Mark Scheme

Q1.

Question	Scheme	Marks	AOs
(a)	Substitution of both $t = 0$ and $t = 10$	M1	2.1
	$s = 0$ for both $t = 0$ and $t = 10$	A1	1.1b
	Explanation ($s > 0$ for $0 < t < 10$) since $s = \frac{1}{10}t^2(t-10)^2$	A1	2.4
		(3)	
(b)	Differentiate displacement s w.r.t. t to give velocity, v	M1	1.1a
	$v = \frac{1}{10}(4t^3 - 60t^2 + 200t)$	A1	1.1b
	Interpretation of 'rest' to give $v = \frac{1}{10}(4t^3 - 60t^2 + 200t) = \frac{2}{5}t(t-5)(t-10) = 0$	M1	1.1b
	$t = 0, 5, 10$	A1	1.1b
	Select $t = 5$ and substitute their $t = 5$ into s	M1	1.1a
	Distance $= 62.5$ m	A1 ft	1.1b
		(6)	
			(9 marks)

Notes

(a) M1 for substituting $t = 0$ and $t = 10$ into s expression

A1 for noting that $s = 0$ at both times

A1 Since s is a perfect square, $s > 0$ for all other t- values.

(b) 1st M1 for differentiating s w.r.t. t to give v (powers of t reducing by 1)

1st A1 for a correct v expression in any form

2nd M1 for equating v to 0 and factorising

2nd A1 for correct t values

3rd M1 for substituting their intermediate t value into s

3rd A1 **ft** following an incorrect t-value.

Q2.

Question Number	Scheme	Marks	
a	At rest when $v=0$: $\left(2t^2-9t+4\right)=0$	M1	
	$=(2t-1)(t-4)$,	DM1	Solve for t. Dependent on the previous M1
	$t=\frac{1}{2},\ 4$	A1	Incorrect answers with no method shown score M0A0
		[3]	
b	$a=\frac{dv}{dt}=4t-9$	M1	Differentiate v to obtain a (at least one power of t going down)
		A1	Correct derivative
	$t=5,\ \ a=11\,(\text{m s}^{-2})$	A1	
		[3]	
c	$s=\int v\,dt=\frac{2}{3}t^3-\frac{9}{2}t^2+4t\,(+C)$	M1	Integrate v to obtain s (at least one power of t going up)
		A1	
	Use of $t=0$, $t=\frac{1}{2}$, $t=4$, $t=5$ (and $t=0, s=15$) as limits in integrals	DM1	Correct strategy for their limits - requires subtraction of the negative distance. Dependent on the previous M1 and at least one positive solution for t in $(0,5)$ from (a)
	$\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t(+15)\right]_0^{\frac{1}{2}}$ $-\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t(+15)\right]_{\frac{1}{2}}^4$ $+\left[\frac{2}{3}t^3-\frac{9}{2}t^2+4t(+15)\right]_4^5$	A1	NB: $\int_0^5 v\,dt$ scores M0A0A0
	$(0,\ \frac{23}{24},\ -\frac{40}{3},\ \frac{-55}{6})$ $=\frac{23}{24}+\frac{343}{24}+\frac{100}{24}=19.4\ (\text{m})$ $\left(15,\ 15\frac{23}{24}\left(\frac{383}{24}\right),\ \frac{5}{3},\ 5.8\dot{3}\left(\frac{35}{6}\right)\right)$	A1 [5]	$19\frac{5}{12}\ \left(\frac{233}{12}\right)$ or better
		(11)	

Q3.

