# DOR Project Part 2

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## I. Introduction and Approach

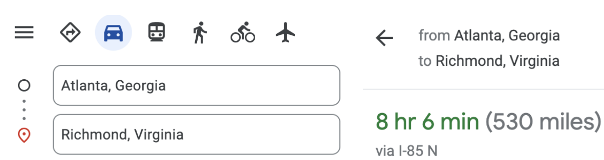
A contagious virus has spread across the country, but a vaccine has been developed by a company named *Phazer*. The vaccine now needed to be fairly distributed from three supplying cities in Atlanta, Richmond, and Albany to 17 different cities. The three cities had 1000, 500, and 500 supply trucks respectively with 1000 vaccines in each truck for a total supply of 2,000,000 vaccines. Cost of transportation is roughly $3.00 per mile for each truck factoring in all related expenses such as paying the driver, weight of the truck, etc..The vaccines expire after 14.5 hours, but can be repacked with coolant at Denver, Boise, Pierre, or Springfield for an additional fee of $3,000 per truck as well of having an additional 8 hour wait to repack. It is important to note, a city is considered vaccinated when half of the city's population is vaccinated. However, the total number of people that needs to be vaccinated to consider every target city vaccinated vastly outnumbers the number of vaccines available. The purpose of this report is to explore different factors that would affect the "fairness" of distribution. Our approach to finding the solution is to first isolate the shortest paths from each supplying city to the destination cities, and from that spanning tree create models to minimize cost and time. The benefits from minimizing each factor are discussed later in the paper. We will then assist *Phazer* make an educated decision on how to fairly distribute the vaccine through a final recommendation. When solving this dilemma, we have three main goals, which will be reflected in our objective models: **1. All trucks are utilized 2. All cities receive vaccines 3. The vaccines are distributed in a fashion we accept as fair**

## II. Assumptions Made For Our Research

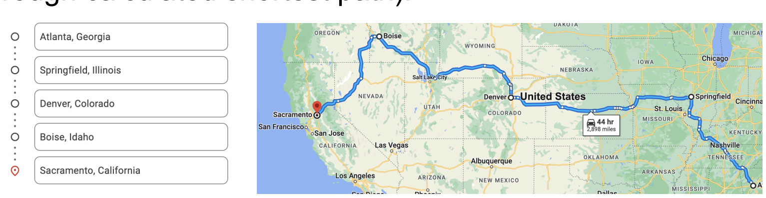
An important assumption we are making is that the supplying cities have the resources to repack coolant supply in a truck since they would have to initially pack coolant anyways. We are also assuming *Phaser* is distributing the vaccine and requires a recommendation instead of the government. The government is simply advising *Phazer* on what cities they believe are important to vaccinate. For this reason in the rest of the paper, we assume *Phazer* is the client we are researching for. The last assumption made was to assume that the data for population, time, and distance we found in February 2022 did not change throughout the process. More on data collection is discussed in the next section.

## III. Data Collection

The raw data used in the following paper for distances and time to travel from city to city were collected through Google Maps [1]. For example, the distance and time to travel from Atlanta to Richmond was found like so:




The time taken to travel through the shortest was taken the same way but with the repackaging centers added in between to ensure we got an accurate estimation of time taken to travel, like so (for Atlanta to Sacramento through calculated shortest path):



The population data for each city was searched on World Population Review and was the most recent data as we could find (February 2022) [2].

## IV. Dijkstra Algorithm: Shortest Path From Supply to Destination

To figure out the shortest path from the supplying cities to the destination cities, the first step we took was finding the spanning tree for the city connections. Each supplying and repackaging city was connected to every city within 14.5 hours (before it has to be repacked). For example, this is the data for all the cities under 14.5 hours from Atlanta:

**CITIES UNDER 14.5 HOURS FROM:**
**Atlanta**

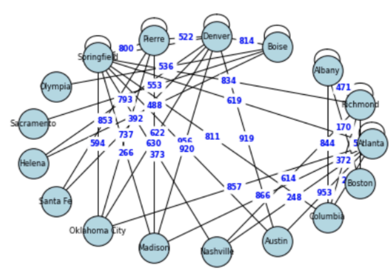
| **Destination** | **Distance** | **Time** |
|---|---|---|
| Atlanta | 0 | 0 |
| Oklahoma City | 857 | 12 hr 45 min |
| Madison | 866 | 12 hr 53 min |
| Springfield | 619 | 9 hr 53 min |
| Nashville | 248 | 3 hr 56 min |
| Austin | 953 | 14 hr 10 min |
| Columbia | 214 | 3 hr 10 min |
| Richmond | 530 | 8 hr 26 min |

The gathered data was then plotted through Python:

```
import networkx as nx

nodes = ['Atlanta','Richmond','Albany','Boise','Denver','Pierre', 'Springfield','Olympia','Sacramento','Helena'
         ,'Santa Fe','Oklahoma City','Madison','Nashville','Austin','Columbia','Boston']

G = nx.Graph()

for v in nodes:
    G.add_node(v)

G.add_edge('Atlanta','Atlanta',weight = 0)
G.add_edge('Atlanta','Oklahoma City',weight = 857)
G.add_edge('Atlanta','Madison',weight = 866)
G.add_edge('Atlanta','Springfield',weight = 619)
G.add_edge('Atlanta','Nashville',weight = 248)
G.add_edge('Atlanta','Austin',weight = 953)
G.add_edge('Atlanta','Columbia',weight = 214)
```

