Unit 04: Curve Sketching

Unit Objectives

- Create detailed sketches of graphs
- Determine key features of f, f’, and f’’ given one of the three curves

Unit 04 Lesson 01: Given f, Sketch f’

Lesson Objectives

- Ask Mr. Rose for an eraser

If the graph of f(x) is given on the left, sketch the graph of f’(x) on the right.

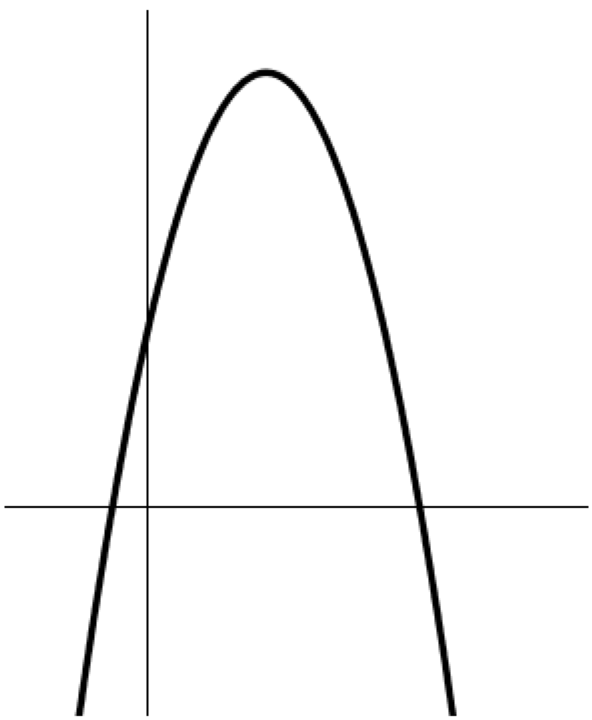
What must be perfect

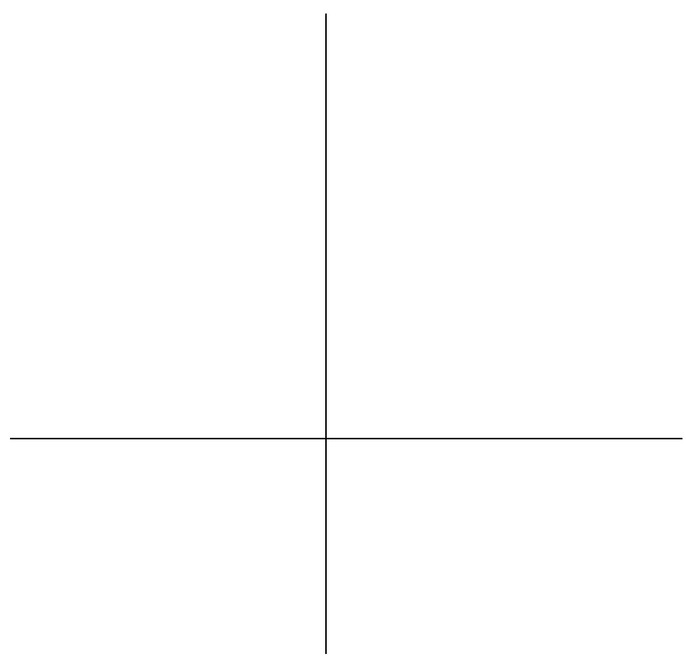
- Label important x values on both x-axes
- Make sure y’ is positive, negative, and zero when it should be

What must be reasonable

- Make sure relative y’-values are reasonable

1.





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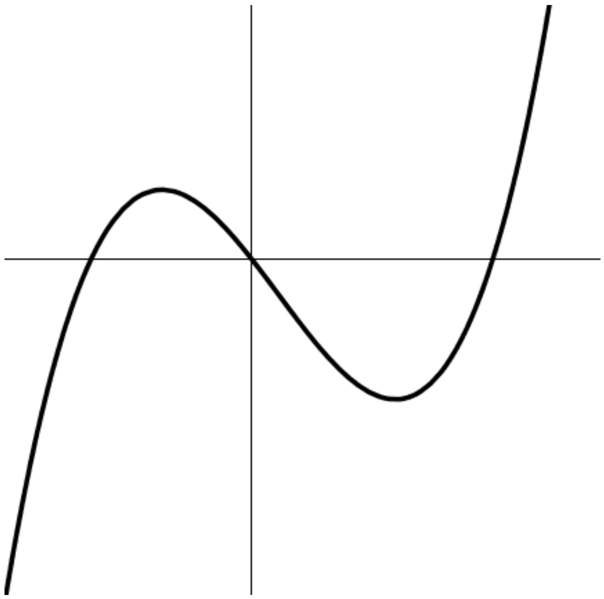

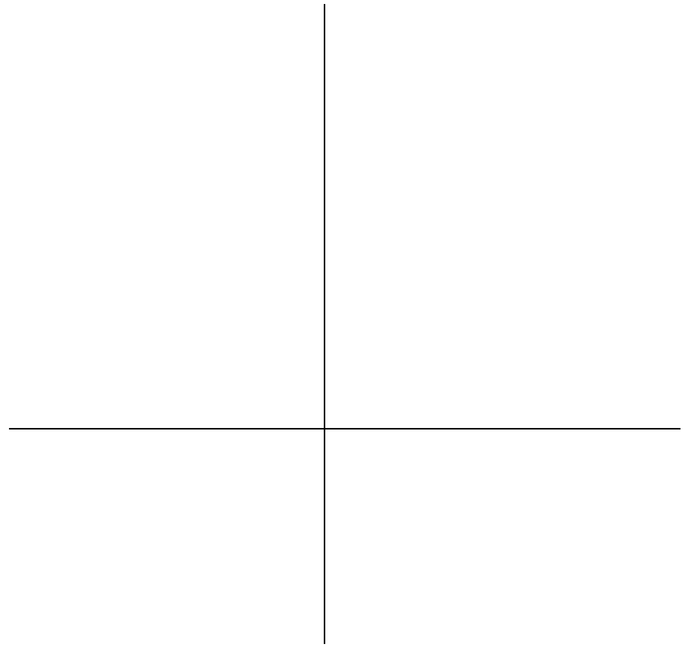

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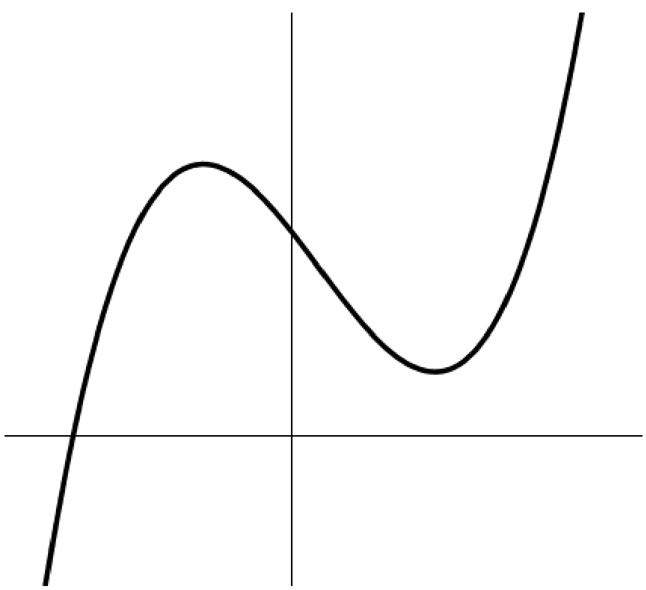

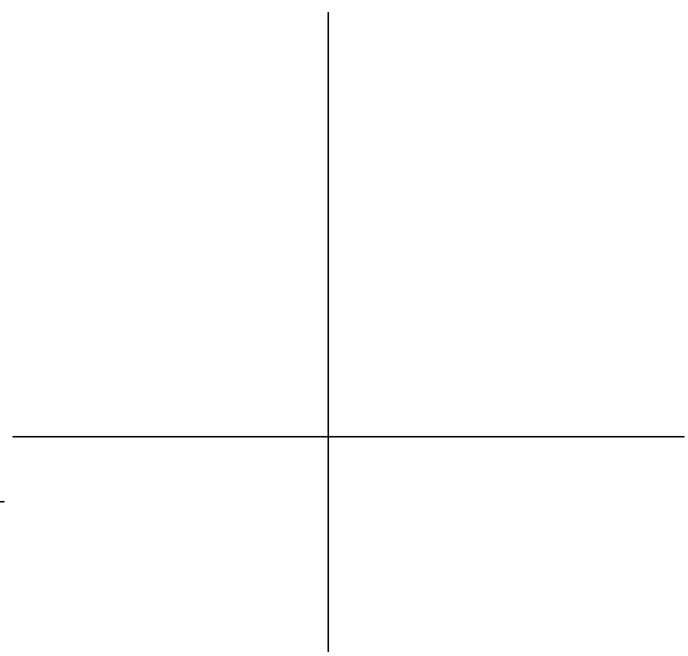

~~~U04L01 Homework~~~

1. [Khan] Applying derivatives to analyze functions: Justification using first derivative
2. [Khan] Contextual applications of differentiation: Motion problems (differential calc)

~~~U04L01 Classwork~~~

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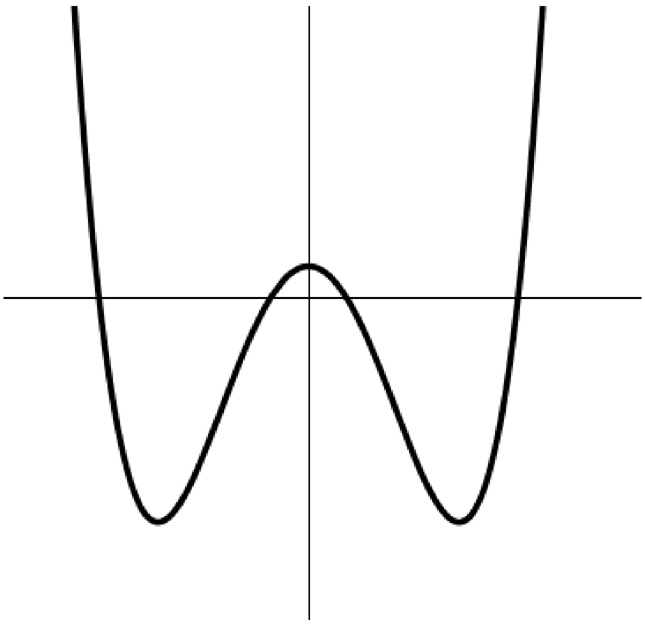

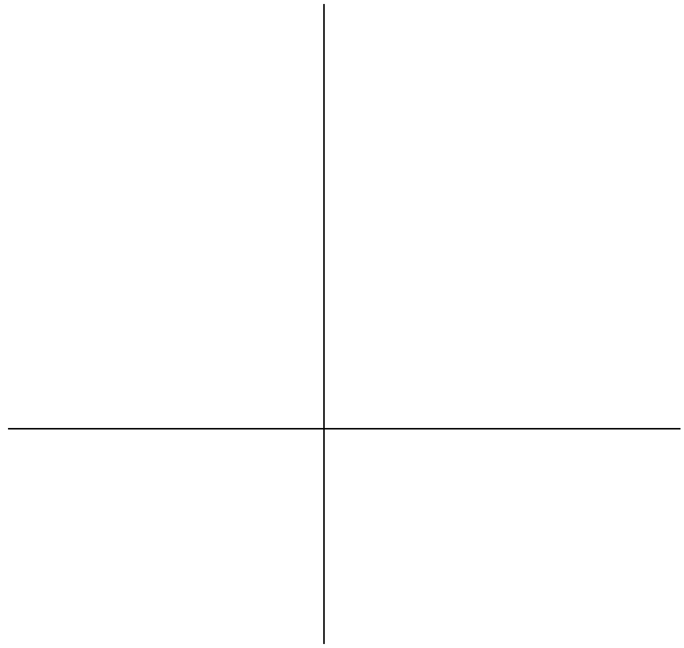

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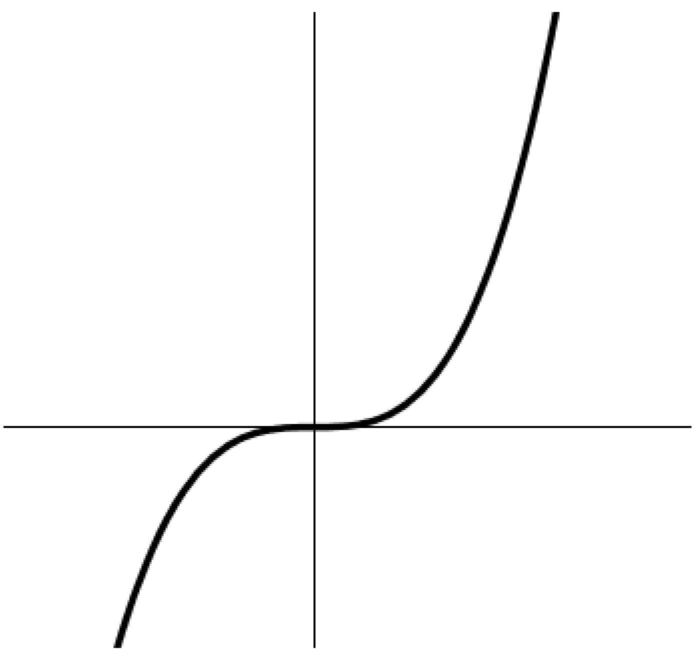

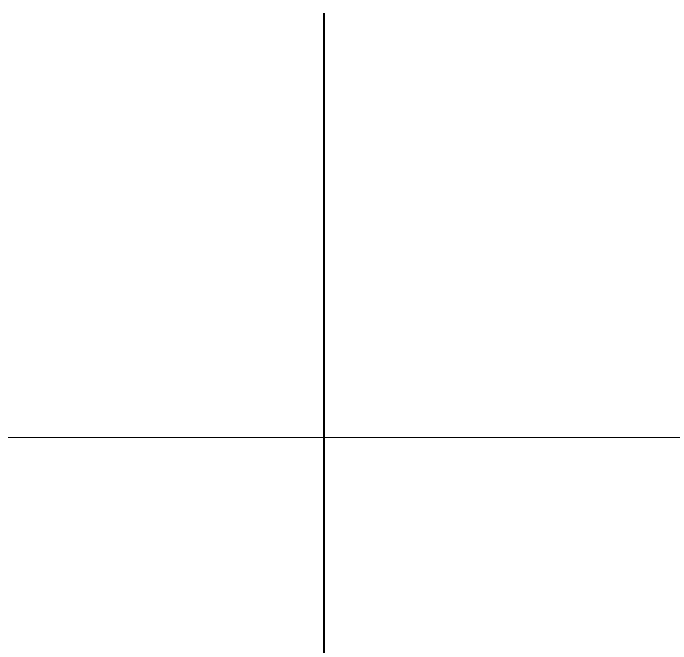

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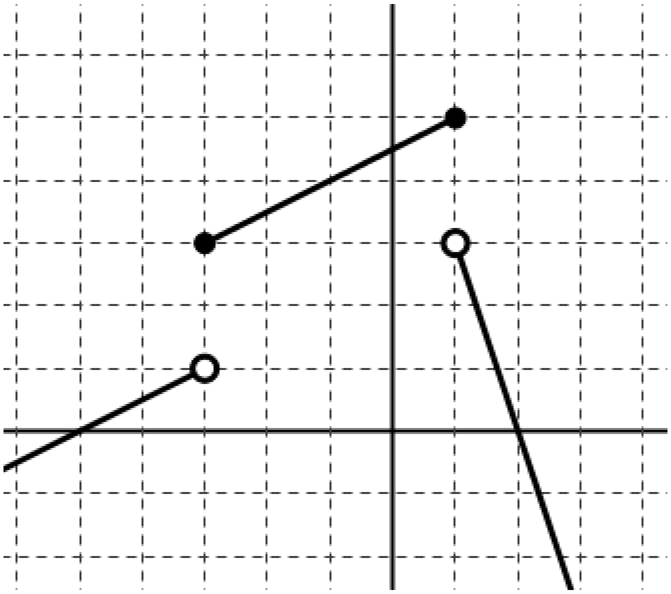

