IRAMBA DC SERIES 02 EXAMINATION
BASIC MATHEMATICS
MARKING GUIDE
SECTION A: (60 MARKS)

1. (a)The traffic lights at three different road crossing changes after every 48 seconds, 72 seconds and 108 seconds respectively. If they change simultaneously at 7a.m at what time they change simultaneously again?

Solution

- Required the LCM of 48, 72 and 108
-

2	48	72	108
2	24	36	54
2	12	18	27
2	6	9	27
3	3	9	27
3	1	3	9
3	1	1	3
	1	1	1

- The LCM $=2^4 \times 3^3 = 432$
- Change 432 seconds to hours
- $1min = 60seconds$
- $x = 432\ seconds$
- $x = 7.2\ minutes$
- $7am + 7.2minutes$
- $\therefore They\ will\ change\ together\ again\ at\ 07{:}\ 07am\ and\ 12\ seconds$ ………….. $03marks$

(b)If x = 0.567567567… and y = $0.8\dot{3}$ by converting these decimals to fractions, find the exact value of $\frac{xy^2}{y}$

Solutions

- Let $x = 0.\dot{5}\dot{6}\dot{7}$……………..(i)
- Multiply 1000 through out
- $1000x = 567.\dot{5}\dot{6}\dot{7}$…………….(ii)
- Subtract equation (ii) by (i)

- $1000x - x = 567.\dot{5}\dot{6}\dot{7} - 0.\dot{5}\dot{6}\dot{7}$
- $999x = 567$
- Divide by 999 througth out
- $\frac{999x}{999} = \frac{567}{999}$
- $x = \frac{567}{999} = \frac{21}{37}$
- Then from $y = 0.8\dot{3}$
- Multiply 10 through out
- $10y = 8.3\dot{3}$……………..(i)
- Subtract equation (ii) by (i)
- $10y - y = 8.3\dot{3} - 0.8\dot{3}$
- $9y = 7.5$
- Divide by 9 through out
- $\frac{9y}{9} = \frac{7.5}{9}$
- $y = \frac{75}{90} = \frac{5}{6}$
- $\frac{xy^2}{y} = \left(\frac{\frac{21}{37}\times\left(\frac{5}{6}\right)^2}{\frac{5}{6}}\right)$
- $\frac{xy^2}{y} = \frac{35}{74}$ …………………….............. $03marks$

2. (a) If $\left(\frac{1}{16}\right)^{n+3}\left(\frac{1}{32}\right)^{-5} = 1$ find the value of

Solution $\left(\frac{1}{16}\right)^{n+3}\left(\frac{1}{32}\right)^{-5} = 1$

- $\left(\frac{1}{2}\right)^{4(n+3)}\left(\frac{1}{2}\right)^{5(-5)} = 1$
- $\left(\frac{1}{2}\right)^{4n+12}\left(\frac{1}{2}\right)^{-25} = 1$ since$a^c \times a^b = a^{c+b}$
- $\left(\frac{1}{2}\right)^{4n+12-25} = \left(\frac{1}{2}\right)^0$
- $\left(\frac{1}{2}\right)^{4n-13} = \left(\frac{1}{2}\right)^0$
- *by comparing the exponents*
- $4n - 13 = 0$

- $4n = 13$
- Divide by 4 both sides
- $n = \frac{13}{4}$

- $\therefore The\ value\ of\ n\ is \frac{13}{4} = 3\frac{1}{4}$ …………. $02marks$

(b)Write "L" in terms of M, N and T from the formula $\frac{M}{N} = \frac{1}{2}\sqrt{\frac{L}{T-L}}$

Solution

- $\frac{M}{N} = \frac{1}{2}\sqrt{\frac{L}{T-L}}$
- Divide by $\frac{1}{2}$ both sides
- $\frac{2M}{N} = \sqrt{\frac{L}{T-L}}$
- Squaring both sides
- $\left(\frac{2M}{N}\right)^2 = \frac{L}{T-L}$
- $\frac{4M}{N^2} = \frac{L}{T-L}$
- Crosing multiplication
- $4M(T - L) = N^2(L)$
- $4MT - 4ML = N^2L$
- Colleting the like terms
- $N^2L + 4ML = 4MT$
- $L\left(N^2 + 4M\right) = 4MT$
- $L = \frac{4MT}{N^2+4M}$
- Therefore, the value is $L = \frac{4MT}{N^2+4M}$ …………. $02marks$

