Most Natural Row Operations

Starting in Section 1.3 of the course you are doing lots of work with reducing matrices using row operations. You will often see the wording "most natural row operations", but what does that mean exactly?

Luckily, it is pretty simple!

Let's say that you have a matrix that looks like this (I've put it in the row operations tool):

2	3	2	5
6	7	8	9
3	4	5	6

My goal is to make it look like this (where the ? will be numbers):

1	0	0	?
0	1	0	?
0	0	1	?

But how do I get there?

For a 3x4 matrix like this, this is the "most natural order":
1) Change the number in R1C1 into a 1
2) Change the rest of column 1 into 0s
3) Change the number in R2C2 into a 1
4) Change the rest of column 2 into 0s
5) Change the number in R3C3 into a 1
6) Change the rest of column 3 into 0s

That's it! Simple and easy. Work one column at a time. Make the 1 where it belongs along the main diagonal, then make the rest into 0s, then move on to the next column, repeat until you are done.

Here I am doing these steps on our example matrix in case you want to follow along to verify that you understand.

2	3	2	5
6	7	8	9
3	4	5	6

Step 1: Change the 2 in R1C1 into a 1. We will use this row operation:
(1/2)R1
That gives us:

1	3/2	1	5/2
6	7	8	9
3	4	5	6

Step 2: Change the rest of column 1 into 0s. We will use these row operations:
R2 - 6R1
R3 - 3R1
That gives us:

1	3/2	1	5/2
0	-2	2	-6
0	-1/2	2	-3/2

Step 3: Change the -2 in R2C2 into a 1. Row operation:
(-1/2)R2

1	3/2	1	5/2
0	1	-1	3
0	-1/2	2	-3/2

Step 4: Change the rest of column 2 into 0s. Row operations:
R1 - (3/2)R2
R3 + (1/2)R2

1	0	5/2	-2
0	1	-1	3
0	0	3/2	0

Step 5: Change the 2 in R3C3 into a 1. Row operation:
(2/3)R3

1	0	5/2	-2
0	1	-1	3
0	0	1	0

Step 6: Change the rest of column 3 into 0s. Row operations:
R1 - (5/2)R3
R2 + R3

1	0	0	-2
0	1	0	3
0	0	1	0

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Final notes:

– This applies to all sizes of matrices, not just 3x4 matrices. The method is the same no matter the size. Work one column at a time, make the 1, make the 0s, move to the next column. If there are "too many rows" you'll end up with some extra rows that are all 0s in the end and then you can ignore them.

– Want to save time? Just type in your matrix and click "reduce completely" and it will do all the work for you! Alternatively, you can use "pivot on selection" to make the 1 and 0s in a single column. The end of the Section 1.3 movie that is 7:55 long covers these shortcuts in detail.