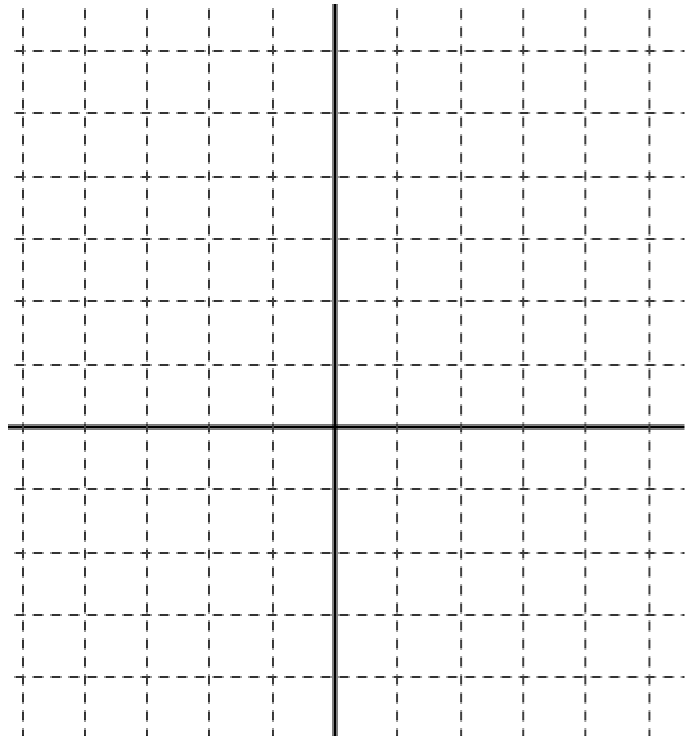

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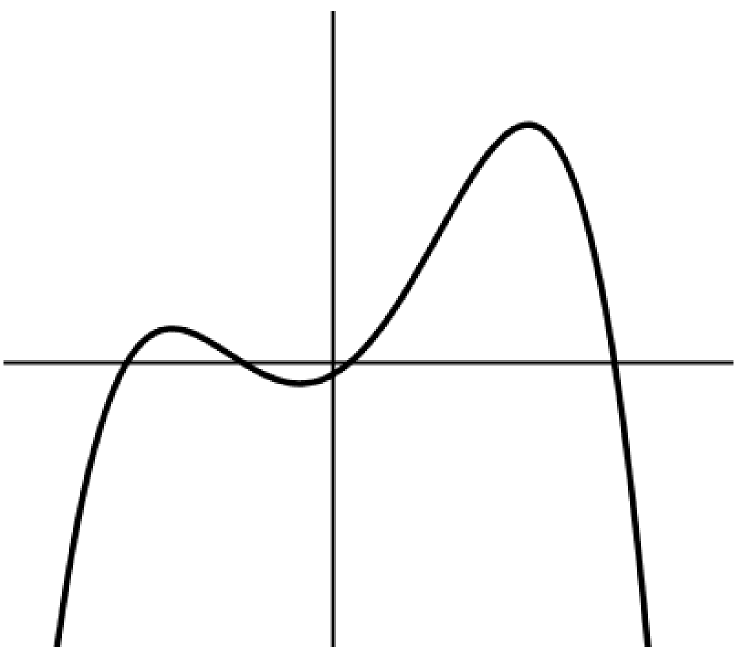

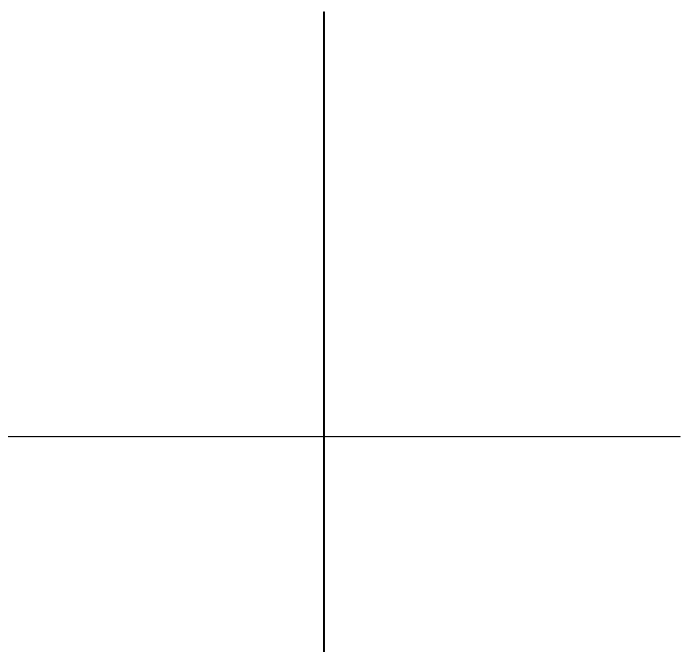

Unit 04 Lesson 02: Sketch Parabolas Using Critical Points

Lesson Objectives

- Use the first derivative to locate a relative maximum or relative minimum

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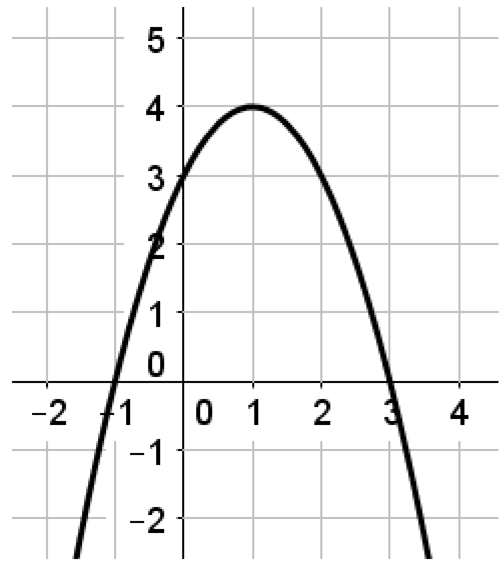
- Zeros (roots) are important, but so are ____________________
- _______________ occur when the _______________ of the

  ________________ ______________ equals _____________
- ______________ give the value of the ______________ of the

  _______________ _______________.
- Therefore, to find the _________________, find the points

  where the _____________________ equals ______________.



- Critical Point: ____
  - Therefore a critical point could be a _________________________, _____________________ or

    ____

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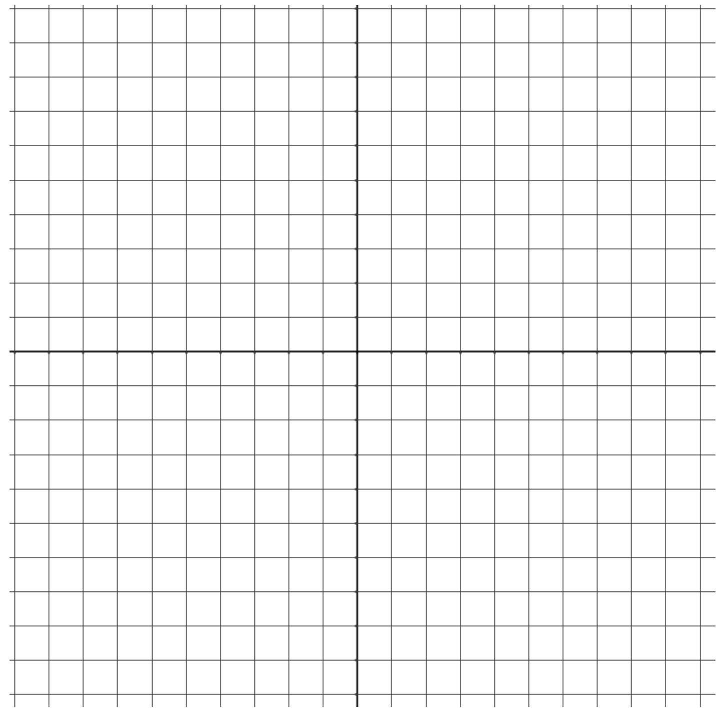
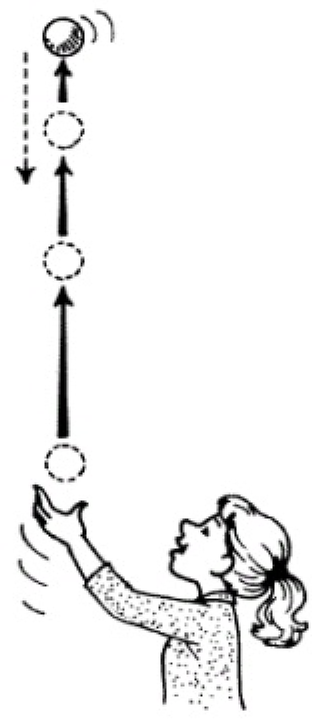
Example: The graph above shows the equation $y = -x^2 + 2x + 3$. Show why calculus would predict this relative extremum.

1. The height, measured in feet, of a ball thrown by a girl is given by the function $f(t) = -16.1t^2 + 20.46t + 4.5$, where t represents the time, measured in seconds, since the ball left the girl's hand.

   a. Find the zeros and vertex of f(t), and use them to sketch the graph. (Quadratic equation looks good here)



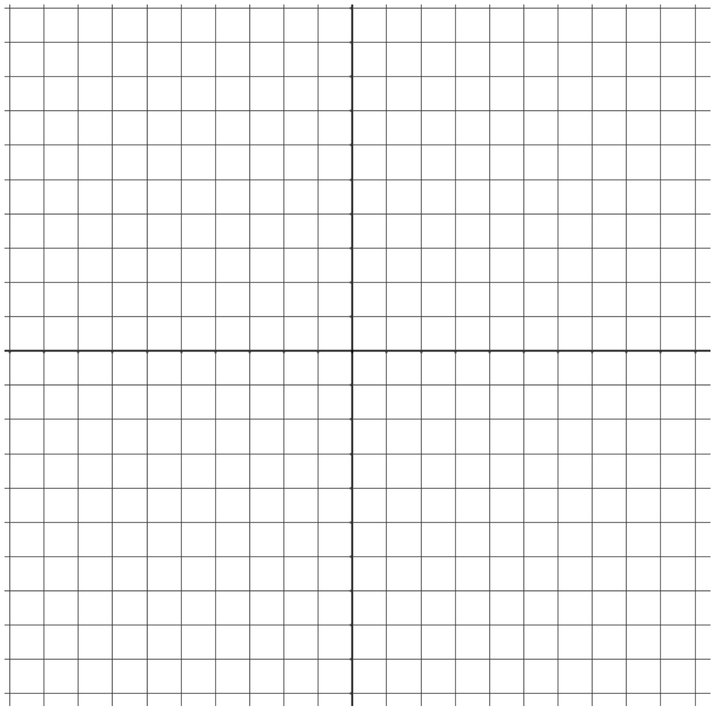


   b. What is the maximum height of the ball?

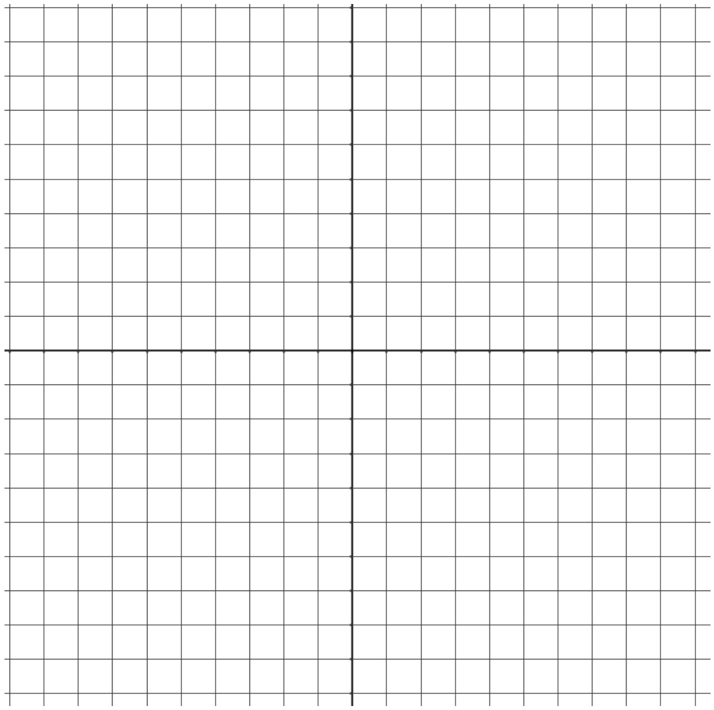
   c. What is the height of the ball when it leaves the girl's hand?

   d. What is the velocity of the ball at the time of the critical point?

2. For the equation $f(x) = \frac{x^2}{3} + \frac{8}{3}x + \frac{7}{3}$, determine the roots and the relative extremum, and use them to sketch the graph.



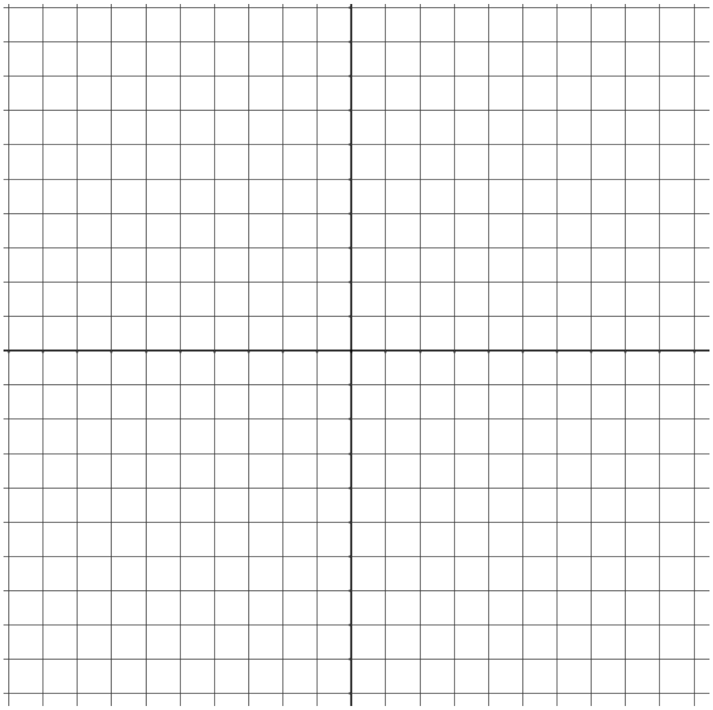
3. For the equation $y = 2x^2 - 8x$, determine the roots and the relative extremum, and use them to sketch the graph.



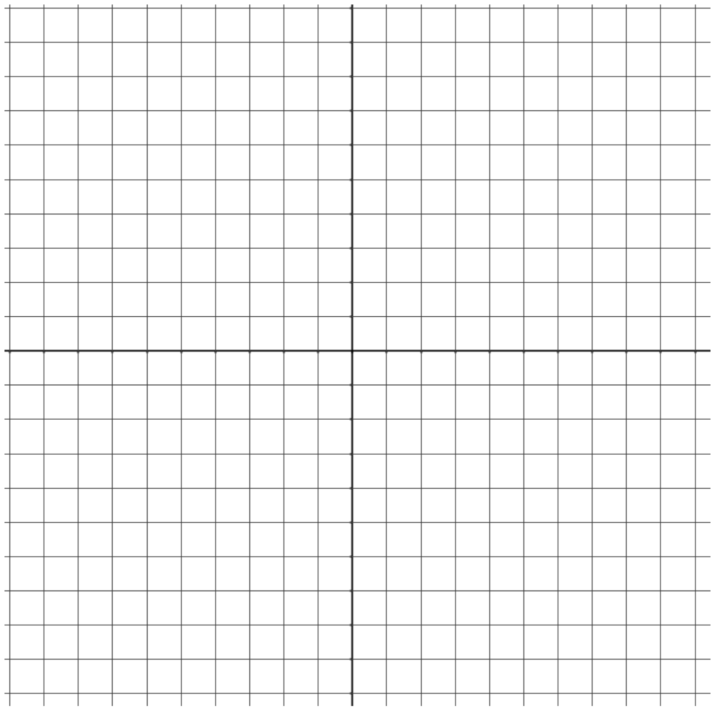
~~~U04L02 Homework~~~ 1. [Khan] Applying derivatives to analyze functions: Find critical points

~~~U04L02 Classwork~~~

1. For the equation $y = -\frac{1}{2}x^2 + 4x - 6$, determine the roots and the relative extremum, and use them to sketch the graph.



