Limits

The **limit** command is used to calculate limits but since MATLAB uses “symbolic computing”, you need to define the variables using “syms”. The number after the comma below indicates the number that *x* is approaching. You may exclude it if it’s 0, as in the third example.

$$\lim_{x \to 0} \frac{x-1}{x+1}$$

input	output
>> syms x limit((x-1)/(x+1),0)	ans = -1

input	output
>> syms x limit((x-1)/(x+1),2)	ans = 1/3

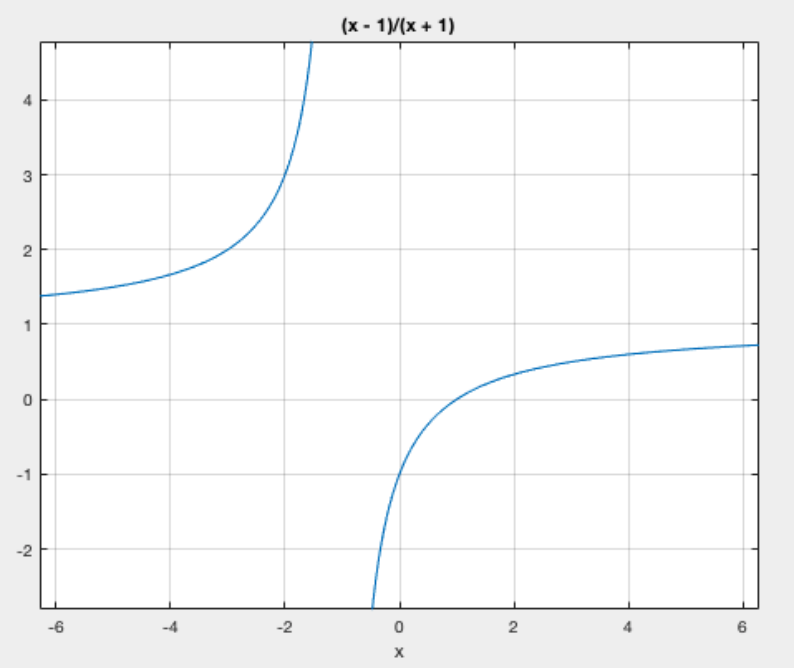
$$\lim_{x \to 0} \frac{sinx}{x}$$

input	output
syms x limit(sin(x)/x)	ans = 1

You can also take one-sided limits and limits at infinity:

input	output
>> syms x limit((x-1)/(x+1),x,-1,'right')	ans = -Inf
>> syms x limit((x-1)/(x+1),x,-1,'left')	ans = Inf
>> syms x limit((x-1)/(x+1), x, inf)	ans = 1

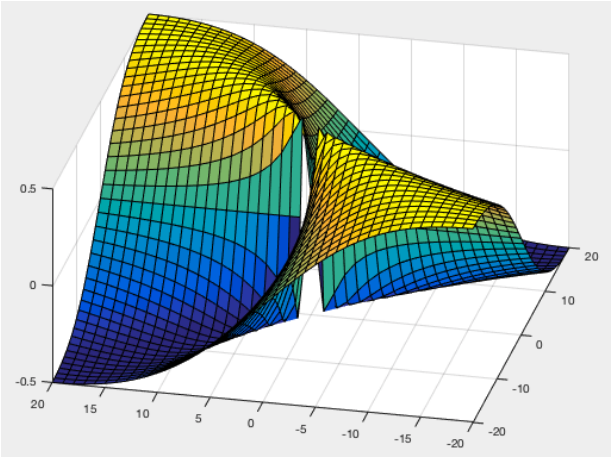
```
>> syms x; ezplot((x-1)./(x+1)), grid on
```

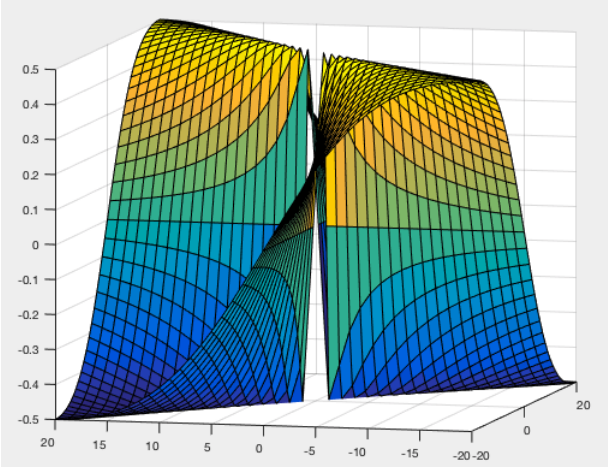


Limits of Multivariable Functions

input	output
>> syms x y limit((x-1)/(y+1),x,y)	ans = (y - 1)/(y + 1)

input	output
>> syms x y limit((x*y)/(x^2+y^2),x,y)	ans = 1/2






```
>> [x,y] = meshgrid(-20:20); g = (x.*y)./(x.^2+y.^2);
surf(x, y, g)
```

Graph the figure as above and rotate to verify that the limit at the origin is indeed ½.