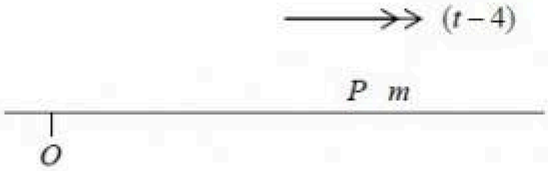
Question Number	Scheme	Marks	Notes
(a)	$\frac{1}{2}t^2-3t+4=0$	M1	Set $v=0$
	$t^2-6t+8=0$		
	$(t-2)(t-4)=0$	DM1	Solve for v
	$t=2\text{ s}\ \ or\ \ 4\text{ s}$	A1 A1	
		(4)	
(b)	$\int \frac{1}{2}t^2-3t+4\mathrm{d}t$	M1	Integration – majority of powers increasing
	$=\frac{1}{6}t^3-\frac{3}{2}t^2+4t(+C)$	A1	Correct (+C not required)
	$s=\int_0^2 \frac{1}{2}t^2-3t+4\ \mathrm{d}t-\int_2^4 \frac{1}{2}t^2-3t+4\ \mathrm{d}t$	DM1	Correct strategy for finging the required distance. Follow their "2". Subtraction/swap limits/modulus signs
	$=\left[\frac{1}{6}t^3-\frac{3}{2}t^2+4t\right]_0^2-\left[\frac{1}{6}t^3-\frac{3}{2}t^2+4t\right]_2^4$		
	$=\frac{8}{6}-6+8-(\frac{64}{6}-24+16-(\frac{8}{6}-6+8))$	A1	Correct unsimplified
	$=\frac{10}{3}-\frac{8}{3}+\frac{10}{3}$		
	$=4$	A1	
		(5)	
		[9]	

Q4.

Question Number	Scheme	Marks	Notes
a	$v=0=2t^2-14t+20$ $=2\ \ t-2\ \ \ t-5$ $t=2\ \text{ or }\ t=5$	M1 M1 A1 [3]	Set $v=0$ Solve for t
	There are many different approaches to part (b). The allocation of the two M marks is M1: A method to find the time when the velocity is a minimum M1: Evaluate the speed at that time		
e.g. b	$t=0\ ,\ v=20\ (\text{m s}^{-1})$ $a=4t-14$ ■ 0 $t=\frac{7}{2},\quad v=2\times\frac{3}{2}\times\frac{-3}{2}=\frac{-9}{2}$ Max speed = 20 ms^{-1}	B1 M1 M1A1 A1 [5]	Must see ±4.5 Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
balt1	$t=0,\ v=20\ (\text{m s}^{-1})$ Sketch with symmetry about their $t=3.5$ v(their 3.5) -4.5 Max speed = 20 ms^{-1}	B1 M1 M1 A1 A1 [5]	Evaluate v at min. Correct work Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
b alt 2	$t=0\ ,\ v=20\ (\text{m s}^{-1})$ Justification of minimum or tabulate sufficient values to confirm location Evaluate v at min. Correct work Correct conclusion. Depends on the two M marks	B1 M1 M1 A1 A1 [5]	Clearly stated & from correct solution only.

Question Number	Scheme	Marks	Notes
b alt 3	$t=0$, $v=20$ (m s^{-1})	B1	
	Complete the square as far as $\left(t-\frac{7}{2}\right)^2$	M1	
	$2\left(t-\frac{7}{2}\right)^2-\frac{9}{2}$	M1A1	
	Max speed = 20 ms^{-1}	A1 [5]	Clearly stated & correct conclusion. Depends on the two M marks. From correct solution only.
c	$\int 2t^2-14t+20\ \mathrm{d}t=\frac{2}{3}t^3-7t^2+20t(+C)$	M1	Integration. Need to see majority of powers going up
		A1	All correct. Condone C missing
	Distance = $\left[\frac{2}{3}t^3-7t^2+20t\right]_0^2-\left[\frac{2}{3}t^3-7t^2+20t\right]_2^4$	M1	Correct method to find the distance, for their 2
		A1	Correct unsimplified
	$=2\times\left[\frac{2}{3}t^3-7t^2+20t\right]^2-\left[\frac{2}{3}t^3-7t^2+20t\right]_4$		
	$=2\left[\frac{16}{3}-7\times4+40\right]-\left[\frac{2\times64}{3}-7\times16+80\right]=24$ (m)	A1 [5]	

Q5.

Question Number	Scheme	Marks
(a)		
	$\frac{dv}{dt}=t-4$	
	$v=\frac{1}{2}t^2-4t(+c)$	M1 A1
	$t=0 \quad v=6 \quad \Rightarrow c=6$	M1
	$\therefore v=\frac{1}{2}t^2-4t+6$	A1
		(4)
(b)	$v=0 \quad 0=t^2-8t+12$	M1
	$(t-6)(t-2)=0$	DM1
	$t=6 \quad t=2$	A1
		(3)
(c)	$x=\frac{t^3}{6}-2t^2+6t+k$	M1 A1 ft
	$x_6-x_2=\frac{6^3}{6}-2\times6^2+6^2+k$	DM1
	$-\left(\frac{2^3}{6}-2\times2^2+6\times2+k\right)$	
	$=-5\frac{1}{3}$	
	$\therefore$ Distance is $5\frac{1}{3}$ m	A1
		(4)
		11

Q6.