2. For the equation $f(x) = 4x^2 - 4$, determine the roots and the relative extremum, and use them to sketch the graph.

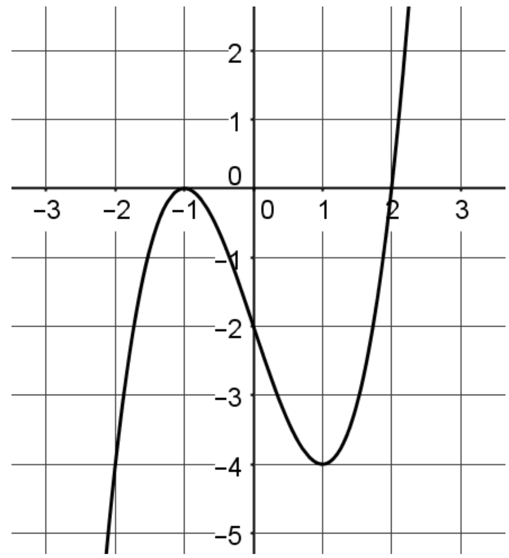


Unit 04 Lesson 03: Sketch Curves Using Critical Points

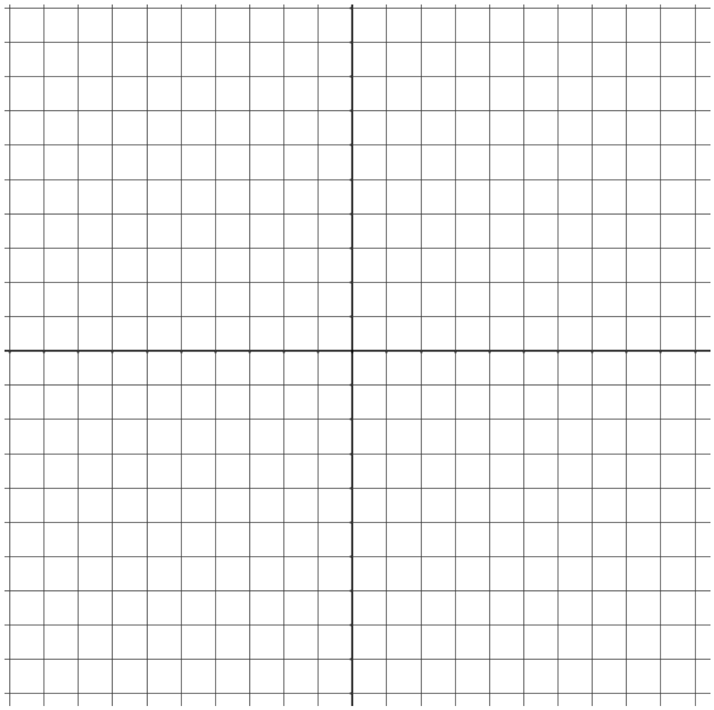
Lesson Objectives

- Use the first derivative to locate local maxima and minima

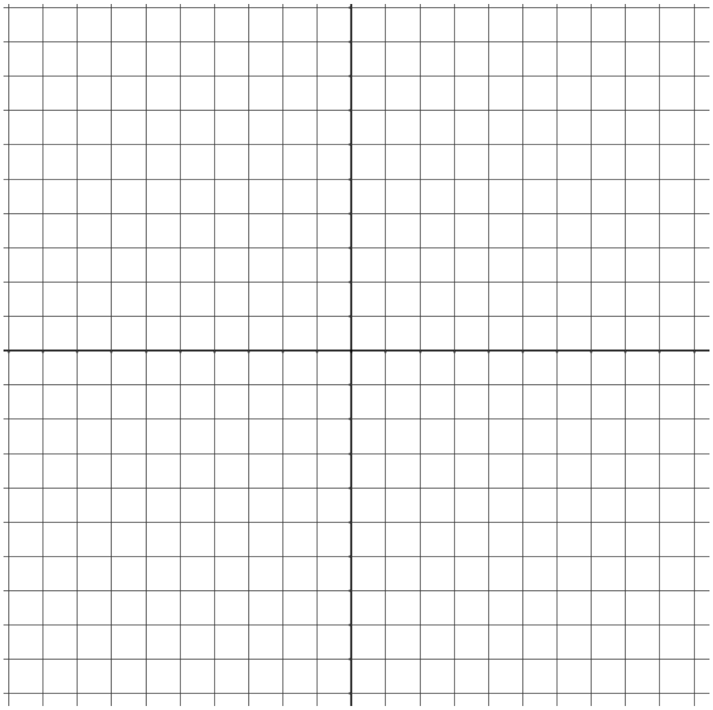
Example: The graph to the right shows the equation $y = x^3 - 3x - 2$. Show why calculus would predict this.



1. For the equation $f(x) = -2x^3 + 6x^2$, determine the roots and the relative extrema, and use them to sketch the graph.



2. For the equation $y = \dfrac{-12x + 24}{x^2 + 12}$, determine the roots and the relative extrema, and use them to sketch the graph.

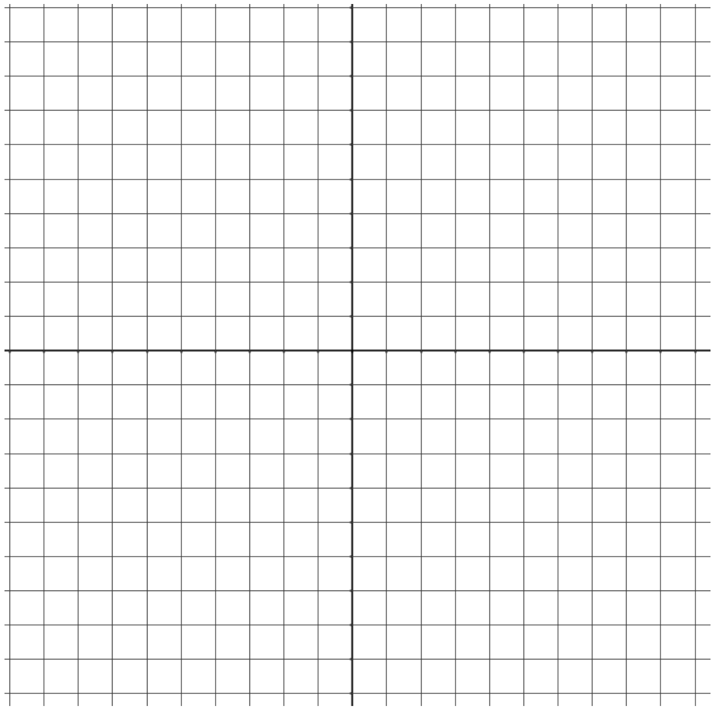


~~~U04L03 Homework~~~

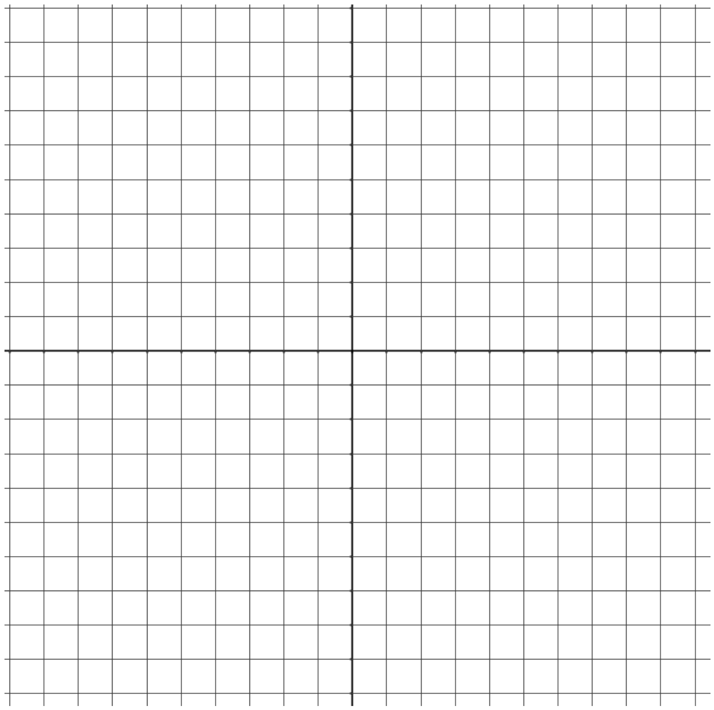
1. [Khan] Applying derivatives to analyze functions: Relative minima and maxima
2. [Khan] Contextual applications of differentiation: Rates of change in other applied contexts (non-motion problems)

~~~U04L03 Classwork~~~

1. For the equation $y = \dfrac{4x}{x^2 + 1}$, determine the roots and the relative extrema, and use them to sketch the graph.



2. For the equation $f(x) = (1/3)x^4 + (4/3)x^3$, determine the roots and the relative extrema, and use them to sketch the graph.



Unit 04 Lesson 04: Evaluate Limits Using L'Hospital's Rule (L'Hôpital's Rule): 0/0 and ∞/∞

Lesson Objectives

- Evaluate Limits Using L'Hospital's Rule
- Use this to help you find horizontal asymptotes

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L'Hospital's Rule

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1. If $f(x) = \dfrac{e^x}{x}$, determine the horizontal asymptotes

2. If $f(x) = \dfrac{x^4 - 3}{(x^3 - 2)(x + 1)}$, determine the horizontal asymptotes

3. $\lim_{x \to \infty} \frac{ln x}{2\sqrt{x}}$

4. $\lim_{x \to 0} \frac{sin x}{x}$

5. $\lim_{x \to 1} \frac{\sqrt[3]{x} - 1}{x - 1}$

6. $\lim_{x \to \infty} \frac{x}{e^x}$

~~~U04L04 Homework~~~ 1. [Khan] Contextual applications of differentiation: L'Hopitals Rule: 0/0

~~~U04L04 Classwork~~~

1. $\lim_{x \to 0} \frac{\sqrt{1+x}-1}{x}$

2. If $f(x) = \frac{ln(-x)}{x}$, determine the horizontal asymptotes

3. If $f(x) = \frac{\sqrt{1+x}-1}{x}$, determine the horizontal asymptotes

4. $\lim_{x \to 0} \frac{sin x}{x^2}$

5. $\lim_{x \to \infty} \frac{ln x}{x}$

6. $\lim_{x \to 0} \frac{sin(5x)}{x}$

7. $\lim_{x \to \infty} \frac{x^{10}}{e^x}$

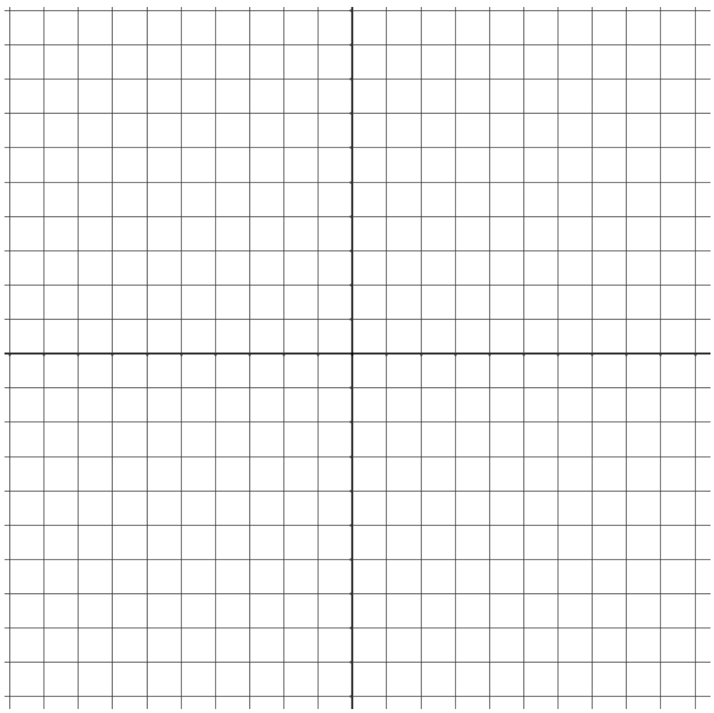
Unit 04 Lesson 05: Sketch Curves Using Asymptotes

Lesson Objectives

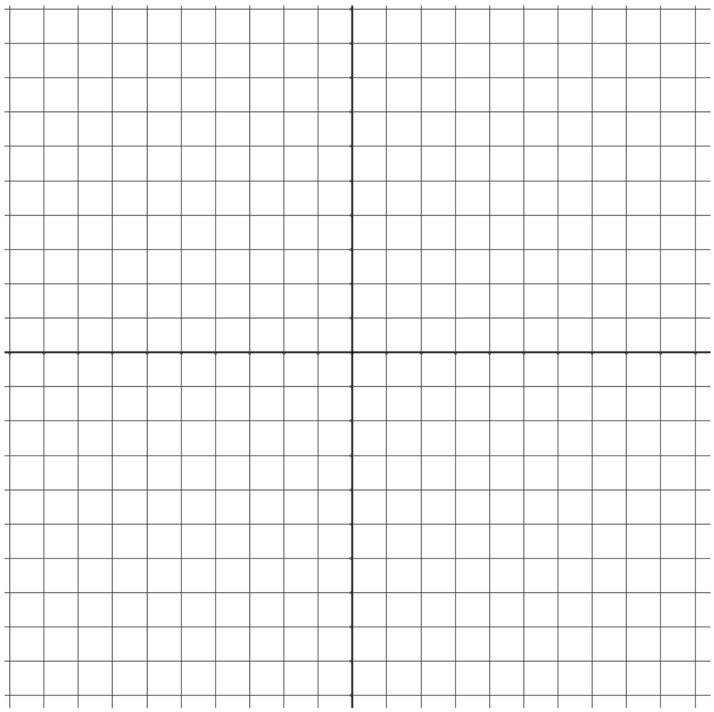
- Use limits to locate asymptotes

Asymptotes:

1. For the equation $y = \dfrac{(x-1)^2}{x-2}$, determine the asymptotes and the relative extrema, and use them to sketch the graph.



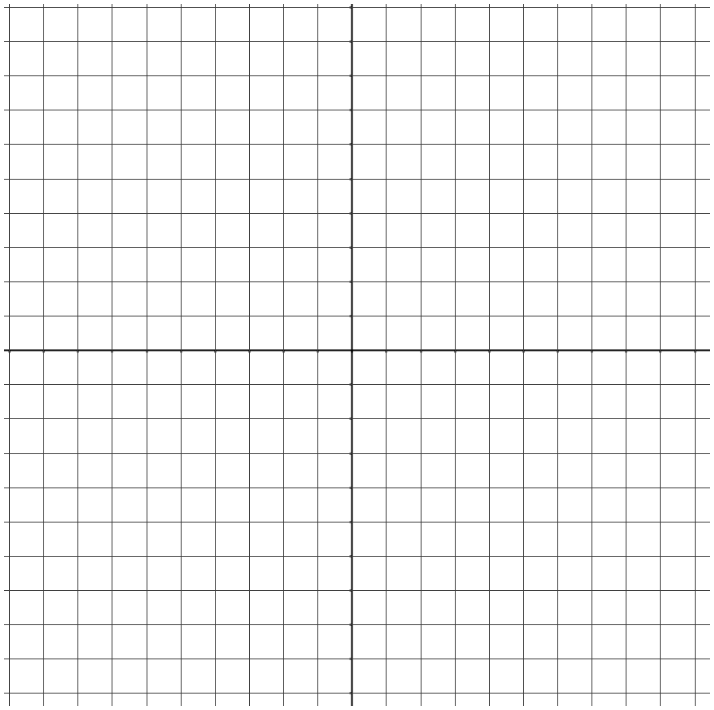
2. For the equation $y = \dfrac{x}{x-4}$, determine the roots, asymptotes, and relative extrema, and use them to sketch the graph.