Doing this for every source city (supplying/repackaging city), we generated a spanning tree with distance as the weights of the arcs:

```
# Plot the graph
edge_labels = nx.get_edge_attributes(G, 'weight')

nx.draw(G,pos)
nx.draw_networkx_edge_labels(G,pos, edge_labels, font_size=1, rotate=False, font_color = 'b', font_weight = 'bold')
nodes = nx.draw_networkx_nodes(G, pos, node_size = 1000, node_color = 'lightblue')
nodes.set_edgecolor('black')
nx.draw_networkx_labels(G, pos,  font_size = 8);
```



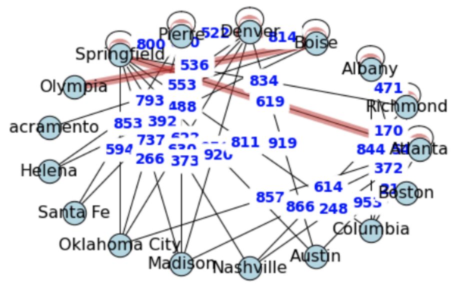
The generated spanning tree could now be solved with Dijkstra's algorithm to find the shortest path from supply to destination cities. Python was used to apply the algorithm, for example this is the python code to find the shortest path to Olympia from a supplying city:

```python
# compute the minimum spanning tree
path = nx.shortest_path_length(G,target='Olympia', weight = 'weight',
                               method='dijkstra')
path
```

```
{'Olympia': 0,
 'Boise': 536,
 'Helena': 1024,
 'Sacramento': 1089,
 'Denver': 1350,
 'Santa Fe': 1742,
 'Pierre': 1872,
 'Oklahoma City': 1972,
 'Springfield': 2200,
 'Austin': 2269,
 'Madison': 2306,
 'Nashville': 2573,
 'Atlanta': 2819,
 'Columbia': 3011,
 'Richmond': 3034,
 'Albany': 3505,
 'Boston': 3581}
```

```python
path = nx.shortest_path(G,source='Atlanta',target='Olympia', weight = 'weight',
                        method='dijkstra')
path
```

```
['Atlanta', 'Springfield', 'Denver', 'Boise', 'Olympia']
```




We repeated this process for all 17 of the destination cities for each supplying city and came to the following data. The cost is calculated by the miles x $3.00 and $3000 per repacking stop. The time is calculated by summing the time it takes to traverse the paths according to the data collected along with 8 additional hours per repacking stocks:

| Time Through Shortest Path (No Repackaging) (hrs) | Miles | Number of Stops | From | Destination | Cost For One Truck | Time For One Truck (hrs) | Shortest Path |
|---|---|---|---|---|---|---|---|
| 44 | 2836 | 3 | Atlanta | Sacramento | $ 17,508.00 | 68 | ['Atlanta', 'Springfield', 'Denver', 'Boise', 'Sacramento'] |
| 43 | 2819 | 3 | Atlanta | Olympia | $ 17,457.00 | 67 | ['Atlanta', 'Springfield', 'Denver', 'Boise', 'Olympia'] |
| 35 | 2283 | 2 | Atlanta | Boise | $ 12,849.00 | 51 | ['Atlanta', 'Springfield', 'Denver', 'Boise'] |
| 42 | 2262 | 3 | Atlanta | Helena | $ 15,786.00 | 66 | ['Atlanta', 'Springfield', 'Denver', 'Helena'] |
| 28 | 1861 | 2 | Atlanta | Santa Fe | $ 11,583.00 | 44 | ['Atlanta', 'Springfield', 'Denver', 'Santa Fe'] |
| 22.6 | 1469 | 1 | Atlanta | Denver | $ 7,407.00 | 30.6 | ['Atlanta', 'Springfield', 'Denver'] |
| 21.28 | 1419 | 1 | Atlanta | Pierre | $ 7,257.00 | 29.28 | ['Atlanta', 'Springfield', 'Pierre'] |
| 17.15 | 1133 | 2 | Atlanta | Boston | $ 9,399.00 | 33.15 | ['Atlanta', 'Columbia', 'Richmond', 'Boston'] |
| 15.73 | 1057 | 2 | Atlanta | Albany | $ 9,171.00 | 31.73 | ['Atlanta', 'Columbia', 'Richmond', 'Albany'] |
| 14.17 | 953 | 0 | Atlanta | Austin | $ 2,859.00 | 14.17 | ['Atlanta', 'Austin'] |
| 12.88 | 866 | 0 | Atlanta | Madison | $ 2,598.00 | 12.88 | ['Atlanta', 'Madison'] |
| 12.75 | 857 | 0 | Atlanta | Oklahoma City | $ 2,571.00 | 12.75 | ['Atlanta', 'Oklahoma City'] |
| 9.88 | 619 | 0 | Atlanta | Springfield | $ 1,857.00 | 9.88 | ['Atlanta', 'Springfield'] |
| 8.43 | 586 | 1 | Atlanta | Richmond | $ 4,758.00 | 16.43 | ['Atlanta', 'Columbia', 'Richmond'] |
| 3.93 | 248 | 0 | Atlanta | Nashville | $ 744.00 | 3.93 | ['Atlanta', 'Nashville'] |
| 3.16 | 214 | 0 | Atlanta | Columbia | $ 642.00 | 3.16 | ['Atlanta', 'Columbia'] |
| 0 | 0 | 0 | Atlanta | Atlanta | $ - | 0 | ['Atlanta'] |
| 47 | 3051 | 3 | Richmond | Sacramento | $ 18,153.00 | 71 | ['Richmond', 'Springfield', 'Denver', 'Boise', 'Sacramento'] |
| 47 | 3034 | 3 | Richmond | Olympia | $ 18,102.00 | 71 | ['Richmond', 'Springfield', 'Denver', 'Boise', 'Olympia'] |
| 38 | 2498 | 2 | Richmond | Boise | $ 13,494.00 | 54 | ['Richmond', 'Springfield', 'Denver', 'Boise'] |
| 38 | 2477 | 2 | Richmond | Helena | $ 13,431.00 | 54 | ['Richmond', 'Springfield', 'Denver', 'Helena'] |
| 32 | 2076 | 2 | Richmond | Santa Fe | $ 12,228.00 | 48 | [Richmond, 'Springfield', 'Denver', 'Santa Fe'] |
| 26 | 1684 | 1 | Richmond | Denver | $ 8,052.00 | 34 | ['Richmond', 'Springfield', 'Denver'] |
| 25 | 1634 | 1 | Richmond | Pierre | $ 7,902.00 | 33 | [Richmond, 'Springfield', 'Pierre'] |
| 22.83 | 1539 | 2 | Richmond | Austin | $ 10,617.00 | 38.83 | ['Richmond', 'Columbia', 'Atlanta', 'Austin'] |
| 21 | 1428 | 2 | Richmond | Oklahoma City | $ 10,284.00 | 37 | ['Richmond', 'Springfield', 'Oklahoma City'] |
| 20 | 1100 | 1 | Richmond | Madison | $ 6,300.00 | 28 | ['Richmond', 'Springfield', 'Madison'] |
| 12.15 | 834 | 0 | Richmond | Springfield | $ 2,502.00 | 12.15 | ['Richmond', 'Springfield'] |
| 9.23 | 614 | 0 | Richmond | Nashville | $ 1,842.00 | 9.23 | ['Richmond', 'Nashville'] |
| 22.03 | 586 | 1 | Richmond | Atlanta | $ 4,758.00 | 30.03 | ['Richmond', 'Columbia', 'Atlanta'] |
| 9.68 | 547 | 0 | Richmond | Boston | $ 1,641.00 | 9.68 | ['Richmond', 'Boston'] |
| 8.27 | 471 | 0 | Richmond | Albany | $ 1,413.00 | 8.27 | ['Richmond', 'Albany'] |
| 5.57 | 372 | 0 | Richmond | Columbia | $ 1,116.00 | 5.57 | ['Richmond', 'Columbia'] |
| 0 | 0 | 0 | Richmond | Richmond | $ - | 0 | ['Richmond'] |
| 54 | 3522 | 4 | Albany | Sacramento | $ 22,566.00 | 86 | ['Albany', 'Richmond', 'Springfield', 'Denver', 'Boise', 'Sacra |
| 54 | 3505 | 4 | Albany | Olympia | $ 22,515.00 | 86 | ['Albany', 'Richmond', 'Springfield', 'Denver', 'Boise', 'Olymp |
| 46 | 2969 | 3 | Albany | Boise | $ 17,907.00 | 70 | ['Albany', 'Richmond', 'Springfield', 'Denver', 'Boise'] |
| 45 | 2948 | 3 | Albany | Helena | $ 17,844.00 | 69 | ['Albany', 'Richmond', 'Springfield', 'Denver', 'Helena'] |
| 39 | 2547 | 3 | Albany | Santa Fe | $ 16,641.00 | 63 | ['Albany', 'Richmond', 'Springfield', 'Denver', 'Santa Fe'] |
| 34 | 2155 | 2 | Albany | Denver | $ 12,465.00 | 50 | ['Albany', 'Richmond', 'Springfield', 'Denver'] |
| 32 | 2105 | 2 | Albany | Pierre | $ 12,315.00 | 48 | ['Albany', 'Richmond', 'Springfield', 'Pierre'] |
| 30 | 2010 | 3 | Albany | Austin | $ 15,030.00 | 54 | ['Albany', 'Richmond', 'Columbia', 'Atlanta', 'Austin'] |
| 29 | 1899 | 2 | Albany | Oklahoma City | $ 11,697.00 | 45 | ['Albany', 'Richmond', 'Springfield', 'Oklahoma City'] |
| 24 | 1571 | 2 | Albany | Madison | $ 10,713.00 | 40 | ['Albany', 'Richmond', 'Springfield', 'Madison'] |
| 20.25 | 1305 | 1 | Albany | Springfield | $ 6,915.00 | 28.25 | ['Albany', 'Richmond', 'Springfield'] |
| 16.58 | 1085 | 1 | Albany | Nashville | $ 6,255.00 | 24.58 | ['Albany', 'Richmond', 'Nashville'] |
| 16.15 | 1057 | 2 | Albany | Atlanta | $ 9,171.00 | 32.15 | ['Albany', 'Richmond', 'Columbia', 'Atlanta'] |
| 12.85 | 843 | 1 | Albany | Columbia | $ 5,529.00 | 20.85 | ['Albany', 'Richmond', 'Columbia'] |
| 7.33 | 471 | 0 | Albany | Richmond | $ 1,413.00 | 7.33 | ['Albany, 'Richmond'] |
| 2.73 | 170 | 0 | Albany | Boston | $ 510.00 | 2.73 | ['Albany', 'Boston'] |
| 0 | 0 | 0 | Albany | Albany | $ - | 0 | ['Albany'] |

Although this process was not an LP, and was a data collection process instead, Dijkstra's algorithm still had similar characteristics (Sets/Parameters) as the following models.