(c) Determine the value of x if $(x + 1) - 1 = (x - 3)$

Solution

- $(x + 1) - 1 = (x - 3)$
- From $(\frac{x}{y}) = x - y$
- $(x + 1) - (x - 3) = 1$
- $\frac{(x+1)}{(x-3)} = 5$

- $5(x - 3) = (x + 1)$
- $5x - 15 = x + 1$
- $4x = 16$
- Divide by 4 both sides
- $x = 4$
- $\therefore The\ value\ of\ x = 4$ …………. $2marks$

3. (a)Let μ be a universal set and A and B be the subsets of μ, if $\mu = \{a, b, c, d, e, f, g, h\}$
   A= {c, g, f} and B= {b, d, h} find
   i. $the\ number\ of\ subsets\ of\ set\ A$
      Solution

      - The number of subsets $= 2^{n}$, $where\ n\ is\ number\ of\ elements$
      - The number of subsets= $2^{3} = 8$

   - $\therefore There\ are$ 8 $subsets\ of\ set\ A$ …………. $02marks$

   ii. $n(A'\cap B)$

      - A= {c, g, f}, $A' = \{a,\ b,\ d,\ e,\ h\}$, B= {b, d, h}
      - $\left(A'\cap B\right) = \{b,\ d,\ h\}$

   - $\therefore n\left(A'\cap B\right) = 3$ …………. $02marks$

   (b)Find the probability that a king appears in drawing single card from an ordinary deck of 52 cards.

   Solution

   - There is KING of heart, spade, club and Diamond
   - Since n(s) = 52
        - n(E) = 4
      - The probability for king to appear = $\frac{n(E)}{n(s)} = \frac{4}{52}$
   - The probability for king to appear is $\frac{1}{13}$…………. $02marks$

4. (a)The coordinate of P, Q and Rare (2, m), (-3, 1) and (6, n) respectively. If the length PQ is $5\sqrt{2}$ units, and the mid-point of QR is $\left(\frac{3}{2}, -1\right)$. Find the possible values of m and n.

Solution

- Length of PQ=$\sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$
  - $= \sqrt{(-3-2)^2 + (1-m)^2}$
    - PQ=$\sqrt{-5^2 + (1-m)^2}$
    - $5\sqrt{2}$ =$\sqrt{25 + (1-m)^2}$
    - $\left(5\sqrt{2}\right)^2$ =$\left(\sqrt{25 + (1-m)^2}\right)^2$
    - $50 = 25 + (1-m)^2$
    - $50 - 25 = 1 - 2m + m^2$
    - $25 = 1 - 2m + m^2$
    - $m^2 - 2m - 24 = 0$
    - $m^2 + 4m - 6m + 24 = 0$
    - $m(m+4) - 6(m+4) = 0$
    - $(m-6)(m+4) = 0$
    - $m - 6 = 0 \text{ or } m + 4 = 0$
    - $\therefore m = 6 \text{ or } m = -4$
- Midpoint of QR
- $From\ m(x, y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$
    - $\left(\frac{3}{2}, -1\right)$=$\left(\frac{-3+6}{2}, \frac{1+n}{2}\right)$
    - $-1 = \frac{1+n}{2}$
    - $-2 = 1 + n$
  - $\therefore n = -3$ ………….. $03marks$

(b)Given vector $\underline{a} = \frac{1}{2}(6\underline{i} + 4\underline{j}), \underline{b} = 8i - 3\underline{j}\ and\ \underline{c} = 2(\underline{i} + 2\underline{j})$

i. The victor $\underline{d} = 3\underline{a} - \underline{b} + \frac{1}{2}\underline{c}$

Solution

- $d = 3\left(\frac{1}{2}(6\underline{i} + 4\underline{j})\right) - (8i - 3\underline{j}) + \frac{1}{2}(2(\underline{i} + 2\underline{j}))$
- $d = 9i + 6j - 8i + 3j + i + 2j$

- $d = 2i + 11j$ …………. $1.5\ marks$

ii. A unit vector in the direction of vector d

- **Solution**
- From unit vector
- $unit\ vector\ \underline{d} = \frac{d}{|\underline{d}|}$
- $d = \frac{2i+11j}{|2i+11j|}$
- $d = \frac{2i+11j}{\sqrt{125}}$

- $\therefore unit\ vector\ is \frac{2i}{\sqrt{125}} + \frac{11j}{\sqrt{125}}$ …………. $1.5 marks$

5. (a) Given $\frac{\overline{AB}}{\overline{KL}} = \frac{\overline{BT}}{\overline{LC}} = \frac{\overline{TA}}{\overline{CK}} = 3$ where $\overline{AB}, \overline{BT}\ and\ \overline{TA}$ are the sides of the triangle ABT and $\overline{KL}, \overline{LC}\ and\ \overline{CK}$ Are sides of the triangle KLC. What does this information imply?