(a)	$a = 4t^3 - 12t$ Convincing attempt to integrate $v = t^4 - 6t^2\ (+c)$ Use initial condition to get $v = t^4 - 6t^2 + 8\left(\text{ms}^{-1}\right)$.		M1 A1 A1 (3)
(b)	Convincing attempt to integrate $s = \frac{t^5}{5} - 2t^3 + 8t\ (+0)$	Integral of their v	M1 A1ft (2)
(c)	Set their $v = 0$ Solve a quadratic in t^2 $(t^2 - 2)(t^2 - 4) = 0 \Rightarrow$ at rest when $t = \sqrt{2},\ t = 2$		M1 DM1 A1 (3) **[8]**

Q7.

Question Number	Scheme	Marks
(a)	$0 \le t \le 4$: $a = 8 - 3t$	M1
	$a = 0 \Rightarrow t = 8/3$ s	DM1
	$\rightarrow v = 8.\frac{8}{3} - \frac{3}{2}\left(\frac{8}{3}\right)^2 = \frac{32}{3}$ (m/s)	DM1 A1
	second M1 dependent on the first, and third dependent on the second.	(4)
(b)	$s = 4t^2 - t^3/2$	M1
	$t = 4$: $s = 64 - 64/2 = \underline{32 \text{ m}}$	M1 A1 (3)
(c)	$t > 4$: $v = 0 \Rightarrow t = \underline{8 \text{ s}}$	B1 (1)
(d)	*Either*	
	$t > 4$ $s = 16t - t^2$ $(+ C)$	M1
	$t = 4, s = 32 \rightarrow C = -16 \Rightarrow s = 16t - t^2 - 16$	M1 A1
	$t = 10 \rightarrow s = 44$ m	M1 A1
	But direction changed, so: $t = 8, s = 48$	M1
	Hence total dist travelled $= 48 + 4 = \underline{52 \text{ m}}$	DM1 A1 (8)
	Or (probably accompanied by a sketch?)	
	t=4 v=8, t=8 v=0, so area under line = $\frac{1}{2} \times (8-4) \times 8$	M1A1A1
	t=8 v=0, t=10 v=-4, so area above line = $\frac{1}{2} \times (10-8) \times 4$	M1A1A1
	$\therefore$ total distance = 32(from b) + 16 + 4 = $\underline{52 \text{ m.}}$	M1A1 (8)

Or M1, A1 for t > 4 $\frac{dv}{dt} = -2$, =constant

t=4, v=8; t=8, v=0; t=10, v=-4

M1, A1 $s = \frac{u+v}{2}t = \frac{32}{2}t$, =16 working for t = 4 to t = 8

M1, A1 $s = \frac{u+v}{2}t = \frac{-4}{2}t$, =-4 working for t = 8 to t = 10

M1, A1 total = 32+14+4, =52

M1 Differentiate to obtain acceleration
DM1 set acceleration. = 0 and solve for t
DM1 use their t to find the value of v
A1 32/3, 10.7oro better

OR using trial an improvement:
M1 Iterative method that goes beyond integer values
M1 Establish maximum occurs for t in an interval no bigger than 2.5<t<3.5
M1 Establish maximum occurs for t in an interval no bigger than 2.6<t<2.8
A1

Or M1 Find/state the coordinates of both points where the curve cuts the x axis.
DM1 Find the midpoint of these two values.
M1A1 as above.