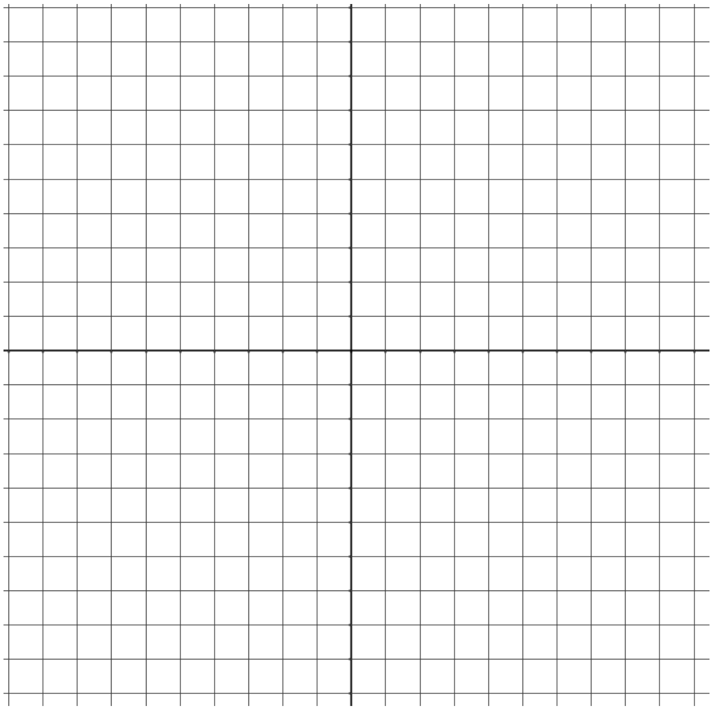
~~~U04L05 Homework~~~

1. [Khan] Contextual applications of differentiation: L’Hôpital’s Rule: ∞/∞
2. [Khan] Applying derivatives to analyze functions: Increasing and decreasing intervals

1. For the equation $y = -\dfrac{8}{x^2 + 6x + 5} - 1$, determine the asymptotes and the relative extrema, and use them to sketch the graph.



2. For the equation $y = \dfrac{2x^2 - 3x}{x - 2}$, determine the asymptotes and the relative extrema, and use them to sketch the graph.



Unit 04 Lesson 06: Given f, Sketch f”

Lesson Objectives

- Ask Mr. Rose for an eraser

If the graph of f(x) is given on the left, sketch the graph of f ’(x) and f ”(x) on the right.

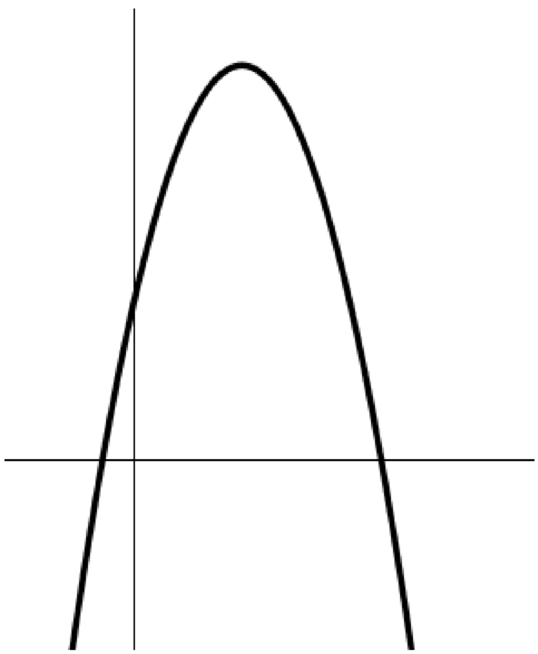
What must be perfect

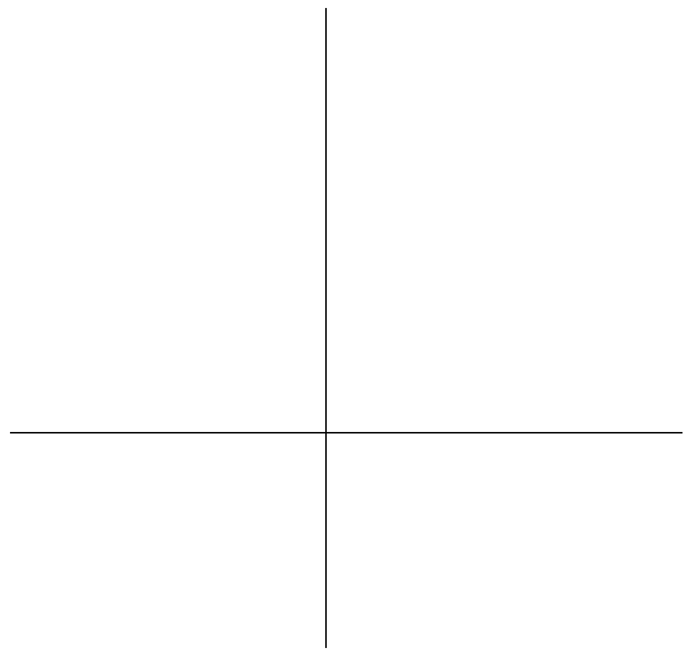
- Label important x values on both x-axes
- Make sure y’ is positive, negative, and zero when it should be (same goes for y”)

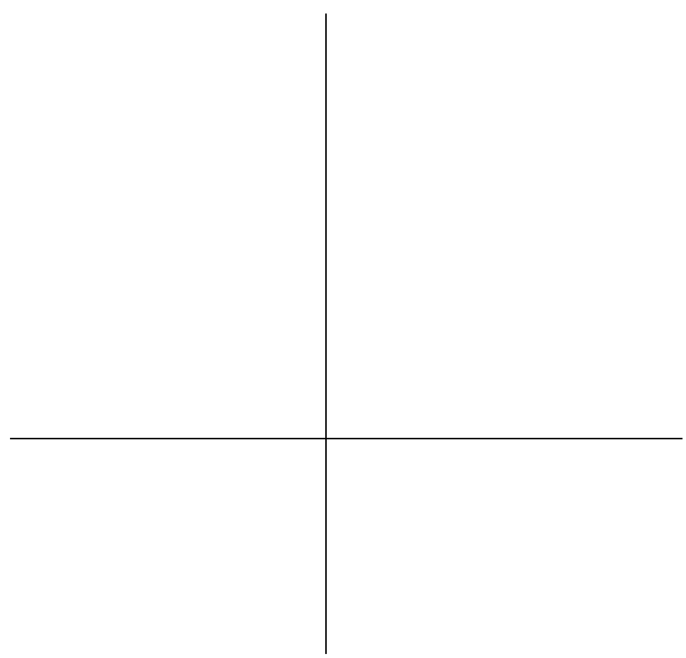
What must be reasonable

- Make sure relative y’-values are reasonable (and y” too)

1.







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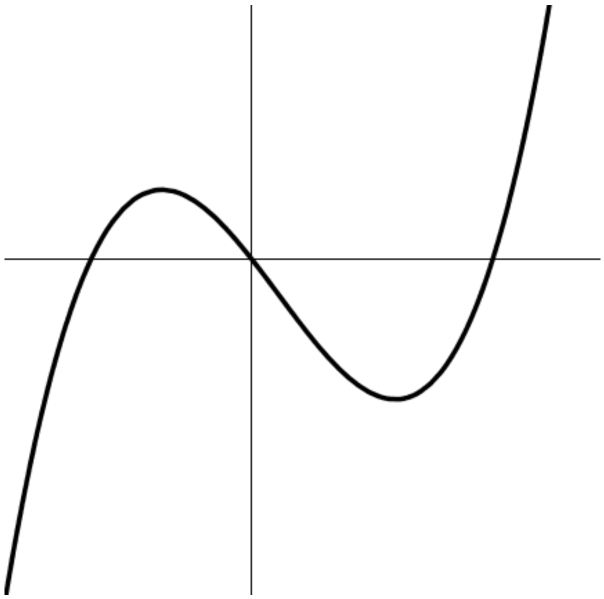

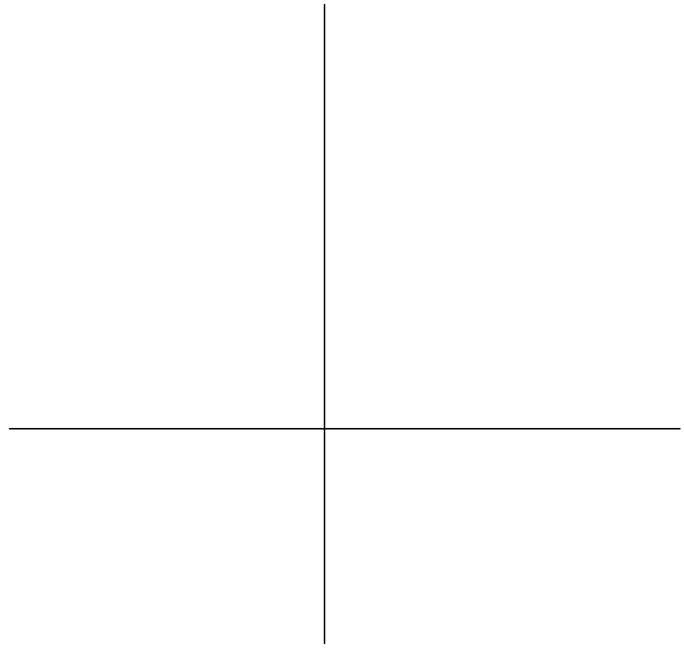

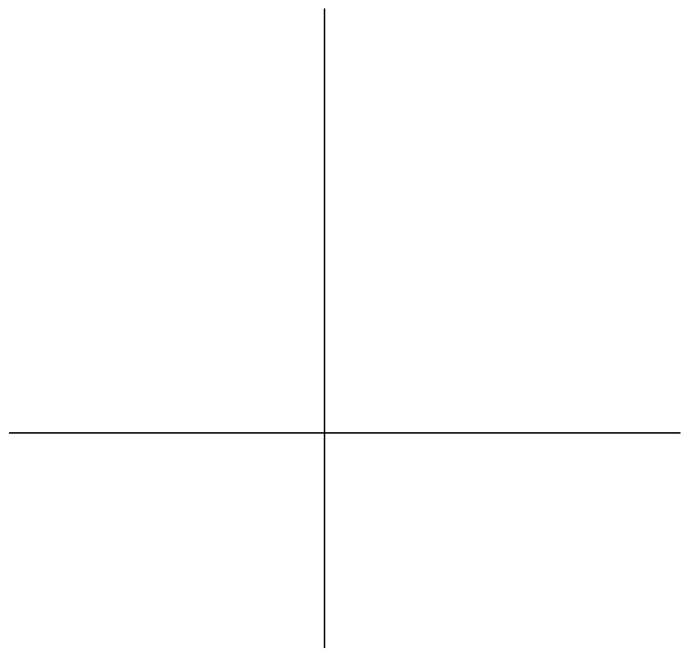

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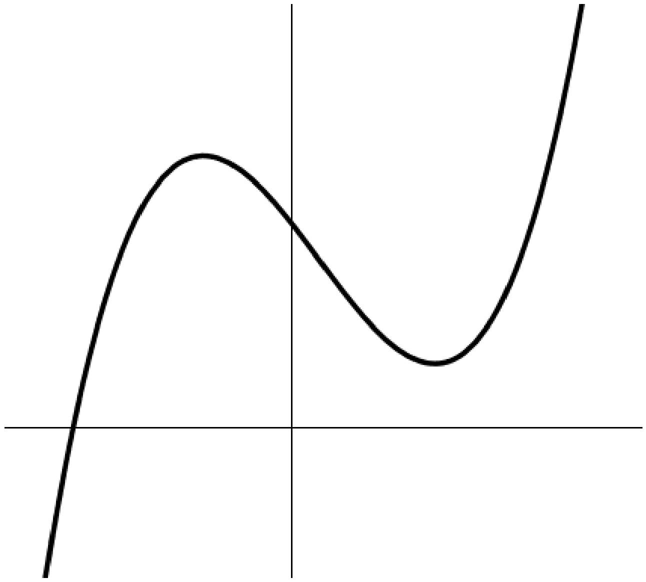

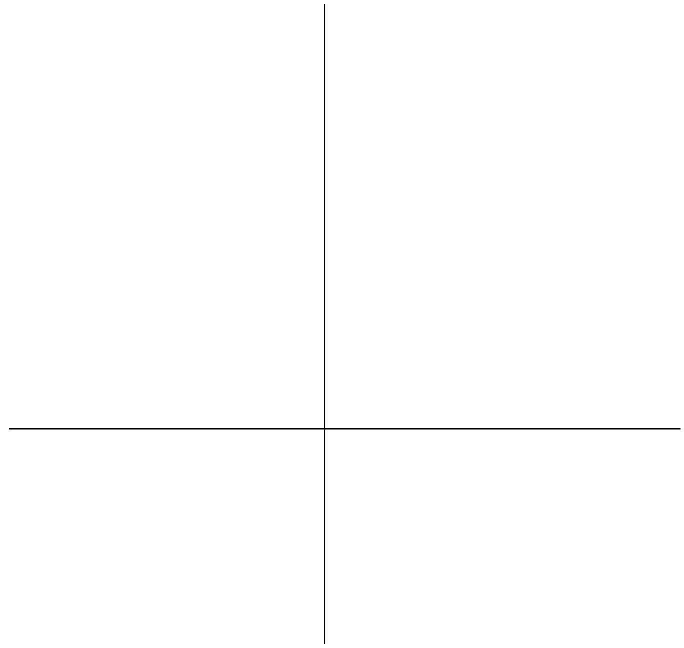

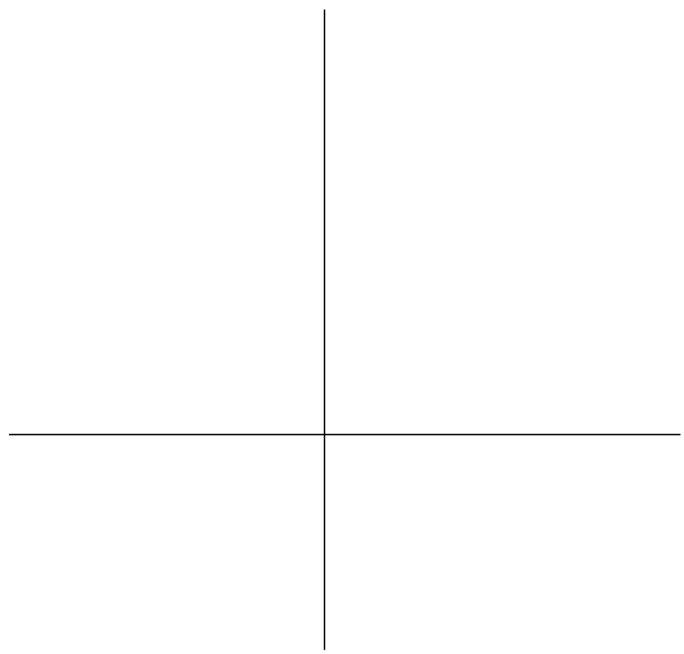