**Sets:**

The sets for this model include the supplying cities the vaccines are coming from and the cities the vaccines need to arrive at as these cities represent the "nodes" in our model.

- Supplying Cities: {Atlanta, Richmond, Albany}
- Repackaging Cities: {Denver, Boise, Pierre, Springfield}
- Destination Cities: {Olympia, Sacramento, Helena, Boise, Austin, Oklahoma City, Springfield, Nashville, Atlanta, Santa Fe, Denver, Pierre, Madison, Richmond, Columbia, Boston, Albany}

**Parameter:**

The relevant parameters are the distances we calculated and collected from Google Maps data collection.

- Distance to travel from a city to another city

Again, this was not an optimization model, and simply a means for data collection. We define the sets and parameters here for more clarification, but since it was not an optimization model it does not have constraints. We will use the information we have gathered from Dijkstra's in the following two optimization model sections.

## V. Optimization Model For Cost

**Motivation:**

Minimizing cost is important as *Phaser* does not have an unlimited budget. We want to ensure our recommendation to *Phaser* has their interest in mind in terms of cost. It is worth noting minimizing total cost of operation is more beneficial for *Phaser* than it is for the people.

**Description:**

The model will be minimizing the total cost of the operation. It will do this by figuring out how many trucks should go to each city from each supplying city to minimize cost. The solver will make this decision through deciding which cities are the most cost efficient to send trucks to while taking into consideration a minimum population that each city needs to be vaccinated by. The minimum population stipulation is discussed more in the constraints section.

**Sets:**

The sets for this model include the supplying cities the vaccines are coming from and the cities the vaccines need to arrive at as these cities represent the "nodes" in our model. The solver will be solving for the number of trucks between these nodes to minimize the total cost of the operation.

- Supplying Cities: {Atlanta, Richmond, Albany}
- Repackaging Cities: {Denver, Boise, Pierre, Springfield}
- Destination Cities: {Olympia, Sacramento, Helena, Boise, Austin, Oklahoma City, Springfield, Nashville, Atlanta, Santa Fe, Denver, Pierre, Madison, Richmond, Columbia, Boston, Albany}

**Parameter:**

The relevant parameters are all the factors that relate to cost that was given from the problem along with any data we solved for including cost.

- Cost of one truck driving one mile is 3 dollars
- Cost of one truck repacking once is 3,000 dollars
- Trucks available from each supplying city (Atlanta has 1000 trucks, Albany has 500, Richmond has 500)
- $c_{ij}$: Cost of one truck traveling from supplying city to destination, calculated previously

**Variables:**

$X_{ij} = Quantity\ of\ trucks\ going\ from\ city\ i\ to\ j$

$i$: $\{Atlanta,\ Albany,\ Richmond\}$

$j$: $\{Olympia,\ Sacramento,\ Helena,\ Boise,\ Austin,\ Oklahoma\ City,\ Springfield,\ Nashville,$
$Atlanta,\ Santa\ Fe,\ Denver,\ Pierre,\ Madison,\ Richmond,\ Columbia,\ Boston,\ Albany\}$

**Relevant Code Snippet:**

```
var XAtlanta_Olympia >= 0;
var XAtlanta_Sacramento >= 0;
var XAtlanta_Helena >= 0;
var XAtlanta_Boise >= 0;
var XAtlanta_Austin >= 0;
```

**Objective:**

Conceptually, the objective is:

$$Min\, Z,\ Z\ =\ Sum\ of\ all:\ (Cost\ of\ one\ truck\ going\ from\ city\ i\ to\ j)\ X_{i,j}$$

Where Z is the total cost of the operation. With the numbers calculated from the shortest path section:

$$Min\, Z,\ Z\ =\ \Sigma c_{ij} X_{ij}$$

**Relevant Code Snippet:**

```
minimize z: 17457*XAtlanta_Olympia+17508*XAtlanta_Sacramento+15786*XAtlanta_Helena +...
```

**Constraints:**

The three constraints to consider are utilizing all the vaccines, supply and demand constraints.
We want to ensure the model uses all 2000 of the trucks available so we can distribute as many vaccines as possible. This is modeled with the following constraint:

**Utilizing all the vaccines:**

$$\Sigma X_{ij}\ =\ 2000$$

The model needs to be constrained by the supply of vaccine trucks available from each city and how many vaccine trucks each city demands. The supply is straightforward to constrain, each supplying city has either 1,000 or 500 available trucks.