Or M1 Convincing attempt to complete the square:

DM1 substantially correct $8t - \frac{3t^2}{2} = -\frac{3}{2}(t - \frac{8}{3})^2 + \frac{3}{2} \times \frac{64}{9}$

DM1 Max value = constant term
A1 CSO

M1 Integrate the correct expression
DM1 Substitute t = 4 to find distance (s=0 when t=0 - condone omission / ignoring of constant of integration)
A1 32(m) only

B1 t = 8 (s) only

M1 Integrate 16-2t
M1 Use t=4, s= their value from (b) to find the value of the constant of integration.
or 32 + integral with a lower limit of 4 (in which case you probably see these two marks
occurring with the next two. First A1 will be for 4 correctly substituted.)
A1 $s = 16t - t^2 - 16$ or equivalent
M1 substitute t = 10
A1 44
M1 Substitute t = 8 (their value from (c))
DM1 Calculate total distance (M mark dependent on the previous M mark.)
A1 52 (m)

	OR the candidate who recognizes v = 16 – 2t as a straight line can divide the shape into two triangles: M1 distance for t = 4 to t = candidates's 8 = ½ x change in time x change in speed. A1 8-4 A1 8-0 M1 distance for t = their 8 to t = 10 =½ x change in time x change in speed. A1 10-8 A1 0-(-4) M1 Total distance = their (b) plus the two triangles (=32 + 16 + 4). A1 52(m) NB: This order on epen grid (the A's and M's will not match up.)	

Q8.

(a)	$v = 10t - 2t^2,\ s = \int v dt$	M1	
	$= 5t^2 - \frac{2t^3}{3}(+C)$	A1	
	$t = 6 \Rightarrow\ s = 180 - 144 = \underline{36}$ (m)	A1	
			(3)
(b)	$\underline{s} = \int v\,dt = \frac{-432t^{-1}}{-1}(+K) = \frac{432}{t}(+K)$	$\underline{\text{B1}}$	
	t = 6, s = "36" $\Rightarrow 36 = \frac{432}{6} + K$	M1*	
	$\Rightarrow K = -36$	A1	
	At $t = 10,\ s = \frac{432}{10} - 36 = \underline{7.2}$ (m)	d*M1	
		$\underline{\text{A1}}$	
			(5) **[8]**

Q9.

Question Number	Scheme	Marks
	$\frac{dv}{dt} = 6t - 4$	M1 A1
	$6t - 4 = 0 \Rightarrow t = \frac{2}{3}$	M1 A1
	$s = \int 3t^2 - 4t + 3\ dt = t^3 - 2t^2 + 3t\ (+c)$	M1 A1
	$t = \frac{2}{3} \Rightarrow s = -\frac{16}{27} + 2$ so distance is $\frac{38}{27}$ m	M1 A1
		[8]

Q10.

Question Number	Scheme	Marks
(a)	$\frac{dv}{dt} = 8 - 2t$	M1
	$8 - 2t = 0$	M1
	Max $v = 8 \times 4 - 4^2 = 16\,(\text{ms}^{-1})$	M1A1
		(4)
(b)	$\int 8t - t^2\,dt = 4t^2 - \frac{1}{3}t^3\,(+c)$	M1A1
	(t=0, displacement = 0 $\Rightarrow$ c=0)	
	$4T^2 - \frac{1}{3}T^3 = 0$	DM1
	$T^2(4 - \frac{T}{3}) = 0 \Rightarrow T = 0, 12$	DM1
	$T = 12$ (seconds)	A1
		(5)
		[9]

Q11.

Question	Scheme	Marks	AOs	Notes
(a)	$v = 12 + 4t - t^2 = 0$ and solving	M1	3.1a	Equating v to 0 and solving the quadratic If no evidence of solving, and at least one answer wrong, M0
	$t = 6$ (or -2)	A1	1.1b	6 but allow -2 as well at this stage
	Differentiate v wrt t	M1	1.1a	For differentiation (both powers decreasing by 1)
	$(a = \frac{\mathrm{d}v}{\mathrm{d}t} =)\ 4 - 2t$	A1	1.1b	Cao; only need RHS
	When $t = 6$, $a = -8$; Magnitude is 8 $(\mathrm{m\,s^{-2}})$	A1	1.1b	Substitute in $t = 6$ and get 8 $(\mathrm{m\,s^{-2}})$ as the answer . Must be **positive.** (A0 if two answers given)
		(5)		
(b)	Integrate v wrt t $(s =) 12t + 2t^2 - \frac{1}{3}t^3 (+C)$	M1 A1	3.1a 1.1b	For integration (at least two powers increasing by 1) Correct expression (ignore C) only need RHS Must be used in part (b)
	$t = 3$ => distance $= 45$ (m)	A1	1.1b	Correct distance. Ignore units
		(3)		
			(8 marks)	

Q12.