Solution

- $\frac{\overline{AB}}{\overline{KL}} = \frac{\overline{BT}}{\overline{LC}} = \frac{\overline{TA}}{\overline{CK}} = 3$ (ratio of corresponding sides is equal)
- Them $\Delta ABT \sim \Delta KLM$(SSS similarity theorem)

- (Triangle ABT is similar to KLM) …………. $03 marks$

(b)A regular hexagon is inscribed in a circle. If the perimeter of the hexagon is 42 cm, find the radius of the circle and its area.

Solution

Let the sides of the hexagon be x

Perimeter = 6x

42cm = 6x, then x = 7cm and

$Area\ of\ the\ = \pi r^2$

22/7 x 7cm x 7cm = 154cm^2

- Radius = 7cm and Area of the circle is 154cm^2…………. $03 marks$

6. (a)Twelve people can dig a trench in 15 days for 8 hours daily. How long can they take to finish the same work, working for 10 hours daily?
Solution
From

- $w_1 h_1 d_1 = w_2 h_2 d_2$
- $12\times15\times8 = 12\times10\times d_2$
- $1440 = 120d_2$
- $d_2 = 12days$
- $it\ will\ take\ 12\ days\ to\ finish\ the\ work$ …………. $03marks$

(b)A variable V varies jointly as the variable A and h. when A=63 and h=4, v=84.
Find;
Solution

- $V \propto \frac{A}{h}$
- $V = \frac{kA}{h}$
- But $A = 63\ and\ h = 4,\ v = 84$
- $84 = \frac{k(63)}{4}$
- $\frac{336}{63} = k$
- $k = 5.3$ ………….. $01marks$

i. V when A=9 and h=7
- $V = k\frac{A}{h}$
- $V = 5.3\times\frac{9}{7}$
- $V = 6.857$………….. $01marks$

ii. A when V=4.5 and h=0.5
- Also,
- $V = \frac{kA}{h}$
- $4.5 = 5.3\times\frac{A}{0.5}$
- $4.5 = 10.6A$
- $A = 0.424$………….. $01marks$

7. (a)If a:b=2:3 and b:c=5:6. Find a:c and a:b:c

Solution

$$\frac{5}{3}\{a: b = 2: 3\ b: c = 5: 6$$

$$\{10: 15\ 15: 18$$

- $a: b: c = 10: 15: 18$ ………….. $0.5 marks$
- $a: c = 10: 18$ *or* $5: 6$ ………….. $05 marks$

(b) A hat is bought for Tsh 2,500/- and sold for Tsh 2,200/-. What is the percentage loss?

Solution

- Buying price =2,500
- Selling price =2,200
- Loss made = buying price-selling price t

$$\rightarrow 2500 - 2200 = 300$$

$$\rightarrow \%loss = \frac{loss\ made}{buying\ price} \times 100\%$$

$$= \frac{300}{2500} \times 100\%$$

$$= 12\%$$

$$\rightarrow therefore,\ the\ percentage\ loss\ was\ 12\%$$

(c)From the following information given by Mbeya Co. ltd for the year ended 31st December 2021.

Stock (01.01.2021) …………………………Three quarter of the closing stock
Stock (31.12.2021) ………………………… $\frac{2}{9}$ of net purchase
Net purchases during 2021 …………………. 432,000.
Gross margin ………………………… 15%
Expenses …………………………….. 20% of Net purchases
Calculate;

i. Cost of goods sold
   Solution
   - $Cost\ of\ goods\ sold = Cost\ of\ goods\ available\ for\ sale - closing\ stock$
   - $Cost\ of\ goods\ available\ for\ sale = opening\ stock + net\ purchases$
   - $= \frac{3}{4}\left(\frac{2}{9} \times 432,000.\right) + 432,000.$
   - $= 73000 + 432000$
   - $COGAS = 505,000$
   - $COGS = 505,000 - 96000$

- $COGS = 409,000$ ………….. $1.5 marks$

ii. Gross profit
   - $gross\ profit = Net\ sales - COGS$

- $Gross\ profit\ = \frac{15}{100} \times COGS$
- $Gross\ profit\ = \frac{15}{100} \times 409,000$

- $Gross\ profit\ = \ 6,1350$…………. $01marks$

iii. Net profit

- $Net\ profit\ = Gross\ profit\ -\ Expenses$
- $=\ 6135\ - \frac{20}{100} \times 432,000$
- $=\ 61,350\ -\ 86,400.$

- $Net\ profit\ =-\ 25050$…………. $01marks$

8. (a)If the 5th term of an arithmetic progression is 23 and the 12th term is 37, find the first term and the common difference.

