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B.Sc. (Hons. Math) (Semester – 3rd)

DIFFERENTIAL EQUATIONS - I

Subject Code: BMATS1-301

Paper ID: [19131211]

Time: 03 Hours

Maximum Marks: 60

Instruction for candidates:

1. Section A is compulsory. It consists of 10 parts of two marks each.
2. Section B consist of 5 questions of 5 marks each. The student has to attempt any 4 questions out of it.
3. Section C consist of 3 questions of 10 marks each. The student has to attempt any 2 questions.

Section – A

(2 marks each)

Q1. Attempt the following:

- a. Define Clairaut's differential equation with the help of an example.
- b. Solve the differential equation: $xy y' = 1 + x + y + xy$, where $y' = \frac{dy}{dx}$.
- c. Form the differential equation of $y = ae^x + be^{2x} + ce^{-3x}$, where a, b, c are constants.
- d. Find the integrating factor of $x^2 y dx - (x^3 + y^3) dy = 0$.
- e. Solve the differential equation: $\frac{d^3 y}{dx^3} - 5 \frac{d^2 y}{dx^2} + 7 \frac{dy}{dx} - 3y = 0$.
- f. Find the Particular integral of $4 \frac{d^2 y}{dx^2} - 4 \frac{dy}{dx} + y = e^{\frac{x}{2}}$.
- g. Define Euler's differential equation. Give an example.
- h. Prove that: $J_{-n}(x) = (-1)^n J_n(x)$.
- i. Prove that: $P'_n(1) = \frac{n(n+1)}{2}$.
- j. State Legendre's differential equation.

Section – B

(5 marks each)

Q2. Solve the differential equation: $y' + 4xy + yx^3 = 0$, where $y' = \frac{dy}{dx}$.

Q3. For what value of k, the differential equation: $\left(1 + e^{\frac{kx}{y}}\right) dx + e^{\frac{x}{y}} \left(1 - \frac{x}{y}\right) dy = 0$ is exact.

Q4. Solve $\tan y \frac{dy}{dx} + \tan x = \cos y \cos^2 x$

Q5. Solve the differential equation: $\frac{d^2 y}{dx^2} + 7 \frac{dy}{dx} + 12y = x \sin x$

Q6. State and Prove Rodrigue's formula

Section – C

(10 marks each)

Q7. Solve:

$$(x \sin x + \cos x) \frac{d^2 y}{dx^2} - x \cos x \frac{dy}{dx} + y \cos x = \sin x (x \sin x + \cos x)^2$$

Q8. Apply the method of variation of parameter to solve:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} - y = x^2 e^x.$$

Q9. Obtain the series solution of Bessel's differential equation of order two:

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - 4)y = 0 \text{ near } x = 0.$$