Day 4.2: Separation of Variables

1) Solve the differential equation $\frac{dy}{dt} = -y^2$ given $y(0) = 0.5$

2) Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. Let $y = f(x)$ be the particular solution to this differential equation with the initial condition $f(-1) = 2$.

a) Write an equation for the line tangent to the graph of f at $x = -1$.

b) Find $y = f(x)$.

3) Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$. Let $y = f(x)$ be the particular solution to the differential equation with the initial condition $f(1) = -1$.

a) Write an equation for the line tangent to the graph of f at $(1, -1)$ and use it to approximate $f(1.1)$.

b) Find $y = f(x)$ and compute the actual value of $f(1.1)$.

4) Consider the differential equation $\frac{dy}{dx} = \frac{1}{2}x + y - 1$.

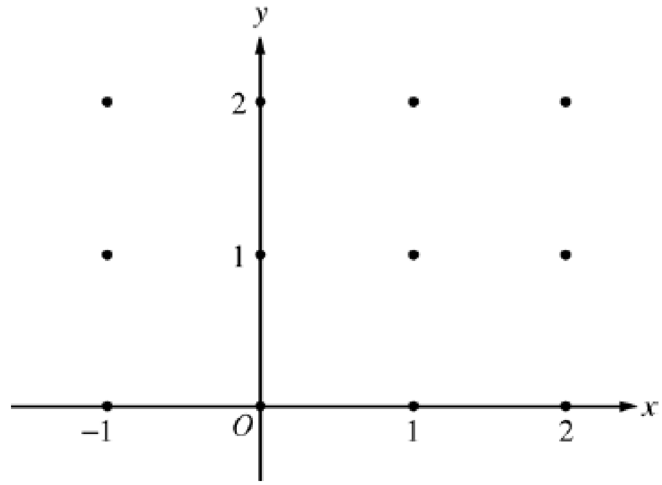
a) Find $\frac{d^2y}{dx^2}$ in terms of x and y. Describe the region in the xy-plane in which all solution curves to the differential equation are concave up.

b) Let $y = f(x)$ be a particular solution to the differential equation with the initial condition $f(0) = 1$. Does f have a relative minimum, a relative maximum, or neither at $x = 0$? *Justify your answer.*

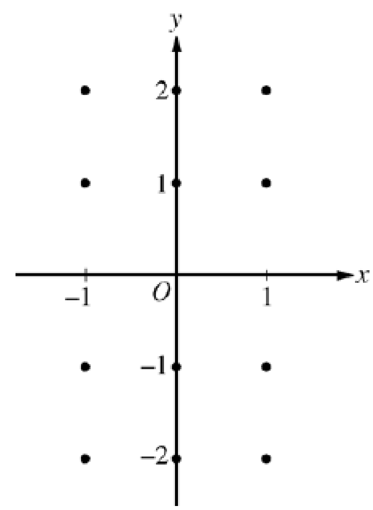
c) Find the values of the constants m, and b, for which $y = mx + b$ is a solution to the differential equation.

The Method of Slope Fields

5) Consider the differential equation $\frac{dy}{dx} = \frac{-xy^2}{2}$. On the axes provided, sketch a slope field for this differential equation at the twelve points indicated, and sketch the solution curve that passes through the point (-1,2)

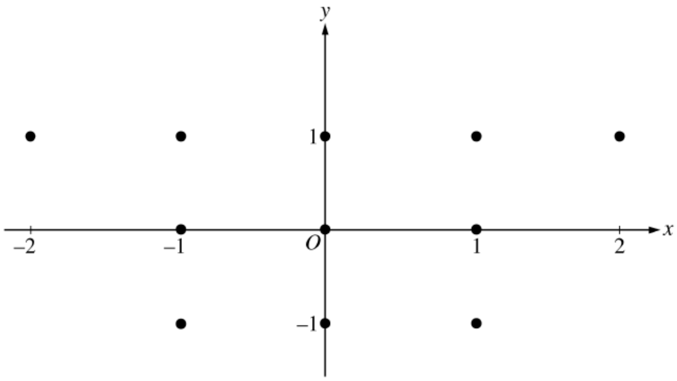


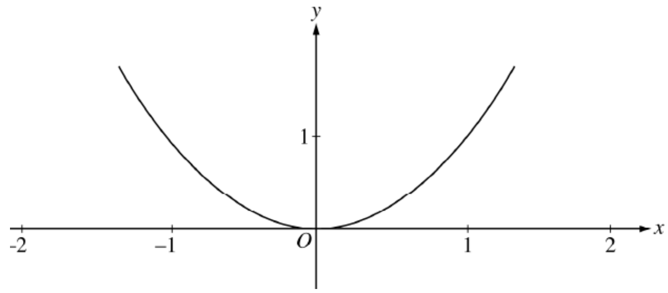
6) Consider the differential equation $\frac{dy}{dx} = -\frac{2x}{y}$. On the axes provided, sketch a slope field for this differential equation at the twelve points indicated, and sketch the solution curve that passes through the point (0, 1).



7) Consider the differential equation given by $\frac{dy}{dx} = x\left(y-1\right)^2$.

a) On the axes provided below at left, sketch a slope field for the given differential equation at the eleven points indicated.





b) Use the slope field for the given differential equation to explain why a solution could not have the graph shown above, at right.

c) Solve the given differential equation and graph the solution curve that passes through $(0, 0)$.

8) Consider the differential equation $\frac{dy}{dx} = \frac{3 - x}{y}$.

(a) Let $y = f(x)$ be the particular solution to the given differential equation for $1 < x < 5$ such that the line $y = -2$ is tangent to the graph of f. Find the x-coordinate of the point of tangency, and determine whether f has a local maximum, local minimum, or neither at this point. Justify your answer.

(b) Let $y = g(x)$ be the particular solution to the given differential equation for $-2 < x < 8$, with the initial condition $g(6) = -4$. Find $y = g(x)$.