- Given
- $A_5\ =\ 23,\ A_{12}\ =\ 37$
- Required to find A_1 $and\ Common\ difference\ =\ d$
- From
- $A_5 = A_1\ +\ 4d\ =\ 23$
- $A_{12} = A_1\ +\ 11d\ =\ 37$
- Solving the simultaneous equations
- $A_1\ +\ 4d\ =\ 23$……………(i)
- $A_1\ +\ 11d\ =\ 37$……………(ii)
- $Minus\ equation\ (i) and\ (ii)$
- $7d\ =\ 14$
- Divide by 7d through out
- $d\ =\ 2$
- Then substute equation $d\ =\ 2\ into\ equation$ (i)
- $A_1\ +\ 4d\ =\ 23$
- $A_1\ +\ 4(2) =\ 23$
- $A_1\ +\ 8\ =\ 23$

- $A_1 = 15$
- $\therefore The\ first\ term\ is\ 15\ and\ the\ common\ difference\ is\ 2$ ………….. $03marks$

(b) In how many years would one double one's investment if Tshs 2500 is invested at 8% compounded semi –annually.

- **Solution**
- $Pricipal\ = 2500/=$
- Compounded semi-annually
- Required number of years when the principal double itself
- From
- $A_n = p\left(1 + \frac{RT}{100}\right)^{nt}$
- $2(2500) = 2500\left(1 + \frac{8\times\frac{1}{2}}{100}\right)^{\frac{1}{2}n}$
- $2 = \left(1 + \frac{1}{25}\right)^{\frac{1}{2}n}$
- $2 = (1.25)^{\frac{1}{2}n}$ $hint$: $Apply$ log base 10 throughout
- $\frac{log(1.25)}{log2} = \frac{1}{2}n$
- $\frac{0.097}{0.3010} = \frac{1}{2}n$
- $0.3222 = \frac{1}{2}n$
- $0.644 = n$
- $n \approx 1\ year$
- Therefore 2500 will take 1 year to double itself at interest of 8% ………….. $03marks$

9. (a) Without the use of mathematical tables and calculators, find the value of

   (i) $cos75^0$

   - **Solution**
   - $\cos cos\ 75° = \cos cos\ (45^0 + 30^0)$
   - $from\ \cos cos\ (A + B) = \cos cos\ A\ \cos cos\ B - \sin sin\ A\ \sin sin\ B$
   - $\cos cos\ (45^0 + 30^0) = \cos cos\ 45°\ \cos cos\ 30° - \sin sin\ 45°\ sin30^0$

$= \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \times \frac{1}{2}$

$\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$\cos cos\,(45^0 + 30^0) = \frac{\sqrt{6}-\sqrt{2}}{4}$ .... $1.5marks$

(ii) $\sin sin\, 27°\ \cos cos\, 18°\ + \cos cos\, 27°\ \sin sin\, 18°$.

Solution

$18°\ + \cos cos\, 27°\ \sin sin\, 18° = sin45^0$.

$sin45^0 = \frac{\sqrt{2}}{2}$ .... $1.5marks$

(b) A and B are two points on the ground level and both lie West of flagstaff. The angles of elevation of the top of the flagstaff from A are 56° and from B is 43°. If B is 28m from the foot of the flagstaff. How far apart are the point A and B?

Solution

- (Diagram is important)
- $Tan\ \theta = \frac{Opposite}{Adjacent}$
  - $Tan\ 43^0 = \frac{OP}{OB} = \frac{OP}{28}$
    - $OP = 28Tan\ 430$ ....(i)
  - $Also\ Tan\ 56^0 = \frac{OP}{OA}$
  - $OP = OA\ Tan\ 56^0$ ....(ii)
- $Since\ eqn\ (i) = (ii)\ Then\ we\ get$
    - $OA\ Tan\ 56^0 = 28Tan\ 430$
    - $OA = \frac{28Tan\ 43^0}{Tan\ 56^0} = \frac{28\ X\ 0.9325}{1.483}$
    - $OA = 17.6\ m$
    - $Thus,\ BO = AB + OA = X + 17.6$
    - $28 = X + 17.6$
    - $x = 28 - 17.6 = 10.4$ m

$\therefore Point\ A\ to\ B\ is\ 10.4\ m$ .... $03marks$

10. (a) When the length of a rectangle whose perimeter is 90cm is increased by 20% and its width is decreased by 20% the area decreases by $20cm^2$. Find the dimensions of the rectangle.