EXERCISES.

Calculus I

Evaluate each limit.

3. $\lim_{x \to 1} e^{x^3 - x}$

4. $\lim_{x \to 3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

5. $\lim_{x \to -3} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

6. $\lim_{x \to 1^+} \dfrac{x^2 - 9}{x^2 + 2x - 3}$

7. $\lim_{h \to 0} \dfrac{(h - 1)^3 + 1}{h}$

8. $\lim_{t \to 2} \dfrac{t^2 - 4}{t^3 - 8}$

9. $\lim_{r \to 9} \dfrac{\sqrt{r}}{(r - 9)^4}$

10. $\lim_{v \to 4^+} \dfrac{4 - v}{|4 - v|}$

11. $\lim_{u \to 1} \dfrac{u^4 - 1}{u^3 + 5u^2 - 6u}$

12. $\lim_{x \to 3} \dfrac{\sqrt{x + 6} - x}{x^3 - 3x^2}$

13. $\lim_{x \to \infty} \dfrac{\sqrt{x^2 - 9}}{2x - 6}$

14. $\lim_{x \to -\infty} \dfrac{\sqrt{x^2 - 9}}{2x - 6}$

15. $\lim_{x \to \pi^-} \ln(\sin x)$

16. $\lim_{x \to -\infty} \dfrac{1 - 2x^2 - x^4}{5 + x - 3x^4}$

17. $\lim_{x \to \infty} \left(\sqrt{x^2 + 4x + 1} - x\right)$

18. $\lim_{x \to \infty} e^{x - x^2}$

19. $\lim_{x \to 0^+} \tan^{-1}(1/x)$

20. $\lim_{x \to 1} \left(\dfrac{1}{x - 1} + \dfrac{1}{x^2 - 3x + 2}\right)$

Calculus II

Evaluate each improper integral.

9. $\int_2^\infty e^{-5p}\,dp$

10. $\int_{-\infty}^0 2^r\,dr$

11. $\int_0^\infty \frac{x^2}{\sqrt{1+x^3}}\,dx$

12. $\int_{-\infty}^\infty (y^3 - 3y^2)\,dy$

13. $\int_{-\infty}^\infty xe^{-x^2}\,dx$

14. $\int_1^\infty \frac{e^{-1/x}}{x^2}\,dx$

15. $\int_0^\infty \sin^2\alpha\,d\alpha$

16. $\int_0^\infty \sin\theta\, e^{\cos\theta}\,d\theta$

17. $\int_1^\infty \frac{1}{x^2+x}\,dx$

18. $\int_2^\infty \frac{dv}{v^2+2v-3}$

19. $\int_{-\infty}^0 ze^{2z}\,dz$

20. $\int_2^\infty ye^{-3y}\,dy$

21. $\int_1^\infty \frac{\ln x}{x}\,dx$

22. $\int_1^\infty \frac{\ln x}{x^2}\,dx$

23. $\int_{-\infty}^0 \frac{z}{z^4+4}\,dz$

24. $\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$

25. $\int_0^\infty e^{-\sqrt{y}}\,dy$

26. $\int_1^\infty \frac{dx}{\sqrt{x}+x\sqrt{x}}$

27. $\int_0^1 \frac{1}{x}\,dx$

28. $\int_0^5 \frac{1}{\sqrt[3]{5-x}}\,dx$

29. $\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}}$

30. $\int_{-1}^2 \frac{x}{(x+1)^2}\,dx$

31. $\int_{-2}^3 \frac{1}{x^4}\,dx$

32. $\int_0^1 \frac{dx}{\sqrt{1-x^2}}$

Calculus III

Evaluate each limit.

7. $\lim_{(x,y)\to(\pi,\,\pi/2)} y\sin(x-y)$

8. $\lim_{(x,y)\to(3,2)} e^{\sqrt{2x-y}}$

9. $\lim_{(x,y)\to(0,0)} \frac{x^4-4y^2}{x^2+2y^2}$

10. $\lim_{(x,y)\to(0,0)} \frac{5y^4\cos^2 x}{x^4+y^4}$

11. $\lim_{(x,y)\to(0,0)} \frac{y^2\sin^2 x}{x^4+y^4}$

12. $\lim_{(x,y)\to(1,0)} \frac{xy-y}{(x-1)^2+y^2}$

13. $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$

14. $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{x^2+xy+y^2}$

15. $\lim_{(x,y)\to(0,0)} \frac{xy^2\cos y}{x^2+y^4}$

16. $\lim_{(x,y)\to(0,0)} \frac{xy^4}{x^4+y^4}$