# Exam 3 Review

Sec 5.3, 5.4, 5.6, 5.7, 7.1, 7.2, 7.3

The problems on this review sheet will help you practice some of the algorithmic and computational skills required for this exam. However, these technical skills are not a substitute for conceptual understanding! On the exam you may be asked to write and explain your thinking (not just solve problems).

**What do you need to know? Some sample questions and important concepts:**

1. Give the general form for the solution of a second order linear differential equation, and explain what each of the parts represents.
2. In solving a nonhomogeneous equation, one step is to make a guess about what a solution might look like. In the case of problem 2 below, based on the right hand side $-578\sin 5t$ we would guess a solution of the form $y(t) = A\sin 5t + B\cos 5t$. Why does our guess involve both sine and cosine (why not just sine)?
3. Consider the method of reduction of order. What type(s) of equation does it solve? What type(s) of solution does it produce? What information do you need to have in order to use it?
4. Consider the method of variation of parameters. What type(s) of equation does it solve? What type(s) of solution does it produce? What information do you need to have in order to use it?
5. Plugging a value of x into a power series results in the sum of infinitely many numbers, each given by a term in the power series. Is it possible to add up infinitely many numbers, and get a finite number for an answer? Explain.
6. Given a function f(x), the theory of MacLaurin and Taylor series shows us how to create polynomials that approximate f(x) near a point. We can make that approximation better by adding more terms, so nearby values of our polynomial get closer and closer to the actual value of f(x) (more decimal digits correct). Is it ever possible to obtain the exact value of f(x) using this strategy?

**Review Problems**

1. Find the general solution: $y'' - 2y' - 3y = 3e^{2t}$
2. Find the general solution: $y'' + 6y' + 9y = -578\sin 5t$
3. Use the method of reduction of order to find the general solution to $x^2y'' - xy' + y = x$ given that $y_1 = x$ is a solution to the complementary equation.
4. Use the method of reduction of order to find the general solution to $xy'' - (2x + 2)y' + (x + 2)y = 0$ given that $y_1 = e^x$ is a solution.
5. Given the differential equation $y'' - xy' - y = 0$:
   a. Suppose that $y(x)$ has a Taylor series about $x = 0$,
   $$y(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 + ...$$
   Substitute into the differential equation and simplify by grouping together terms with

similar powers of $x$.

b. Given the initial conditions $y(0) = 16,\ y'(0) = 15$, find the first five terms of the Taylor series solution $y(x)$.

c. Use the answer to part b to find an approximation of $y(2)$

6. Given the differential equation $y'' + x^2 y = 0$ with initial conditions $y(0) = 1,\ y'(0) = 0$, use the first five terms of the Taylor series about $x = 0$ to find an approximate value of the solution at $x = 1.2$.
7. Suppose y is the solution to a given initial value problem and y is given to you in the form of a MacLaurin series, $y(x) = 11 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{7}{12}x^3 + \frac{51}{24}x^4 + \ldots$.
   a. Find the values of $y(0),\ y'(0),\ y''(0),\ y'''(0),\ y^{(4)}(0)$ (Note that the notation $y^{(4)}(x)$ indicates the fourth derivative of y).
   b. The next term in the MacLaurin series would be $a_5 x^5$. Find the value of the coefficient $a_5$, given that $y^{(5)}(0) = \frac{3}{7}$ (that is, the fifth derivative of y evaluated at x=0 is $\frac{3}{7}$).
8. Find a particular solution of $x^2 y'' + xy' - y = 2x^2 + 2$ given that $y_1 = x$ and $y_2 = \frac{1}{x}$ are solutions of the complementary equation.
9. Find a particular solution of $xy'' + (2 - 2x)y' + (x - 2)y = e^{2x}$ given that $y_1 = e^x$ and $y_2 = \frac{e^x}{x}$ are solutions of the complementary equation.
10. Find the general solution: $y'' - 2y' + y = 14x^{3/2}e^x$

# Exam 3 Review ANSWER KEY

*If you discover an error please let me know, either in class, on the OpenLab, or by email to [jreitz@citytech.cuny.edu](mailto:jreitz@citytech.cuny.edu). Corrections will be posted on the OpenLab.*

Answers to Review Problems:

1. $y(t) = c_1 e^{-t} + c_2 e^{3t} - e^{2t}$
2. $y(t) = c_1 e^{-3t} + c_2 t e^{-3t} + 8 \sin 5t + 15 \cos 5t$
3. $y = c_1 x + c_2 x \ln(x) + \frac{1}{2} x[\ln(x)]^2$
4. $y = c_1 e^x + c_2 e^x x^3$
5. a.
   $(2a_2 - a_0) + (6a_3 - 2a_1)x + (12a_4 - 3a_2)x^2 + (20a_5 - 4a_3)x^3 + (30a_6 - 5a_4)x^4 + ... = 0$
   b. Taylor series at $x = 0$ given initial conditions $y(0) = 16,\ y'(0) = 15$:
   $y(x) \approx 16 + 15x + 8x^2 + 5x^3 + 2x^4 + ...$
   c. $y(2) \approx 150$
6. Taylor series at $x = 0$: $y(x) \approx 1 - \frac{1}{12} x^4$. Approximate value at $x = 1.2$ is
   $y(1.2) \approx 0.8272$
7. a. $y(0) = 11,\ y'(0) = \frac{1}{2},\ y''(0) = \frac{3}{4},\ y'''(0) = \frac{7}{2},\ y^{(4)}(0) = 51$
   b. $a_5 = \frac{1}{280}$
8. $y_p = \frac{2}{3}(x^2 - 3)$
9. $y_p = \frac{e^{2x}}{x}$
10. *HINT: First solve the complementary equation. Then use variation of parameters.*
    $y = \frac{8}{5} x^{7/2} e^x + c_1 e^x + c_2 x e^x$