Question	Scheme	Marks	AOs
(a)	Differentiate v w.r.t. t	M1	3.1a
	$a = \frac{dv}{dt} = 10 - 2t$ isw	A1	1.1b
		(2)	
(b)	Solve problem using $v = 0$ when $t = 6$	M1	3.1a
	$0 = 10t - t^2 - 24$	A1	1.1b
	Solve quadratic oe to find other value of t	M1	1.1b
	$t = 4$	A1	1.1b
		(4)	
(c)	Integrate v or $-v$ w.r.t. t	M1	3.1a
	$5t^2 - \frac{1}{3}t^3 - 24t$	A1	1.1b
	Total distance = $-\left[5t^2 - \frac{1}{3}t^3 - 24t\right]_0^4 + \left[5t^2 - \frac{1}{3}t^3 - 24t\right]_4^6$	M1	2.1
	$\frac{116}{3}$ (m)	A1	1.1b
		(4)	
		(10 marks)	

Notes:		
a	M1	Differentiate, with both powers decreasing by 1
	A1	Correct expression
b	M1	Put $t = 6$ **OR** use $(t-6)(t-x) = t^2 - 10t + k$ oe
	A1	Correct expression (unsimplified) for v **OR** $v = (t-6)(t-4)$
	M1	Put $v = 0$ to give quadratic in t and solve for other value of t
	A1	$t = 4$
c	M1	Integrate, with at least two powers increasing by 1 (allow if only two terms integrated)
	A1	Correct expression
	M1	Complete method to find the total distance
	A1	Accept 39(m) or better

Q13.

Q.	Scheme	Marks	Notes
a	$v=0 \Rightarrow 3t^2-16t+21=0$	M1	Set $v=0$ and attempt to solve
	$((3t-7)(t-3)=0) \quad t_1=\frac{7}{3}, \quad t_2=3$	A1	
		(2)	
b	$a=\frac{\mathrm{d}}{\mathrm{d}t}(3t^2-16t+21)$	M1	Differentiate v to obtain a
	$=6t-16$	A1	
	$t=t_1, \quad a=6\times\frac{7}{3}-16=-2$ (m s^{-2}) Magnitude 2 (m s^{-2})	A1	No errors seen. Must be positive - the Q asks for magnitude.
		(3)	
c	$s=\int(3t^2-16t+21)\mathrm{d}t$	M1	Integrate v to find s
	$=t^3-8t^2+21t(+C)$	A1	
	$\pm\left((3^3-8\times9+21\times3)-\left(\left(\frac{7}{3}\right)^3-8\times\frac{49}{9}+21\times\frac{7}{3}\right)\right)$	M1	Correct use of their limits
	$s=0.148$ (m) $\quad \left(\frac{4}{27}\right)$	A1	Final answer must be positive. 0.15 or better
		(4)	
d	Return to $O \Rightarrow s=0=t(t^2-8t+21)$	B1	seen or implied
	Discriminant of quadratic $=64-4\times21(=-20)<0$	M1	Or equivalent. ***given answer so must show some evidence of method***
	No real roots $\Rightarrow$ does not return to O	A1	Sufficient correct working to justify ***given answer***
		(3)	
dalt	Travels away until $t_1=\frac{7}{3}$, turns back at $t_2=3$ then turns away again	M1	Complete story
	$s_3=18$	B1	Seen or implied
	Complete argument	A1	
		(3)	
dalt	Distance time graph	B1	
	Locate min turning point	M1	
	Complete argument	A1	
		(3)	
		[12]	

Q14.

