(x+1)(x+2)^2}{(x+3)(x+4)}$. We differentiate the function as follows:

Taking log on both sides, we have

$$\log log\, y = \log log \left(\frac{(x+1)(x+2)^2}{(x+3)(x+4)}\right)$$

$$\log log\, y = \log log \left((x+1)(x+2)^2\right) - log((x+3)(x+4))$$

$$\log log\, y = \log log\,(x+1) + \log log\,(x+2)^2 - [\log log\,(x+3) + \log log\,(x+4)]$$

$$\log log\, y = \log log\,(x+1) + 2\log log\,(x+2) - \log log\,(x+3) - \log log\,(x+4)$$

Differentiating with respect to x on both sides, we get

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{(x+1)} + \frac{2}{(x+2)} - \frac{1}{(x+3)} - \frac{1}{(x-4)}$$

$$\frac{dy}{dx} = y\left[\frac{1}{(x+1)} + \frac{2}{(x+2)} - \frac{1}{(x+3)} - \frac{1}{(x-4)}\right]$$

$$\frac{dy}{dx} = \left(\frac{(x+1)(x+2)^2}{(x+3)(x+4)}\right)\left[\frac{1}{(x+1)} + \frac{2}{(x+2)} - \frac{1}{(x+3)} - \frac{1}{(x-4)}\right]$$

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Questions

Find the derivative of following functions with respect to x

1. $\frac{(x-2)}{(x+4)}$
2. $\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$
3. $(x-1)(x+2)^3$
4. $\log log\, x^{\cos cos\, x}$
5. x^x
6. $x^{\sin sin\, x} + x)^{\cos cos\, x}$
7. If $e^{y-x} = x^y$, prove that $\frac{dy}{dx} = \frac{\log log\, x}{[\log log\, ex]^2}$
8. Find $\frac{dy}{dx}$ for the following functions
   a. $x^y + y^x = 1$
   b. $x)^y = (\cos cos\, y)^x$
   c. $xy = e^{x-y}$

If the question is $x^y = y^x$, then we can write $x^y - y^x = 0$ and proceed as (a)

More questions must be practiced.

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