**Atlanta has 1,000 available trucks:**

$$\Sigma X_{Atlanta,\, j}\ <\ 1000$$

**Relevant Code Snippet:**

```
subject to c11:XAtlanta_Olympia+XAtlanta_Sacramento+XAtlanta_Helena +... <1000
```

**Richmond has 500 available trucks:**

$$\Sigma X_{Richmond,\, j}\ <\ 500$$

**Albany has 500 available trucks:**

$$\Sigma X_{Albany,\, j}\ <\ 500$$

To constrain the model for the demand of each city, we decided to have every city receive enough vaccines to vaccinate at least 10% of the city. We knew we couldn't constrain the model to minimum 50% because there weren't enough trucks available. We chose a minimum of 10% of the population because we decided it was unfair for people in far cities to not receive any vaccines at all because of their location, which they have no control over. We also chose to constrain the demand to a **maximum** of 50% of the population per city. This is because the model would otherwise unfairly prioritize the closest cities automatically when the closest cities only require 50% of the population to be vaccinated before being considered vaccinated. To find the minimum amount of trucks each city needed to receive, we took the population of the city and then divided it by 10 to get the 10% of the population. Then, we divided the resulting population number by 1000 since each truck carries 1000 vaccines and rounded up to the nearest whole number since you cannot have a partial truck. We then did the same procedure to get 50% of the population. Here is a snippet of the calculations:

| City | Total Population | 10% of Population | Minimum Trucks Needed For 10% of Population | Rounded | 50% of Population | Minimum Trucks Needed For 50% of Population | Rounded |
|---|---|---|---|---|---|---|---|
| Olympia, Washington | 54004 | 5400.4 | 5.4004 | 6 | 27002 | 27.002 | 28 |
| Sacramento, California | 531285 | 53128.5 | 53.1285 | 54 | 265642.5 | 265.6425 | 266 |
| Helena, Montana | 28190 | 2819 | 2.819 | 3 | 14095 | 14.095 | 15 |
| Boise, Idaho | 230510 | 23051 | 23.051 | 24 | 115255 | 115.255 | 116 |
| Denver, Colorado | 760049 | 76004.9 | 76.0049 | 77 | 380024.5 | 380.0245 | 381 |
| Santa Fe, New Mexico | 86099 | 8609.9 | 8.6099 | 9 | 43049.5 | 43.0495 | 44 |
| Pierre, South Dakota | 13468 | 1346.8 | 1.3468 | 2 | 6734 | 6.734 | 7 |
| Oklahoma City, Oklahoma | 676492 | 67649.2 | 67.6492 | 68 | 338246 | 338.246 | 339 |
| Madison, Wisconsin | 265158 | 26515.8 | 26.5158 | 27 | 132579 | 132.579 | 133 |
| Springfield, Illinois | 112400 | 11240 | 11.24 | 12 | 56200 | 56.2 | 57 |
| Nashville, Tennessee | 682262 | 68226.2 | 68.2262 | 69 | 341131 | 341.131 | 342 |
| Atlanta, Georgia | 532695 | 53269.5 | 53.2695 | 54 | 266347.5 | 266.3475 | 267 |
| Austin, Texas | 961855 | 96185.5 | 96.1855 | 97 | 480927.5 | 480.9275 | 481 |
| Columbia, South Carolina | 136632 | 13663.2 | 13.6632 | 14 | 68316 | 68.316 | 69 |
| Richmond, Virginia | 226610 | 22661 | 22.661 | 23 | 113305 | 113.305 | 114 |
| Albany, New York | 97856 | 9785.6 | 9.7856 | 10 | 48928 | 48.928 | 49 |
| Boston, Massachusetts | 696959 | 69695.9 | 69.6959 | 70 | 348479.5 | 348.4795 | 349 |

### Each City Needs Trucks Delivered For 10% Of Population:

$X_{Atlanta, Olympia} + X_{Richmond, Olympia} + X_{Albany, Olympia} >= 6$
$X_{Atlanta, Sacramento} + X_{Richmond, Sacramento} + X_{Albany, Sacramento} >= 54$
$X_{Atlanta, Helena} + X_{Richmond, Helena} + X_{Albany, Helena} >= 3$
$X_{Atlanta, Boise} + X_{Richmond, Boise} + X_{Albany, Boise} >= 24$
$X_{Atlanta, Denver} + X_{Richmond, Denver} + X_{Albany, Denver} >= 77$
$X_{Atlanta, Santa Fe} + X_{Richmond, Santa Fe} + X_{Albany, Santa Fe} >= 9$
$X_{Atlanta, Pierre} + X_{Richmond, Pierre} + X_{Albany, Pierre} >= 2$
$X_{Atlanta, Oklahoma City} + X_{Richmond, Oklahoma City} + X_{Albany, Oklahoma City} >= 68$
$X_{Atlanta, Madison} + X_{Richmond, Madison} + X_{Albany, Madison} >= 27$
$X_{Atlanta, Springfield} + X_{Richmond, Springfield} + X_{Albany, Springfield} >= 12$
$X_{Atlanta, Nashville} + X_{Richmond, Nashville} + X_{Albany, Nashville} >= 69$
$X_{Atlanta, Atlanta} + X_{Richmond, Atlanta} + X_{Albany, Atlanta} >= 54$
$X_{Atlanta, Austin} + X_{Richmond, Austin} + X_{Albany, Austin} >= 97$
$X_{Atlanta, Columbia} + X_{Richmond, Columbia} + X_{Albany, Columbia} >= 14$
$X_{Atlanta, Richmond} + X_{Richmond, Richmond} + X_{Albany, Richmond} >= 23$
$X_{Atlanta, Albany} + X_{Richmond, Albany} + X_{Albany, Albany} >= 10$
$X_{Atlanta, Boston} + X_{Richmond, Boston} + X_{Albany, Boston} >= 70$