~~~U04L06 Homework~~~

1. [Khan] Apply derivatives to analyze functions: Concavity intro
2. [Khan] Apply derivatives to analyze functions: Inflection points intro

~~~U04L06 Classwork~~~

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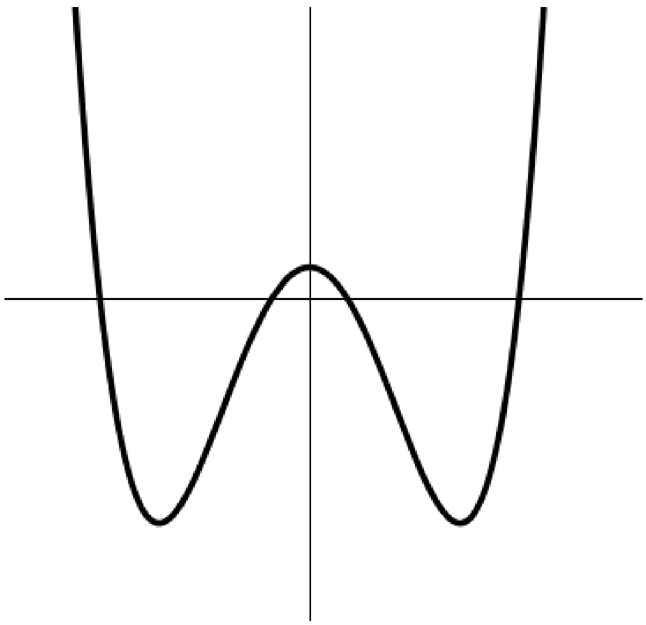

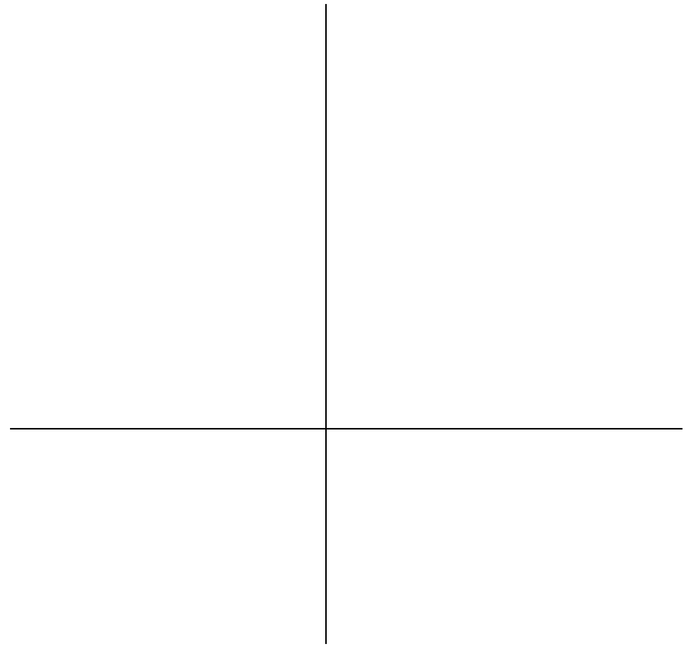

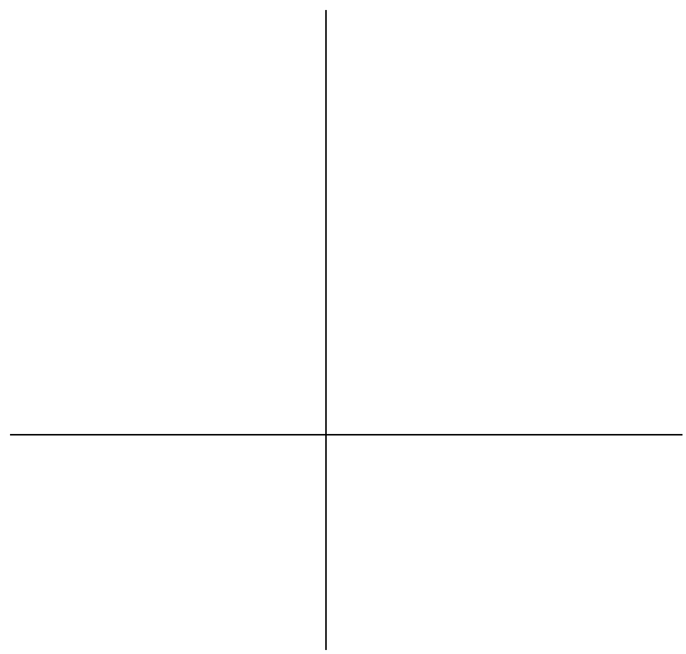

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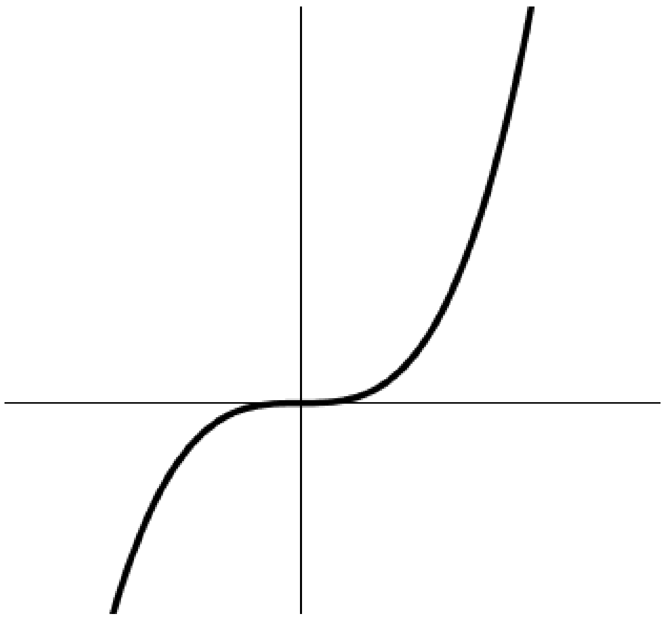

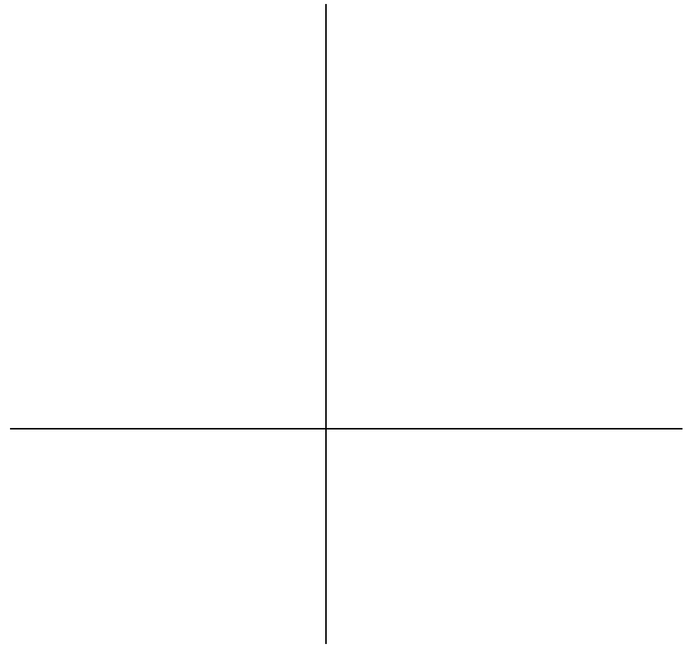

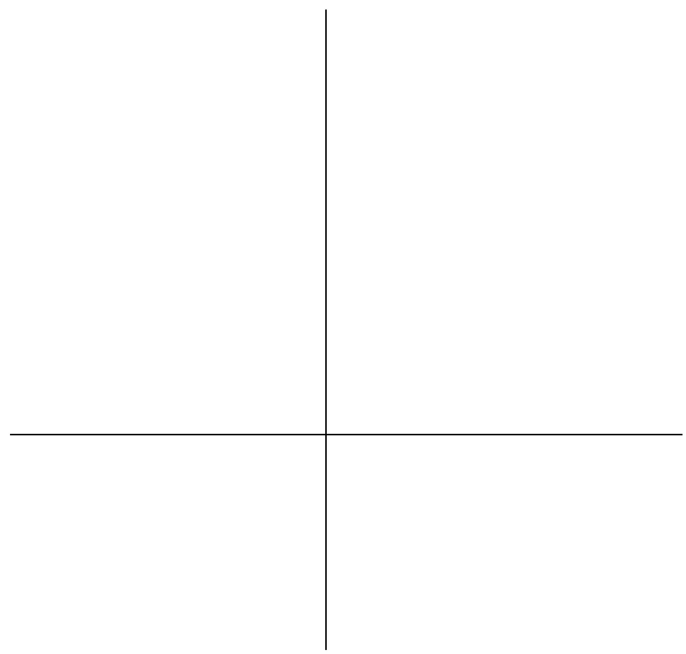

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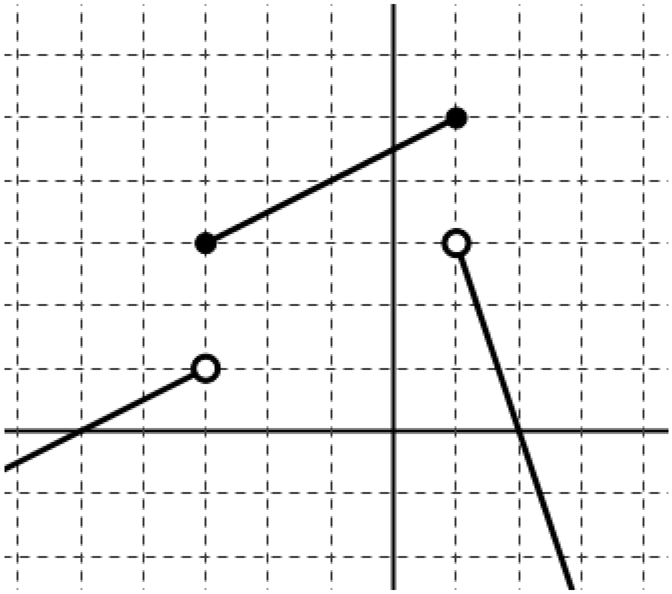

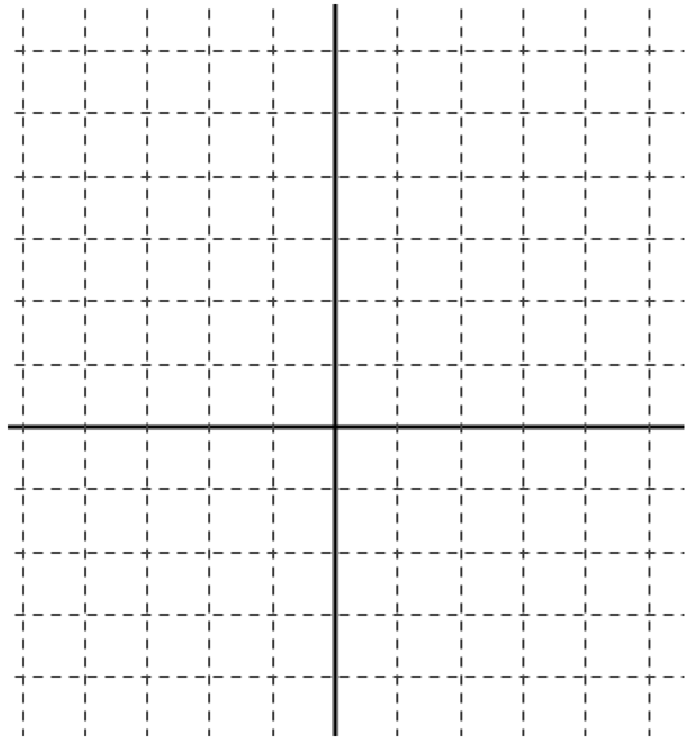

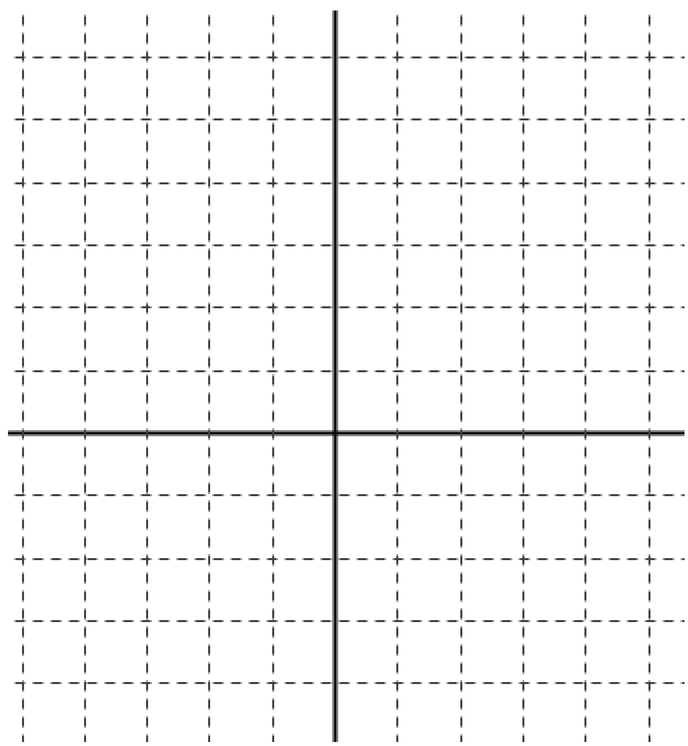

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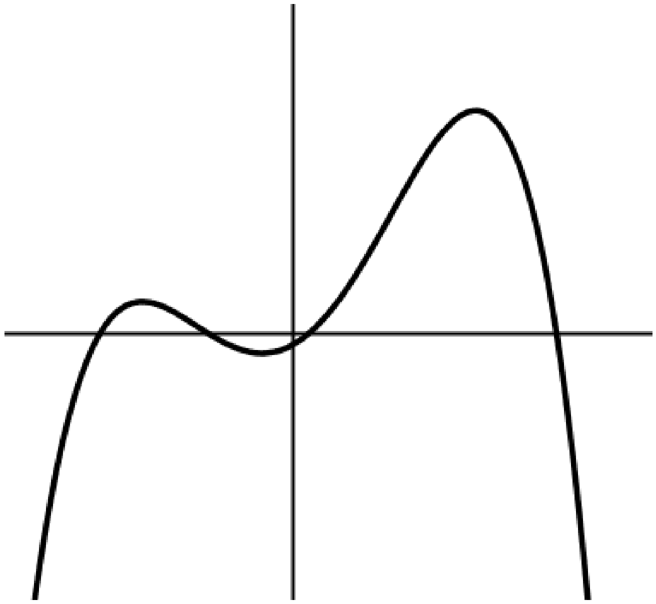
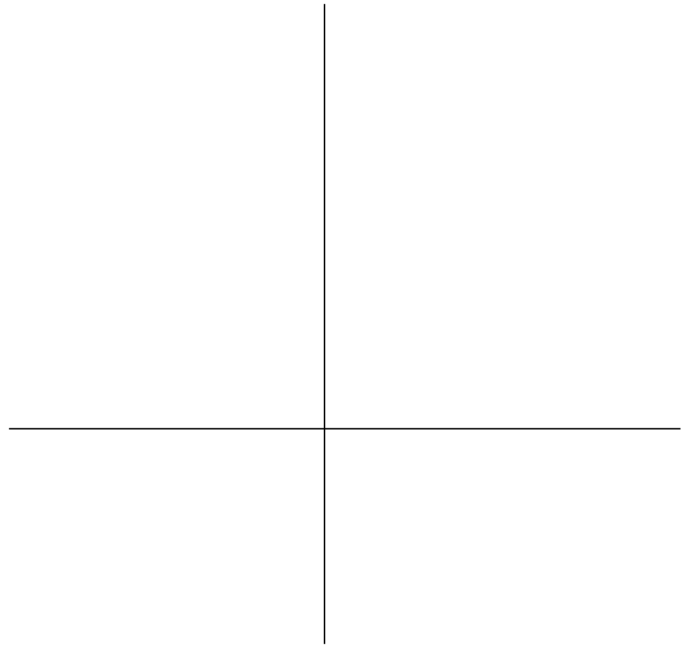
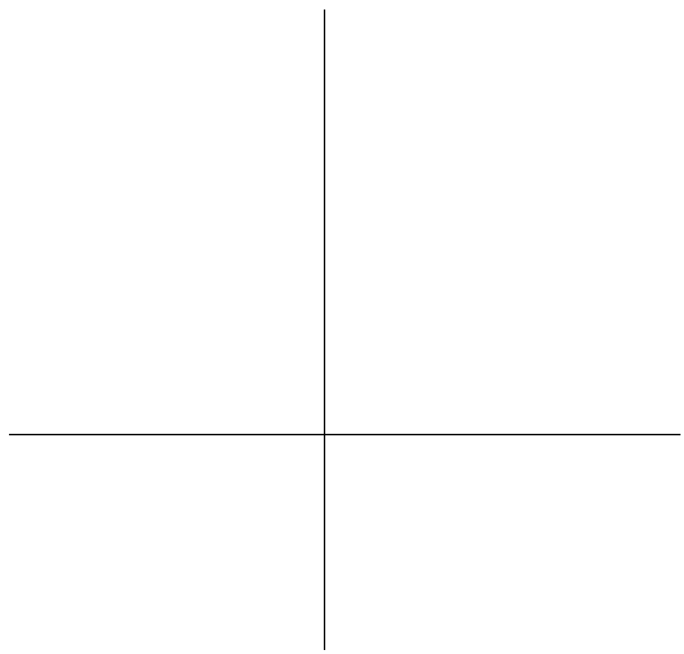






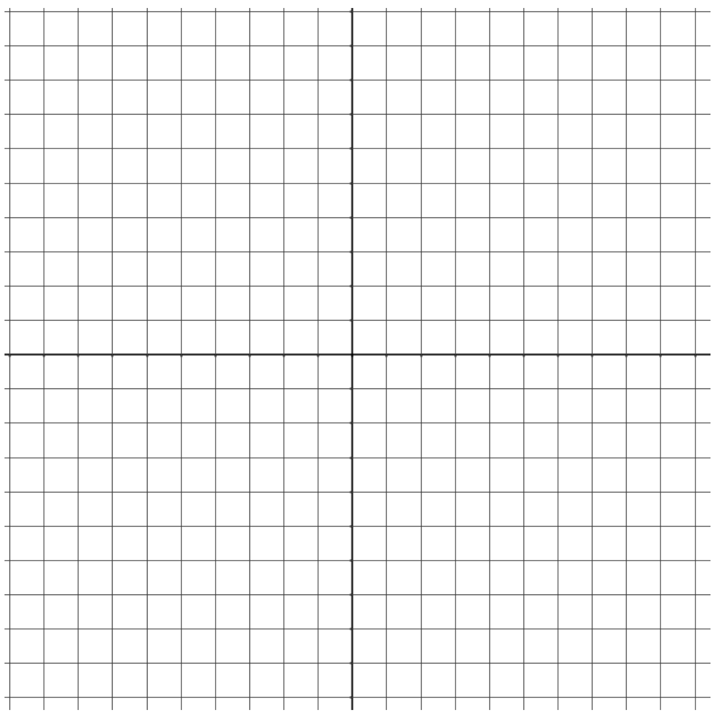
Unit 04 Lesson 07: Sketch Curves Using Inflection Points

Lesson Objectives

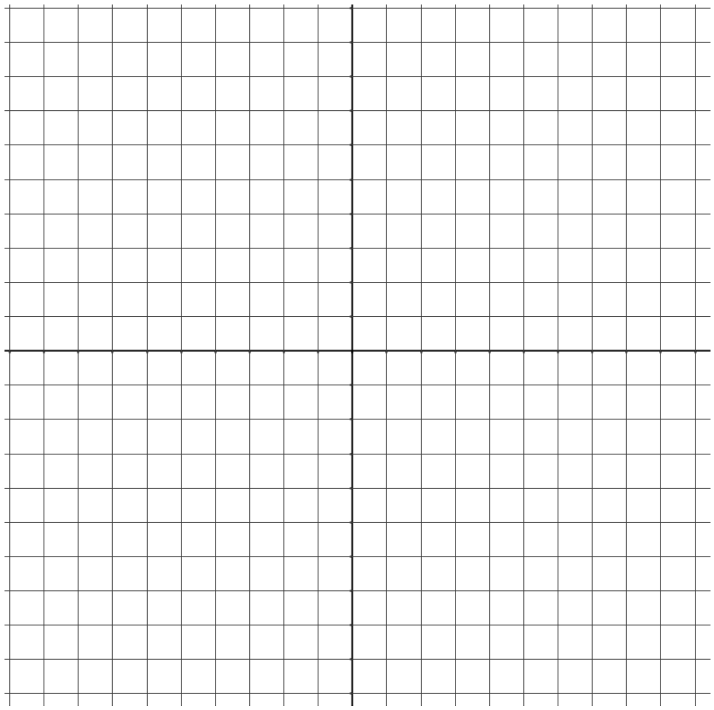
- Use inflection points to help sketch curves

Concavity:

1. For the equation $y = 5e^{-\frac{x^2}{2}}$, determine the asymptotes, relative extrema, and points of inflection and use them to sketch the graph.



2. For the equation $x^3 - 12x^2 + 45x - 49$, determine the relative extrema and the points of inflection, and use them to sketch the graph.



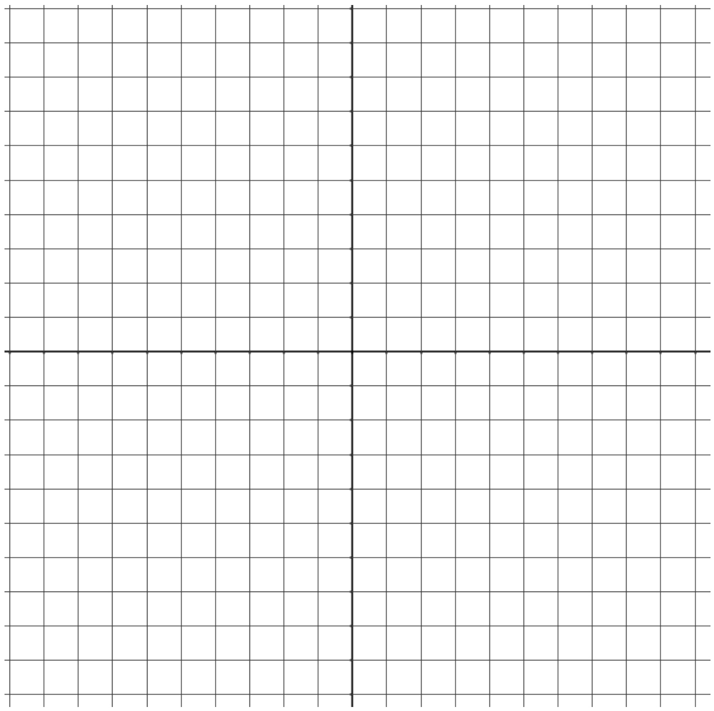
~~~U04L07 Homework~~~

