1.

Solve the equation

$$7\operatorname{sech} x - \tanh x = 5$$

Give your answers in the form $\ln a$, where a is a rational number.

(5)

2.

a. Starting from the definitions of $\sinh x$ and $\cosh x$ in terms of exponentials, prove that

$$\cosh 2x = 1 + 2\sinh^2 x$$

(3)

b. Solve the equation

$$\cosh 2x - 3\sinh x = 15$$ giving your answers as exact logarithms.

(3)

3.

The curve C_1 has equation $y = 3\sinh 2x$, and the curve C_2 has equation $y = 13 - 3\mathrm{e}^{2x}$

a. Sketch the graph of the curves C_1 and C_2 on one set of axes, giving the equation of any asymptote and the coordinates of points where the curves cross the axes.

(4)

b. Solve the equation $3\sinh 2x = 13 - 3\mathrm{e}^{2x}$, giving your answer in the form $\frac{1}{2}\ln k$, where k is an integer.

(5)

4.

a. Sketch the graph of $y = \tanh x$

(2)

b. Given that $u = \tanh x$, use the definitions of $\sinh x$ and $\cosh x$ in terms of e^{x} and e^{-x} to show that

$$x = \frac{1}{2}\ln\left(\frac{1+u}{1-u}\right)$$

(6)

c.

i. Show that the equation

$$3\,\mathrm{sech}^2 x + 7\tanh x = 5$$

can be written as

$$3\tanh^2 x - 7\tanh x + 2 = 0$$

(2)

ii. Show that the equation

$$3\tanh^2 x - 7\tanh x + 2 = 0$$

has only one solution for *x*.

Find this solution in the form $\frac{1}{2}\ln a$, where *a* is an integer.

(5)

5.

a. Prove that the curve

$$y = 12\cosh x - 8\sinh x - x$$

has exactly one stationary point.

(7)

b. Given that the coordinates of this stationary point are (a, b), show that $a + b = 9$

(4)

6.

a. Using the definition $\sinh\theta = \frac{1}{2}\left(\mathrm{e}^{\theta} - \mathrm{e}^{-\theta}\right)$, prove that identity

$$4\sinh^3\theta + 3\sinh\theta = 3\sinh 3\theta$$

(3)

b. Given that $x = \sinh\theta$ and $16x^3 + 12x - 3 = 0$, find the value of θ in terms of a natural logarithm.

(4)

c. Hence find the real root of the equation $16x^3 + 12x - 3 = 0$, giving your answer in the form $2^{p} - 2^{q}$, where *p* and *q* are rational numbers.

(2)