### Relevant Code Snippet:

```
subject to c14: XAtlanta_Olympia+XRichmond_Olympia+XAlbany_Olympia >= 6;
```

### Each City's Population Cannot Be More Than 50% Vaccinated:

$X_{Atlanta, Olympia} + X_{Richmond, Olympia} + X_{Albany, Olympia} <= 28$
$X_{Atlanta, Sacramento} + X_{Richmond, Sacramento} + X_{Albany, Sacramento} <= 266$
$X_{Atlanta, Helena} + X_{Richmond, Helena} + X_{Albany, Helena} <= 15$
$X_{Atlanta, Boise} + X_{Richmond, Boise} + X_{Albany, Boise} <= 116$
$X_{Atlanta, Denver} + X_{Richmond, Denver} + X_{Albany, Denver} <= 381$
$X_{Atlanta, Santa Fe} + X_{Richmond, Santa Fe} + X_{Albany, Santa Fe} <= 44$
$X_{Atlanta, Pierre} + X_{Richmond, Pierre} + X_{Albany, Pierre} <= 7$
$X_{Atlanta, Oklahoma City} + X_{Richmond, Oklahoma City} + X_{Albany, Oklahoma City} <= 339$
$X_{Atlanta, Madison} + X_{Richmond, Madison} + X_{Albany, Madison} <= 133$
$X_{Atlanta, Springfield} + X_{Richmond, Springfield} + X_{Albany, Springfield} <= 57$
$X_{Atlanta, Nashville} + X_{Richmond, Nashville} + X_{Albany, Nashville} <= 342$
$X_{Atlanta, Atlanta} + X_{Richmond, Atlanta} + X_{Albany, Atlanta} <= 267$
$X_{Atlanta, Austin} + X_{Richmond, Austin} + X_{Albany, Austin} <= 481$
$X_{Atlanta, Columbia} + X_{Richmond, Columbia} + X_{Albany, Columbia} <= 69$
$X_{Atlanta, Richmond} + X_{Richmond, Richmond} + X_{Albany, Richmond} <= 114$
$X_{Atlanta, Albany} + X_{Richmond, Albany} + X_{Albany, Albany} <= 49$
$X_{Atlanta, Boston} + X_{Richmond, Boston} + X_{Albany, Boston} <= 349$

### Relevant Code Snippet:

```
subject to c54: XAtlanta_Olympia+XRichmond_Olympia+XAlbany_Olympia <= 28;
```

### Output:

Using the GNU MathProg programming language and IDE with the full code demonstrated in the snippets above, the results were as follows:

| Objective | Minimize z |
|---|---|
| Optimal objective value | 4720548 |

| Variable | Value |
| --- | --- |
| XAlbany_Boston | 349 |
| XAtlanta_OklahomaCity | 339 |
| XAtlanta_Atlanta | 267 |
| XRichmond_Nashville | 187 |
| XAtlanta_Nashville | 155 |
| XAtlanta_Madison | 133 |
| XAtlanta_Austin | 106 |
| XAlbany_Richmond | 102 |
| XRichmond_Denver | 77 |
| XRichmond_Columbia | 69 |
| XRichmond_Springfield | 57 |
| XRichmond_Sacramento | 54 |
| XAlbany_Albany | 49 |
| XRichmond_Boise | 24 |
| XRichmond_Richmond | 12 |
| XRichmond_SantaFe | 9 |
| XRichmond_Helena | 3 |
| XRichmond_Pierre | 2 |
| XAtlanta_Olympia | 0 |
| XAtlanta_Sacramento | 0 |
| XAtlanta_Helena | 0 |

*Rest of the variables are 0.*

What these results mean is that to optimize truck distribution for cost, Atlanta should send 349 trucks to Austin, 339 trucks to Oklahoma City, 155 to Nashville, etc. The minimized total cost of the operation is $44,720,548.

## VI. Optimization Model For Time

**Motivation:**

Minimizing the total time of operation is important because the virus is contagious and getting people vaccinated as fast as possible will reduce further spreading of the virus. The faster the vaccine is distributed, the faster the vaccines are being used as well. Comparative to how minimizing cost benefits the distributor, minimizing for time benefits the people receiving and using the vaccines being delivered.

**Description:**

The model will be minimizing the total time of the operation. This model will be extremely similar to the previous model for cost, except the parameters will be related to time and the objective will have different coefficients accordingly.

**Sets:**

The sets for this model are the same as the previous model.The solver will be solving for the number of trucks between these nodes to minimize the total time of the operation this time.

- Supplying Cities: {Atlanta, Richmond, Albany}
- Repackaging Cities: {Denver, Boise, Pierre, Springfield}
- Destination Cities: {Olympia, Sacramento, Helena, Boise, Austin, Oklahoma City, Springfield, Nashville, Atlanta, Santa Fe, Denver, Pierre, Madison, Richmond, Columbia, Boston, Albany}

**Parameter:**

The relevant parameters are all the factors that relate to time that was given from the problem along with any data we solved for including time.