1. [Khan] Apply derivatives to analyze functions: Analyze concavity
2. [Khan] Apply derivatives to analyze functions: Find inflection points

~~~U04L07 Classwork~~~

1. For the equation $f(x) = 2x^3 - 6x^2$, determine the following features and use them to sketch the graph.

   a. roots b. asymptotes c. critical points d. points of inflection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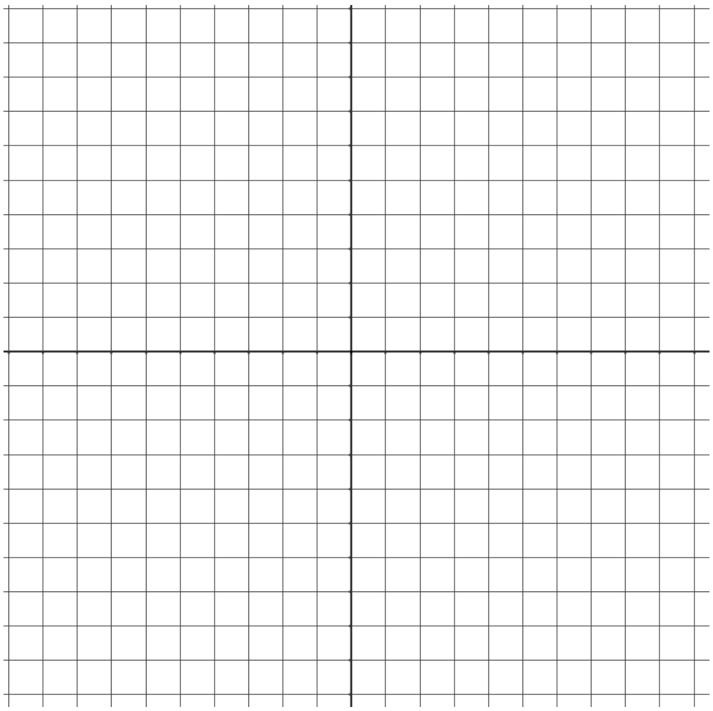
2. For the equation $y = 10\frac{lnx}{x}$, determine the following features and use them to sketch the graph.
   a. roots b. asymptotes c. critical points d. points of inflection



Unit 04 Lesson 08: Determine Absolute Maxima and Minima

Lesson Objectives

- Determine the highest and lowest points on a graph

Relative Maximum

Relative Minimum

Absolute Maximum

Absolute Minimum

Can occur at 2 possible places

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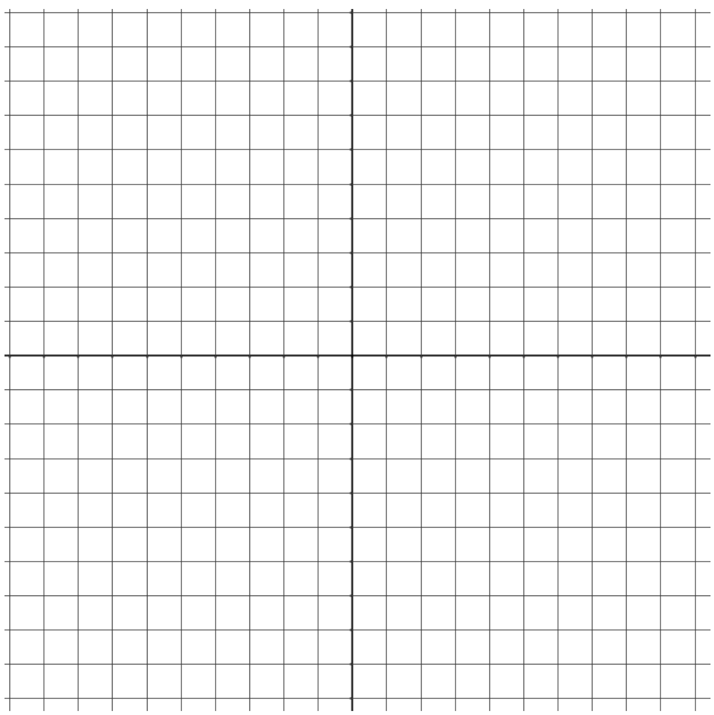
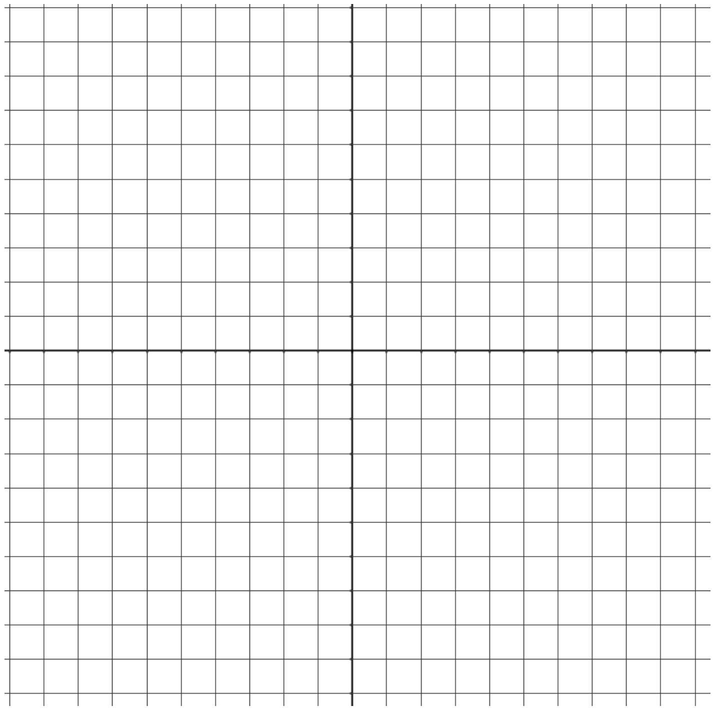
1. For the equation $g(x) = \frac{1}{8}x^3 - \frac{3}{2}x - 1$, determine the absolute maximum and absolute minimum on the interval [-3, 5]. Sketch a graph to verify your answer.



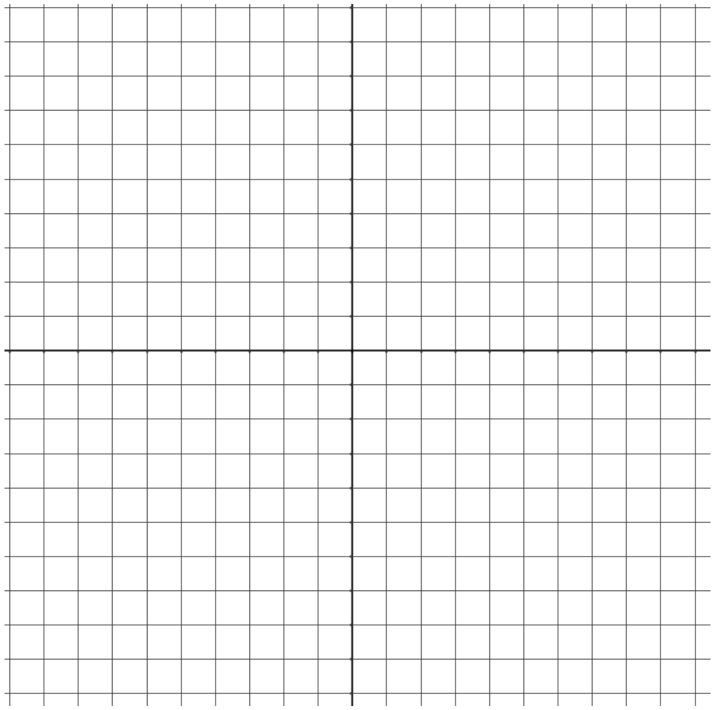
2. For the equation $f(x) = \dfrac{x^2}{(x-3)^2} - 2$, determine the absolute maximum and absolute minimum on the interval [-3, 6]. Sketch a graph to verify your answer.



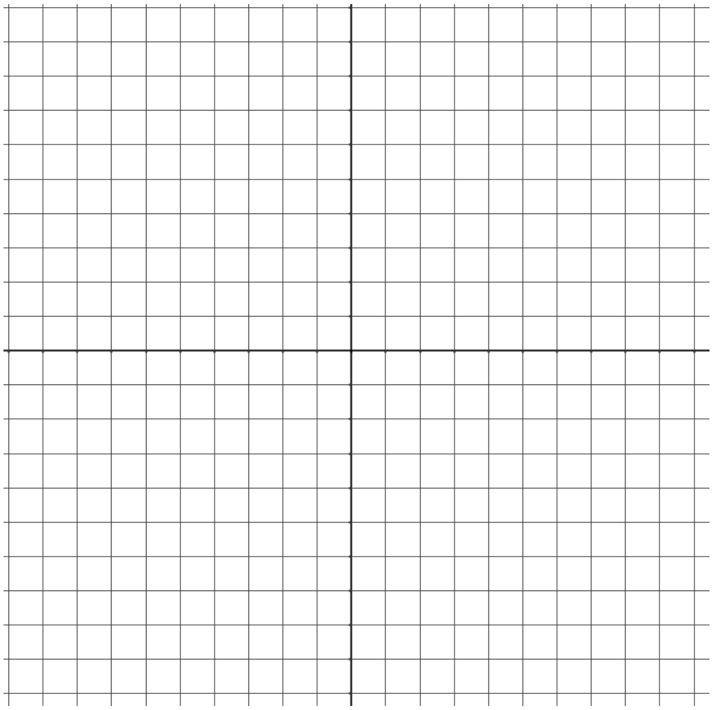
~~~U04L08 Homework~~~ 1. [Khan] Apply derivatives to analyze functions: Absolute minima & maxima (closed intervals)

~~~U04L08 Classwork~~~

1. For the equation $f(x) = -\dfrac{2x^2}{(1-x)^2} + 2$, determine the absolute maximum and absolute minimum on the interval [-1, 2]. Sketch a graph to verify your answer.



2. For the equation $4x^4 - 20x^2 + 16$, determine the absolute maximum and absolute minimum on the interval [-2, 1]. Sketch a graph to verify your answer.



Unit 04 Lesson 09: Sketch Curves Sans Sketching Curves--No Calculators

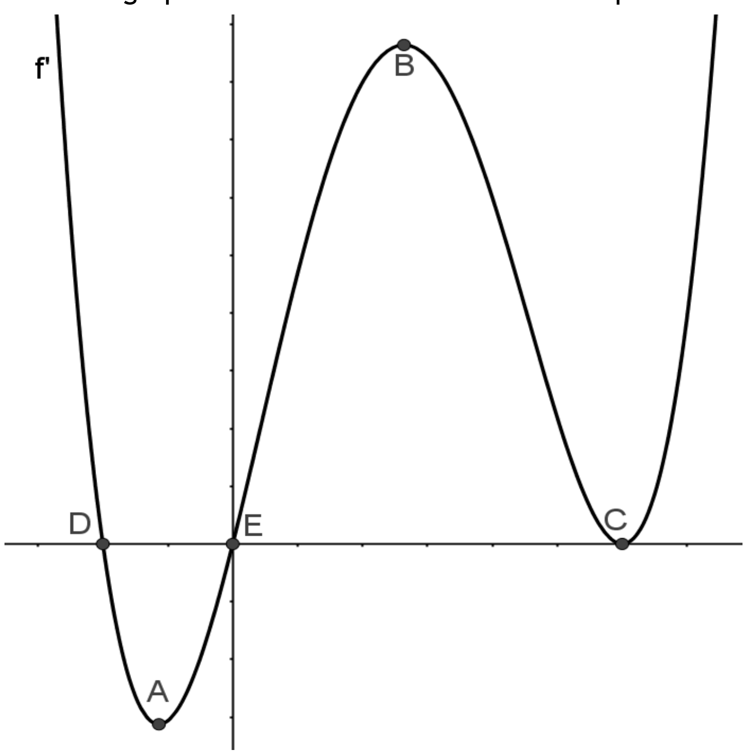
Lesson Objectives

- Answer questions about curve sketching that don't involve actually sketching a curve--no calculators

1. Let *g* be the function defined by $g(x) = 2x^4 + 8x^3$. How many relative extrema does g have?

2. The function *f* has a first derivative given by $f'(x) = x(x+3)^2(x-1)$. At what value(s) of x does *f* have a relative maximum?

3. A graph of f' is shown below. State the points which have x-values that correspond to the following.



(make a table just like we've been doing)

   a. Critical points:

   b. Relative extrema:

   c. Relative minima:

   d. Relative maxima:

   e. Inflection points:

   f. Roots:

4. For what values of x does the graph of $y = 2x^6 + 6x^5$ have a point of inflection?

5. If *g* is the function given by $g(x) = \frac{1}{3}x^3 - \frac{3}{2}x^2 - 70x - 8$
   a. On which interval(s) is *g* decreasing?

   b. On which interval(s) is *g* increasing?

6. Let *f* be the function defined by $f(x) = 2x^3 + 3x^2 - 12x + 2$. On which of the following intervals is the graph of *f* both decreasing and concave up?