Q	Scheme	Marks	Notes
a	$t=0,\ v=11\ \Rightarrow r=11$	B1	
	$t=2,\ v=3\ \Rightarrow 4p+2q+11=3,$	M1	Accept $4p+2q+r=3$
	$4p+2q=-8$	A1	Any equivalent unsimplified form with 11 used
	Differentiate to find acceleration	M1	OR use symmetry, $t=4, v=11$
	$a=2pt+q$	A1	$\Rightarrow 11=16p+4q+11$, $4p+q=0$
	$t=2,\ a=0\ \Rightarrow 4p+q=0$	DM1	2nd eqn in p & q and solve for p & q Dependent on both previous m marks
	$\Rightarrow -q+2q=-8,\ q=-8,\ p=2$	A1	
	$\left(v=2t^2-8t+11\right)$		
	$t=3,\ a=4t-8=4\,(\text{ms}^{-2})$	A1	
		(8)	
a alt	Min speed at $t=2\ \Rightarrow$ $v=\left(pt^2+qt+r\right)=k\left(t-2\right)^2+c$	B1	
		M1	Completed square form.
	$v=k\left(t-2\right)^2+3$	A1	Correct completed square form
	$t=0,\ v=11\ \Rightarrow 4k+3=11,$	M1	Solve for k
	$k=2$	A1	$v=2\left(t-2\right)^2+3\left(=2t^2-8t+11\right)$
	Differentiate to find acceleration	DM1	Dependent on both previous m marks
	$a=4\left(t-2\right)$	A1	
	$t=3,\ a=4\,(\text{m s}^{-2})$	A1	
		(8)	
b	Integrate: $\int 2\left(t-2\right)^2+3\,dt=\frac{2}{3}\left(t-2\right)^3+3t\left(+C\right)$ or $\int 2t^2-8t+11\,dt=\frac{2}{3}t^3-4t^2+11t\left(+C\right)$	M1	follow their coefficients found in (a) Accept in p, q, r
	At most one error seen	A1ft	For their coefficients
	All correct	A1ft	For their coefficients provided $\neq 0$
	$\left[\frac{2}{3}\left(t-2\right)^3+3t\right]_2^3=\left(\frac{2}{3}+9\right)-\left(0+6\right)$ or $\left[\frac{2}{3}t^3-4t^2+11t\right]_2^3$ $=\left(18-36+33\right)-\left(\frac{16}{3}-16+22\right)$	DM1	Use of $t=2, t=3$ as limits on a definite integral (or subtract distances to cancel C). Dependent on having integrated. Allow with p, q, r

Q	Scheme	Marks	Notes
	$3\frac{2}{3}$ (m)	A1	Accept exact equivalent or 3.7 or better
		(5)	
		[13]	

Q15.

Question	Scheme	Marks	AOs
(a)	Multiply out and differentiate *wrt* to time (or use of product rule i.e. must have two terms with correct structure)	M1	1.1a
	$v = 2t^3 - 3t^2 + t$	A1	1.1b
	$2t^3 - 3t^2 + t = 0$ and solve: $t(2t-1)(t-1) = 0$	DM1	1.1b
	$t = 0$ or $t = \frac{1}{2}$ or $t = 1$; any two	A1	1.1b
	All three	A1	1.1b
		(5)	
(b)	Find x when $t = 0, \frac{1}{2}$, 1 and 2 : $(0, \frac{1}{32}, 0, 2)$	M1	2.1
	Distance $= \frac{1}{32} + \frac{1}{32} + 2$	M1	2.1
	$2\frac{1}{16}$ (m) oe or 2.06 or better	A1	1.1b
		(3)	
(c)	$x = \frac{1}{2}t^2(t-1)^2$	M1	3.1a
	$\frac{1}{2}$ perfect square so $x \geq 0$ i.e. never negative	A1 cso	2.4
		(2)	
			(10 marks)

Notes:

(a)

M1: Must have 3 terms and at least two powers going down by 1

A1: A correct expression

DM1: Dependent on first M, for equating to zero and attempting to solve a <u>cubic</u>

A1: Any two of the three values (Two correct answers can imply a correct method)

A1: The third value

(b)

M1: For attempting to find the values of x (at least two) at their t values found in (a) or at $t = 2$ or equivalent e.g. they may integrate their v and sub in at least two of their t values

M1: Using a correct strategy to combine their distances (must have at least 3 distances)

A1: $2\frac{1}{16}$ (m) oe or 2.06 or better

(c)

M1: Identify strategy to solve the problem such as:

- (i) writing x as $\frac{1}{2} \times$ perfect square
- (ii) or using x values identified in (b).
- (iii) or using calculus i.e. identifying min points on x–t graph.
- (iv) or using x-t graph.

A1 cso : Fully correct explanation to show that $x \geq 0$ i.e. never negative