- Time taken by one truck to repacking once is 8 hours
- Trucks available from each supplying city (Atlanta has 1000 trucks, Albany has 500, Richmond has 500)
- $t_{ij}$: Time of one truck traveling from supplying city to destination, calculated in previous section

**Variables:**

$X_{ij} = Quantity\ of\ trucks\ going\ from\ city\ i\ to\ j$

$i$: $\{Atlanta,\ Albany,\ Richmond\}$

$j$: $\{Olympia,\ Sacramento,\ Helena,\ Boise,\ Austin,\ Oklahoma\ City,\ Springfield,\ Nashville,\ Atlanta,\ Santa\ Fe,\ Denver,\ Pierre,\ Madison,\ Richmond,\ Columbia,\ Boston,\ Albany\}$

**Objective:**

Conceptually, the objective is:

$Min\ Z,\ Z\ =\ Sum\ of\ all:\ \ (Time\ taken\ by\ one\ truck\ going\ from\ city\ i\ to\ j)\ X_{i,j}$

Where Z is the total time of the operation. With the numbers gathered from the same table as the previous model for $t_{ij}$:

$Min\ Z,\ Z\ =\ \Sigma t_{ij} X_{ij}$

**Constraints:**

The first two constraints for this model are the same as the previous model since the supply and vaccine truck amount are not different. However for the time model, we purposefully did not set a ceiling constraint on the demand. This is because unlike cost, since we are minimizing time in this model, it is OK for the model to prioritize the closest cities. We still set a floor constraint since as stated in our introduction, one of our goals was making sure every city received some amount of vaccination.

**Utilize All Available Trucks Constraint:**

$\Sigma X_{ij} = 2000$

**Supply Constraints:**

$\Sigma X_{Atlanta,\,j} < 1000$, $\Sigma X_{Richmond,\,j} < 500$, $\Sigma X_{Albany,\,j} < 500$

**Demand Constraints:**

$X_{Atlanta,\,j} + X_{Richmond,\,j} + X_{Albany,\,j} >= 10\%\ of\ Population\ of\ city\ j$

## Output:

| | |
|---|---|
| Objective | Minimize z |
| Optimal objective value | 11503.98 |

| Variable | Value |
|---|---|
| XAtlanta_Atlanta | 541 |
| XRichmond_Richmond | 497 |
| XAlbany_Albany | 430 |
| XAtlanta_Austin | 97 |
| XAtlanta_Denver | 77 |
| XAlbany_Boston | 70 |
| XAtlanta_Nashville | 69 |
| XAtlanta_OklahomaCity | 68 |

| | |
|---|---|
| XAtlanta_Sacramento | 54 |
| XAtlanta_Madison | 27 |
| XAtlanta_Boise | 24 |
| XAtlanta_Columbia | 14 |
| XAtlanta_Springfield | 12 |
| XAtlanta_SantaFe | 9 |
| XAtlanta_Olympia | 6 |
| XRichmond_Helena | 3 |
| XAtlanta_Pierre | 2 |
| XAtlanta_Helena | 0 |
| XAtlanta_Boston | 0 |
| XAtlanta_Albany | 0 |

*Rest of the variables are 0s*

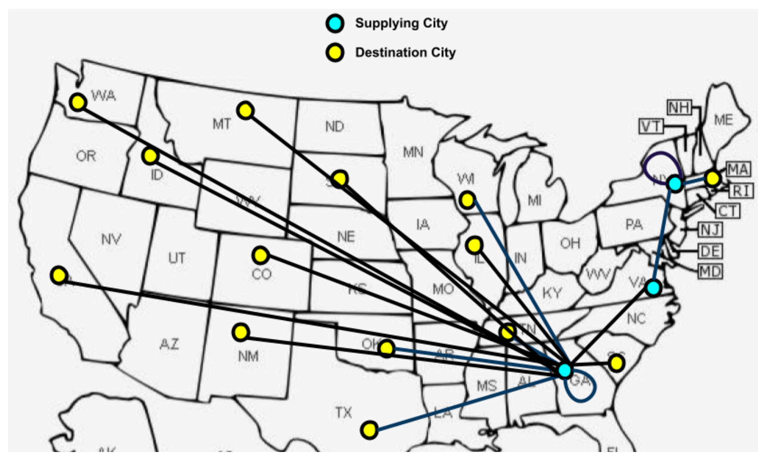
What these results mean is that to optimize truck distribution for time, Atlanta should supply 541 trucks to Atlanta, Richmond should supply 339 trucks to Richmond, etc. The minimized total time of operation is 11, 504 hours. An easier way to think about this is 11,504/2000 or one truck travels 5.75 hours on average.

## VII. Analysis/Pros & Cons Of Models And Their Results

We can visualize the results from these models in the following plots:

Optimal Truck Flow According to Cost Model

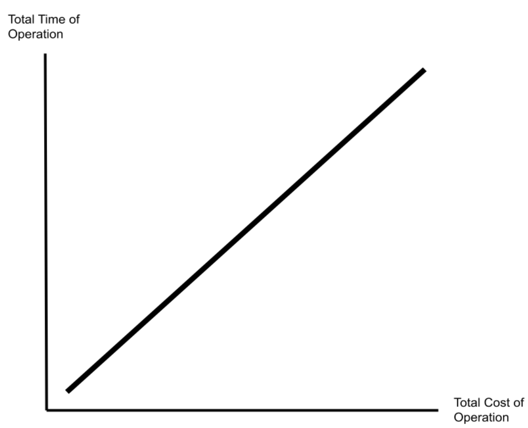

Optimal Truck Flow According to Time Model




Before analyzing the results of the models, we can already find some pros and cons for optimizing either factor that was touched on earlier in the paper. Optimizing for cost is advantageous for the distributor of the vaccine, in our case *Phazer.* Optimizing for time is advantageous for the people who need the vaccine and the population in general. The faster the vaccine is distributed, the faster the virus will be eliminated from society. Another factor to consider is, what are the maximum amount of resources available for each factor. If there is a limit to *Phazer*'s budget for this operation, cost would be more important to consider. The same if *Phazer* had a deadline to distribute the vaccines, time would become much more important to consider.