~~~U04L09 Homework~~~

1. [Khan] Apply derivatives to analyze functions: Justification using second derivative
2. [Khan] Apply derivatives to analyze functions: Connecting f, f', f'' graphically

~~~U04L09 Classwork~~

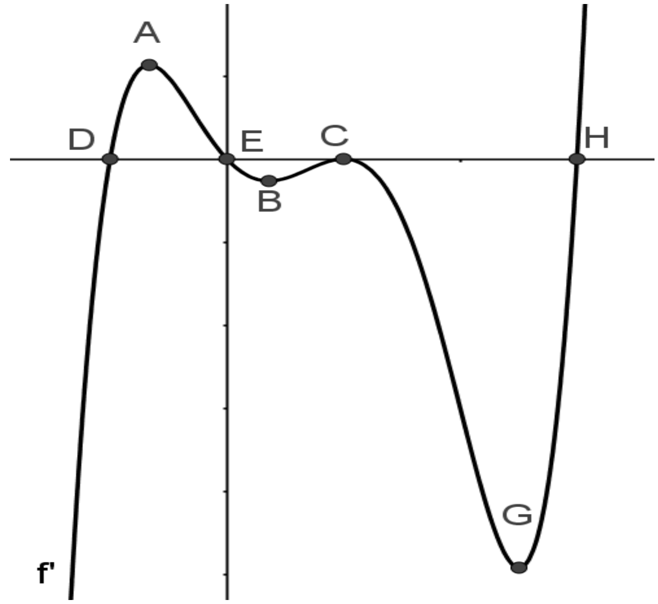
1. The graph below shows f'. State the points which have x-values that correspond to the following.
   a. Critical points:

   b. Relative maxima:

   c. Relative minima:

   d. Inflection points:

   e. Roots:



2. Let g be the function defined by $g(x) = 2x^2 + 8x$. How many points of inflection does g have?

3. The function f has a first derivative given by $f'(x) = x(x-4)^2(x-2)$. At what value(s) of x does f have a relative minimum?

4. For what values of x does the graph of $y = 2x^6 + 6x^5$ have a relative maximum?
~~~

5. If *g* is the function given by $g(x) = -x^3 - 6x^2 + 180x + 5$
   a. On which interval(s) is *g* decreasing?

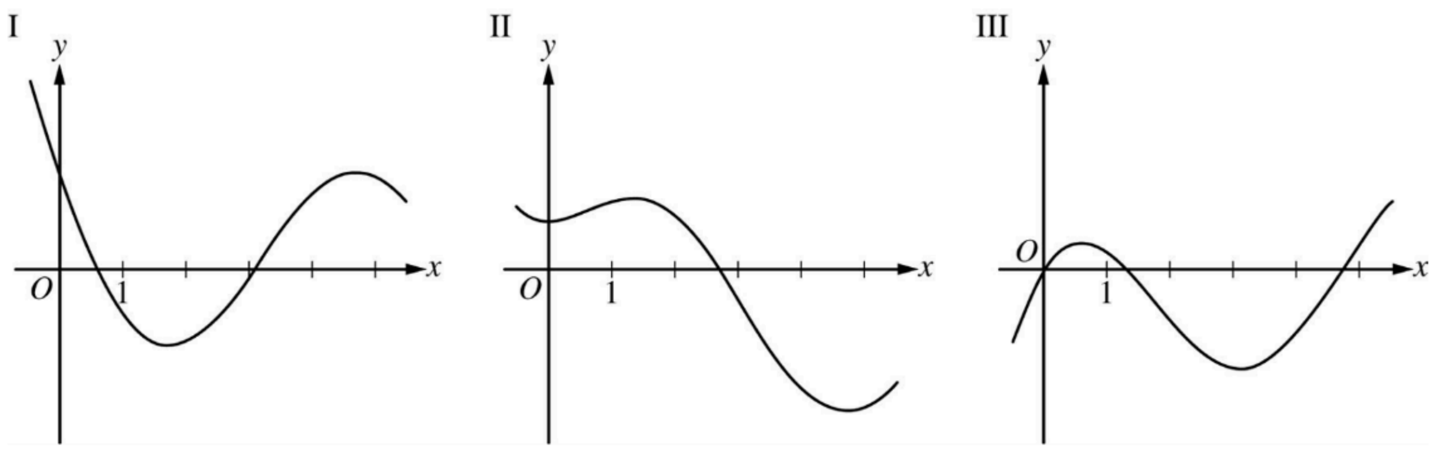
   b. On which interval(s) is *g* increasing?

6. Let *f* be the function defined by $f(x) = x^3 + 3x^2 - 9x + 7$. On which of the following intervals is the graph of *f* both increasing and concave down?

Unit 04 Lesson 10: Compare Graphs of f, f', and f''

Lesson Objectives

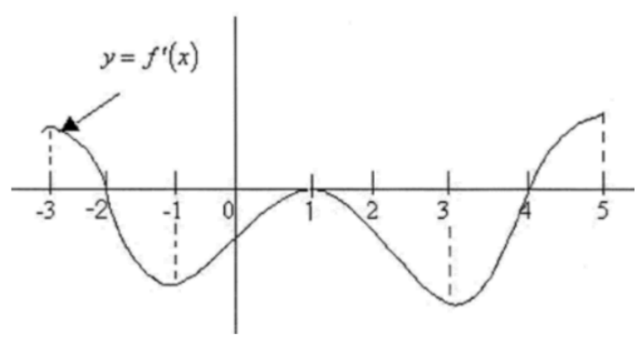
- Answer questions comparing graphs of f, f', f''



Three graphs labeled I, II, and III are shown above. One is the graph of f, one is the graph of f', and one is the graph of f''. Which of the following correctly identifies each of the three graphs?

	f	f'	f''
(A)	I	II	III
(B)	II	I	III
(C)	II	III	I
(D)	III	I	II

For what value(s) of x does f have a relative maximum? Why?
For what value(s) of x does f have a relative minimum? Why?
On what intervals is the graph of f concave up? Why?
On what intervals is f increasing? Why?
For what value(s) of x does f have an inflection point? Why?

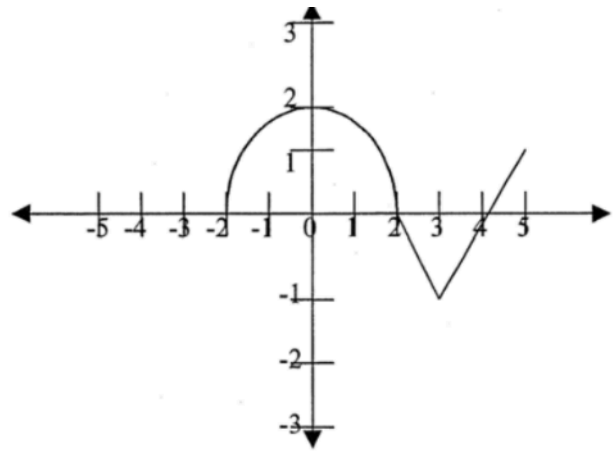


~~~U04L10 Homework~~~ 1. [Khan] Apply derivatives to analyze functions: Analyze functions (calculator-active)

~~~U04L10 Classwork~~~

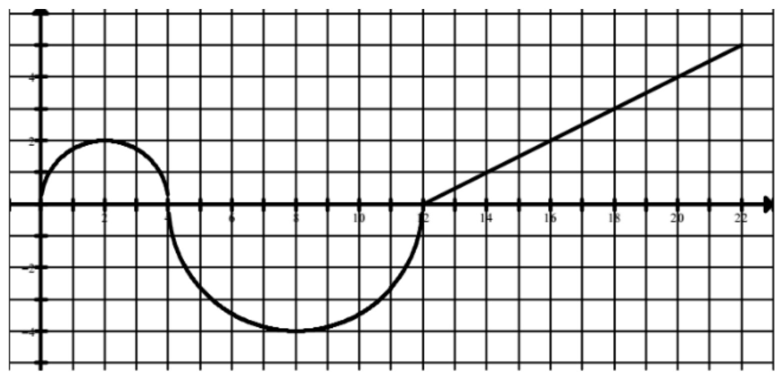
1.

Find g(3).
For what value(s) of x does g have a relative maximum? Why?
For what value(s) of x does g have a relative minimum? Why?
For what value(s) of x does g have an inflection point? Why?
Write an equation for the line tangent to the graph of g at x=3

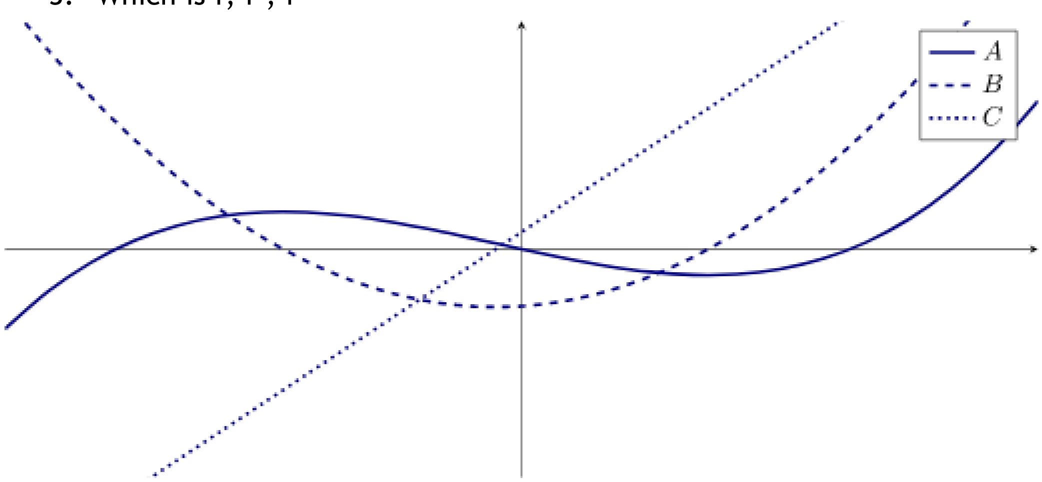


2.

On what intervals is f increasing? Justify your answer.
For what values of x does f have a relative minimum? Justify.
On what intervals is f concave up? Justify.
For what values of x is f'' undefined?
Identify the x−coordinates for all points of inflection of f.
For what value of x does f reach its maximum value? Justify.
If f (4) = 5, find f(12).



3. Which is f, f', f''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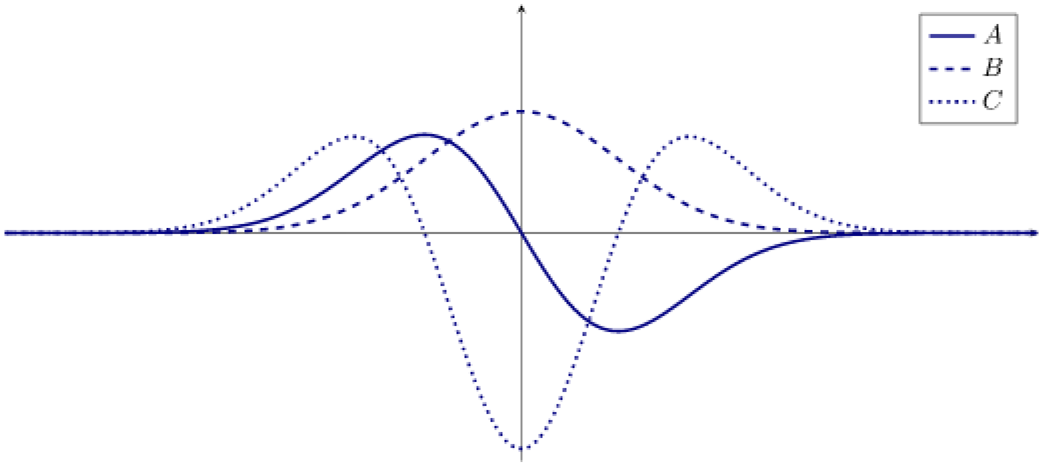
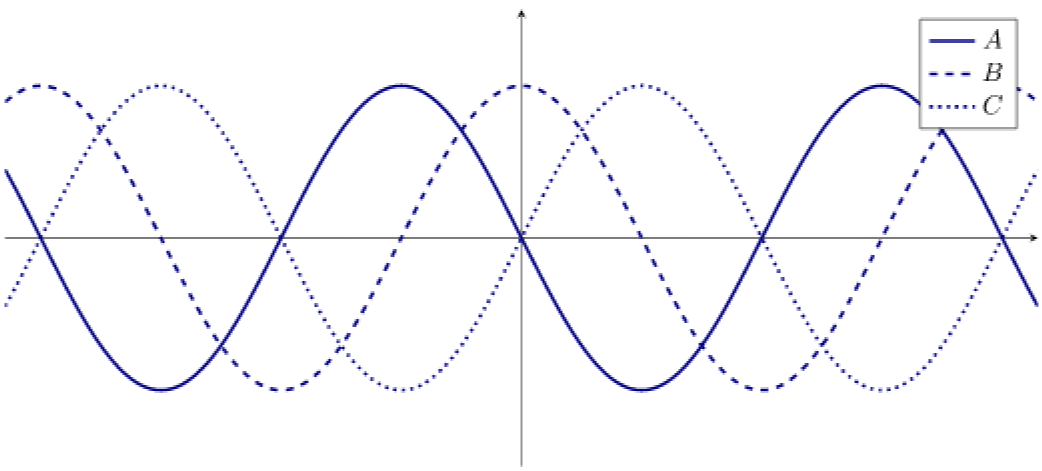
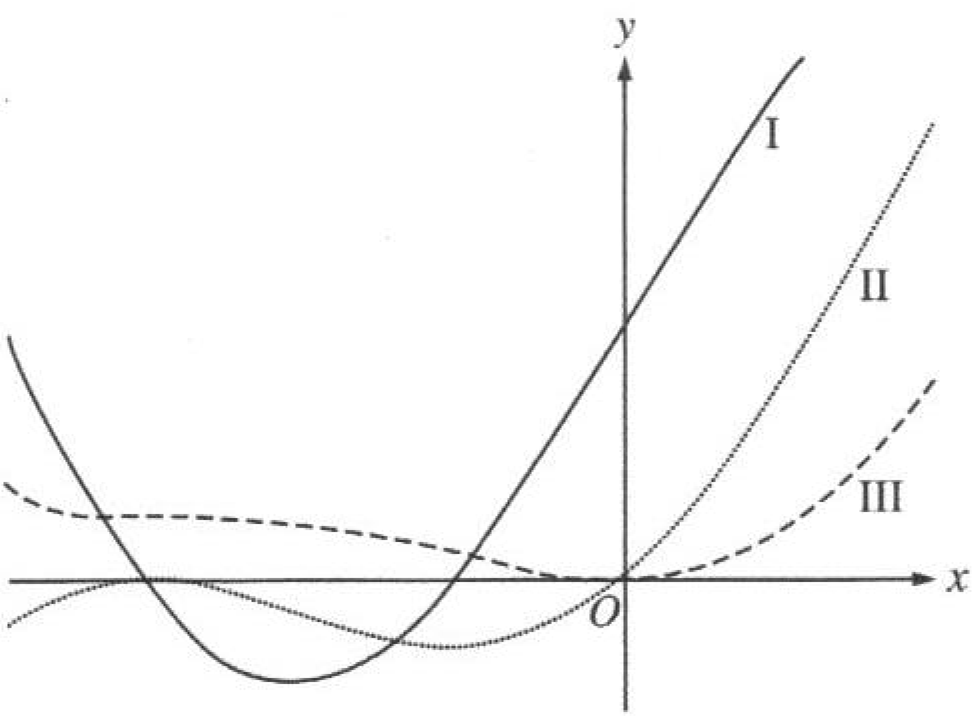
4. Which is f, f', f''



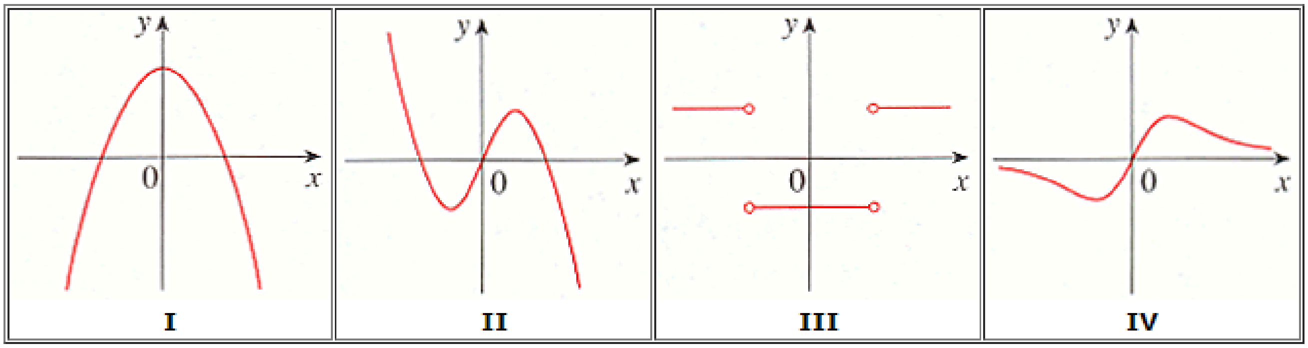
5. Which is f, f', f''



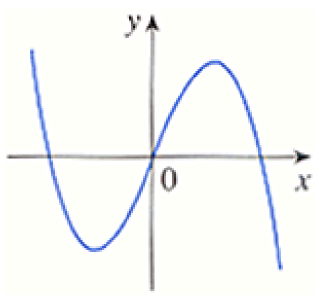
6. Which is f, f', f''



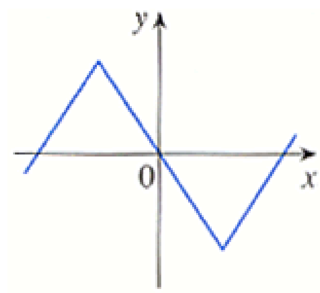
7. Match the graph of f (a, b, c, or d) with the graph of f' (I, II, III, or IV)



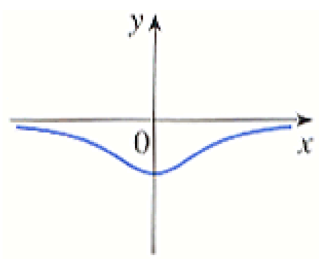
(a)



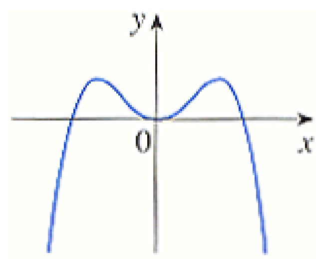
(b)



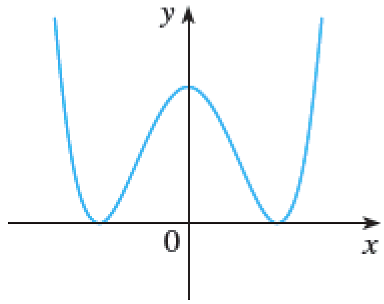
(c)



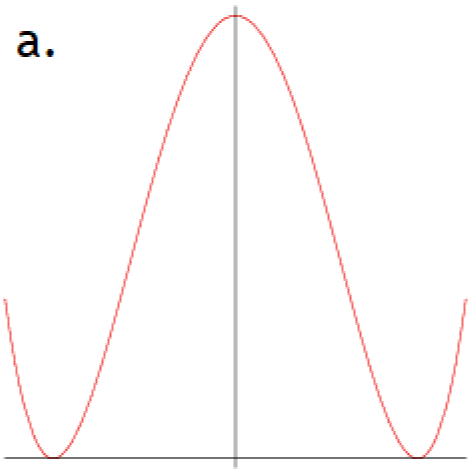
(d)



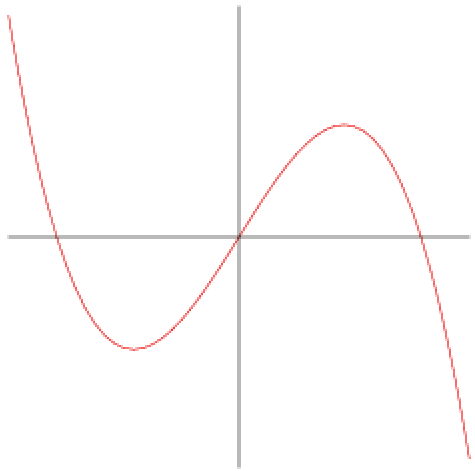
8. Which graph is the derivative of the given graph?



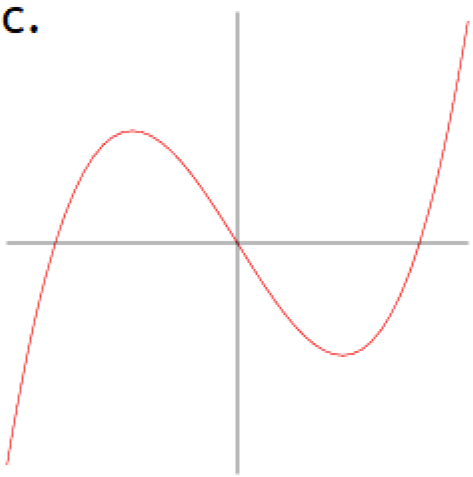
a.



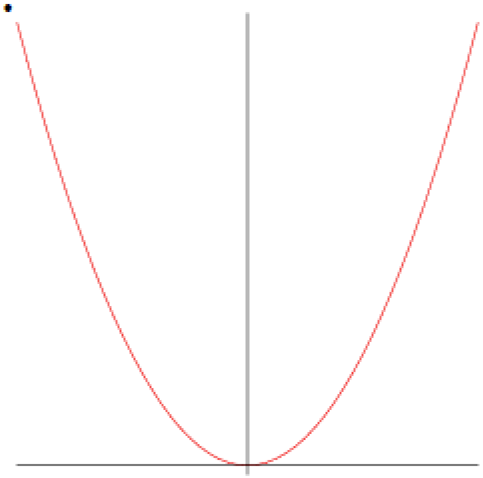
b.



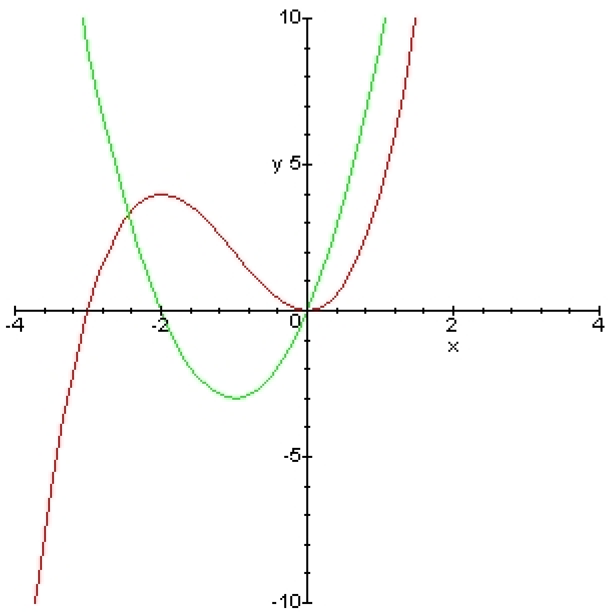
c.



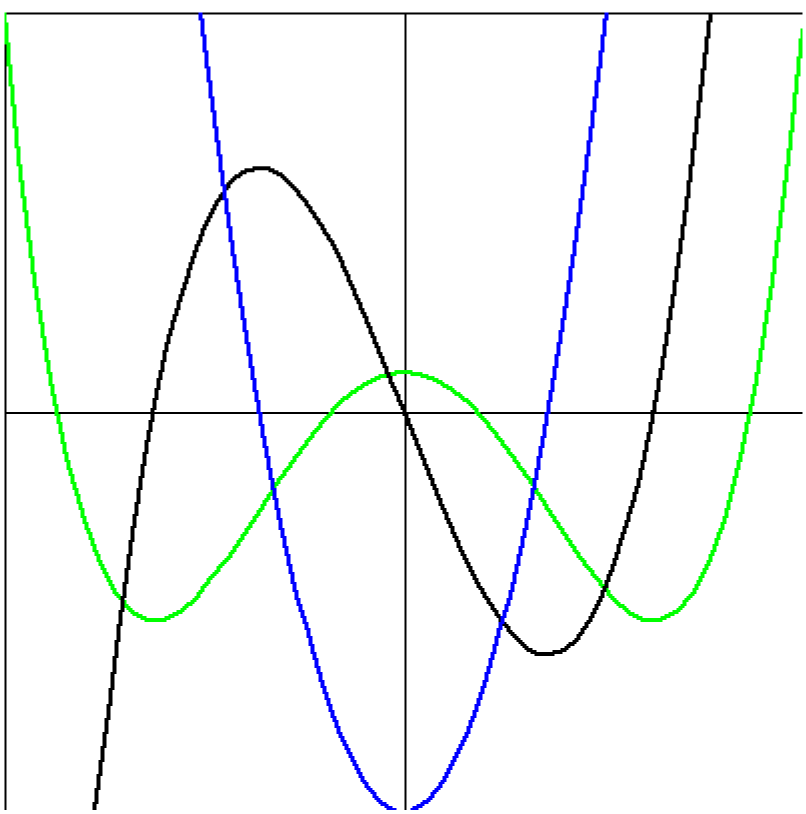
d.



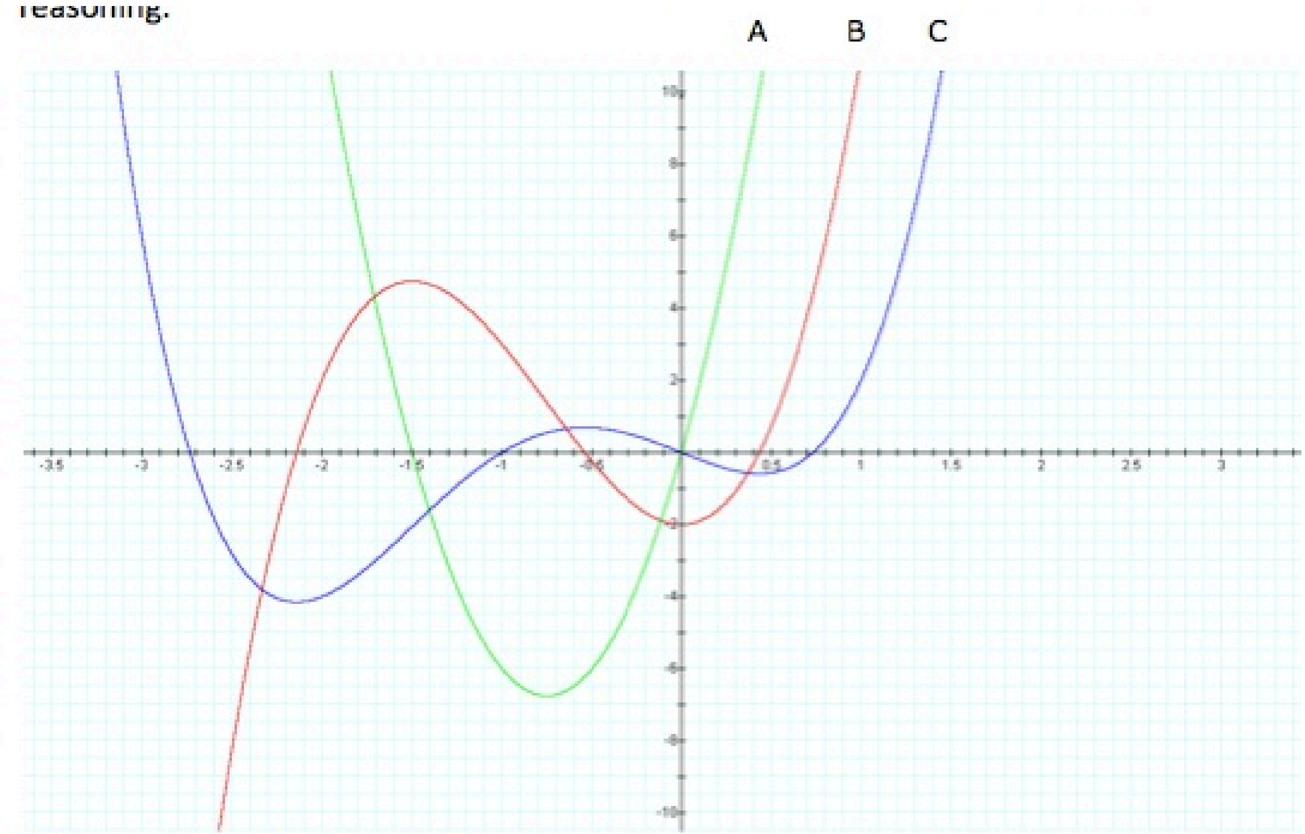
9. Determine who is the derivative of who



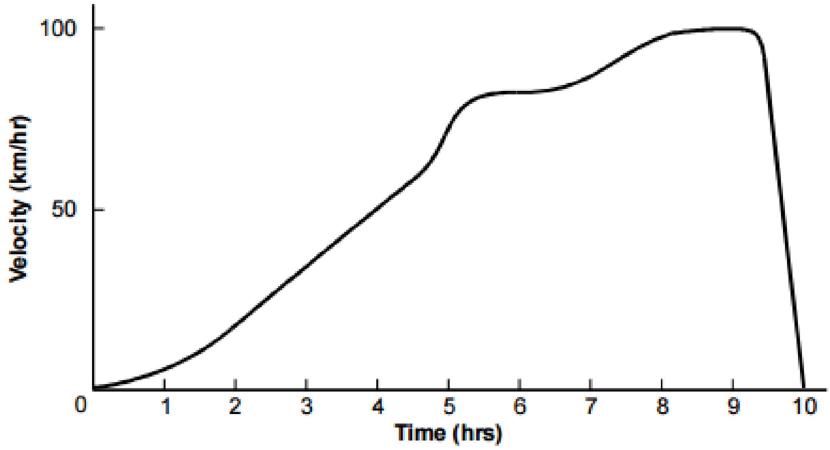
10. Determine who is the derivative of who



11. Determine who is the derivative of who



12.



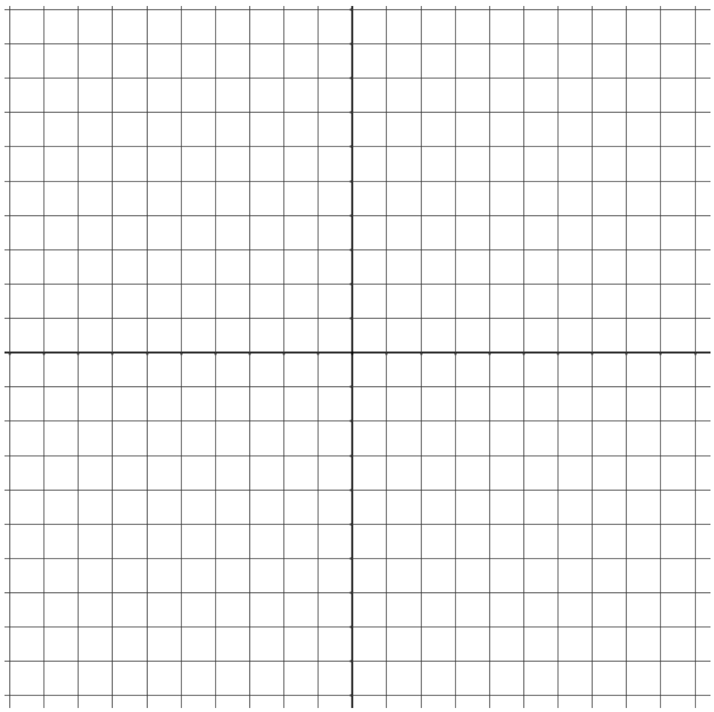
The velocity-time graph above represents a trip made by a train.

Using your knowledge of motion and graphs:

- explain the significance of the area between the line drawn to represent the train's velocity and the horizontal axis of the graph;
- explain the significance of the slope of the velocity-time graph; and
- draw an acceleration-time graph for the same trip, with acceleration on the vertical axis and time on the horizontal axis.

Unit 04 Lesson 11: Prepare for Test On Curve Sketching

1. For the equation $y = 10x/e^x$, determine the following features and use them to sketch the graph.
   a. roots b. asymptotes c. critical points d. points of inflection



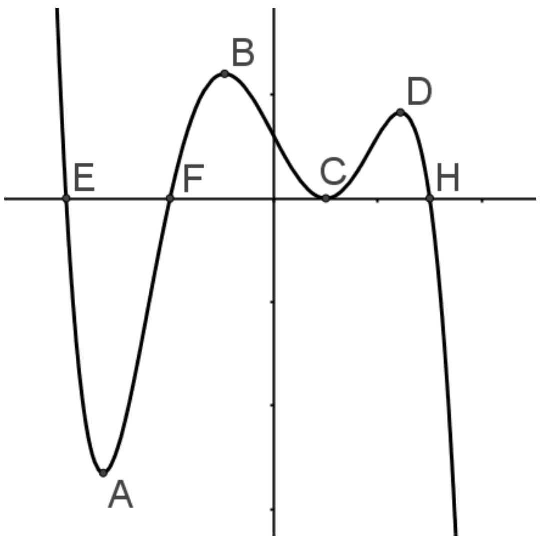
2. The graph below shows f'. State the points which have x-values that correspond to the following.
    a. Critical points:

    b. Relative maxima:

    c. Relative minima:

    d. Inflection points:

    e. Roots:



3. For the function $f(x) = -\frac{x^3}{3} + \frac{3}{2}x^2 + 4x + 1$
    a. On what intervals is the graph both increasing and concave down?
    b. What is the absolute max and absolute min on the interval [-4, 6]?