Now as we analyze the results from our models, we find more pros and cons for each model. As shown in the plots above and on the table on the right, there is a massive difference between what each model prioritized. A big downside that is noticeable about minimizing for time is that most of the vaccines are being distributed from Atlanta. Cost has a similar appearance, but in this case, most of the vaccines are being distributed from Richmond. In the cost plot on the left, we can see there are still plenty of cities being supplied from Atlanta and Boston. The reason diversification of distribution cities is important is for risk management. If the responsibility for the bulk of the operation falls under solely Atlanta, the operation becomes significantly more risky. Especially when this vaccine and its distribution is so new and untested, it would be unwise to rely so heavily on one city for the first ever wave of distribution. Another thing to note from the results is the difference in the range of the number of vaccines distributed to one city. More importantly, the **tradeoff** for optimizing each model. For example, in the time model, some cities receive shipments of 400-500 trucks whereas in the cost model, the truck distribution between cities is much more evenly distributed with most being between 50-350 trucks. As you optimize for time, we can see a compromise is made for how evenly the trucks are being distributed. Thinking about the situation with a multi-objective approach, we can conceptually come to a conclusion that the two factors are related to each other directly. **As total cost increases, total time should also increase. This is because they are both tied to distance**. Stopping to repackage has a penalty for both factors, so that should not affect the linear relationship *greatly*. Due to this, we can visualize the relationship as a positive linear function:

| X City to X City | # of Trucks, Cost Model | # of Trucks, Time Model |
|---|---|---|
| XAlbany_Boston | 349 | 70 |
| XAtlanta_OklahomaCity | 339 | 68 |
| XAtlanta_Atlanta | 267 | 541 |
| XRichmond_Nashville | 187 | 0 |
| XAtlanta_Nashville | 155 | 69 |
| XAtlanta_Madison | 133 | 27 |
| XAtlanta_Austin | 106 | 97 |
| XAlbany_Richmond | 102 | 497 |
| XRichmond_Denver | 77 | 0 |
| XRichmond_Columbia | 69 | 0 |
| XRichmond_Springfield | 57 | 0 |
| XRichmond_Sacramento | 54 | 0 |
| XAlbany_Albany | 49 | 430 |
| XRichmond_Boise | 24 | 0 |
| XRichmond_Richmond | 12 | 0 |
| XRichmond_SantaFe | 9 | 0 |
| XRichmond_Olympia | 6 | 0 |
| XRichmond_Helena | 3 | 3 |
| XRichmond_Pierre | 2 | 0 |
| XAtlanta_Denver | 0 | 77 |
| XAtlanta_Sacramento | 0 | 54 |
| XAtlanta_Madison | 0 | 24 |
| XAtlanta_Columbia | 0 | 14 |
| XAtlanta_Springfield | 0 | 12 |
| XAtlanta_SantaFe | 0 | 9 |
| XAtlanta_Olympia | 0 | 6 |
| XAtlanta_Pierre | 0 | 2 |
| | | |




## VIII. Final Recommendation

As a culmination of our research, subjective factors, objective data collected, and considering our original goals, we recommend:

***Phazer* should distribute the vaccines in line with the recommendation from our COST minimization model as shown on the right. (Albany sends 349 trucks to Boston, Atlanta sends 339 trucks to Oklahoma City, etc.)**

First, this satisfies our original goals of utilizing all trucks available, all cities receive some amount of vaccines, and it is a distribution deemed fair by us. We believe the distribution is fair because the number of vaccines being received by each city is fairly even. No city receives a significantly higher number of vaccines than another. We believe this is a right mix of being advantageous for the people and *Phazer* at the same time. *Phazer* benefits because the distribution process costs as little as possible for them which supports their bottom line. It also benefits *Phazer* because the vaccine is being distributed from a good mix of all three supplying cities, making the distribution less risk averse from an unexpected disaster. For the people, every target city is served, and in an even way as well.

Final Recommendation

| X City to X City | # of Trucks, Cost Model |
|---|---|
| XAlbany_Boston | 349 |
| XAtlanta_OklahomaCity | 339 |
| XAtlanta_Atlanta | 267 |
| XRichmond_Nashville | 187 |
| XAtlanta_Nashville | 155 |
| XAtlanta_Madison | 133 |
| XAtlanta_Austin | 106 |
| XAlbany_Richmond | 102 |
| XRichmond_Denver | 77 |
| XRichmond_Columbia | 69 |
| XRichmond_Springfield | 57 |
| XRichmond_Sacramento | 54 |
| XAlbany_Albany | 49 |
| XRichmond_Boise | 24 |
| XRichmond_Richmond | 12 |
| XRichmond_SantaFe | 9 |
| XRichmond_Olympia | 6 |
| XRichmond_Helena | 3 |
| XRichmond_Pierre | 2 |