Solution

⇨ Perimeter of the rectangle is given by: $P = 2(l + w)$

⇨ $2(l + w) = 90cm$

- ⇨ $l + w = 45cm$............ (i)
- ⇨ $20\%\uparrow on\ length = 1.2l$

------------------01mark

- ⇨ $20\%\downarrow on\ width = 0.8w$
- ⇨ $Area\ to\ decrease\ by\ 20 \rightarrow A - 20$
- ⇨ $\rightarrow 1.2l \times 0.8w = lw - 20$
- ⇨ $0.96lw = lw - 20$
- ⇨ $lw - 0.96lw = 20$

------------------01mark

- ⇨ $0.04lw = 20$
- ⇨ but from eqn (i) $l = 45 - w$
- ⇨ $0.04w(45 - w) = 20$
- ⇨ $0.04w^2 - 1.8w + 20 = 0$ (solving by any relevant method)

------------------01mark

- ⇨ $w = 20\ or\ w = 25$
- ⇨ $\therefore The\ dimensions\ of\ the\ rectangle\ are\ Length = 25cm\ and\ Width = 20cm$

(b)Solve for x and y if $\{x^2 - y^2 = 12\ x + y = 6$

Solution

- $\{x^2 - y^2 = 12\ x + y = 6$
- Then $\{x^2 - y^2 = 12$..........................(i) $x + y = 6$.............................. (ii)
- Consider equation (i) and (ii)
- $x + y = 6$

- $x = 6 - y$
- $Substute\ eqution\ (iii) into\ equation\ (i)$

- $x^2 - y^2 = 12$
- $(6 - y)^2 - y^2 = 12$
- $36 - 12y + y^2 - y^2 = 12$
- $12y = 24$
- $y = 2$
- Then
- $x = 6 - y$
- $x = 6 - 2$
- $x = 4$
- Therefore $x = 4\ and\ y = 2$ ………… $03marks$

SECTION B (40 MARKS)

11. (a)The marks in basic Mathematics terminal Examination obtained by 40 students in one of the secondary schools in Katavi were as follows;

60	54	48	43	37	61	43	66	65	52
81	70	48	63	74	67	52	48	37	48
43	52	52	22	27	37	38	29	19	37
42	47	36	52	50	28	44	32	40	42

i. Prepare a frequency distribution table with class intervals 10 – 19, 20 – 29, etc.

FREQUENCY DISTRIBUTION TABLE

Class interval (C.I)	Frequency (f)	Class mark (x)	fx	$C.F$
10-19	01	14.5	14.5	1
20-19	04	24.5	98	5
30-39	07	34.5	241.5	12
40-49	12	44.5	534	24
50-59	07	54.5	381.5	31
60-69	06	64.5	387	37
70-79	02	74.5	149	39
80-89	01	84.5	84.5	40

	$\sum f$ =40		$\sum fx = 1890$	

- ………….. $02 marks$

ii. Find the class which contain the median

- Median class is obtained by taking $\frac{N}{2} = 20,$ where is $40 - 49$ ………….. $01 marks$

iii. Find the mean

- From $mean = \sum \frac{fx}{f}$

- $mean = \frac{1890}{40} = 47.25$ ………….. $02 marks$

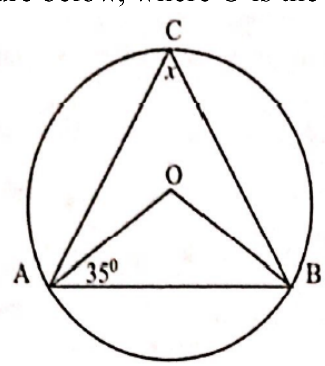
iv. Calculate the median.

- From $median = l + (\frac{\frac{n}{2} - fb}{fw})c$
- Where $l = 39.5$, $N = 20$, $Fb = 12$, $Fw = 12,$ $c = 10$

- $median = 39.5 + (\frac{20-12}{12})10$
- $39.5 + 6.667$
- $= 46.167$

- $Median = 46.167$ ………….. $02 marks$

(b) Find the value of angle x in the figure below, where O is the center of the circle



Solution

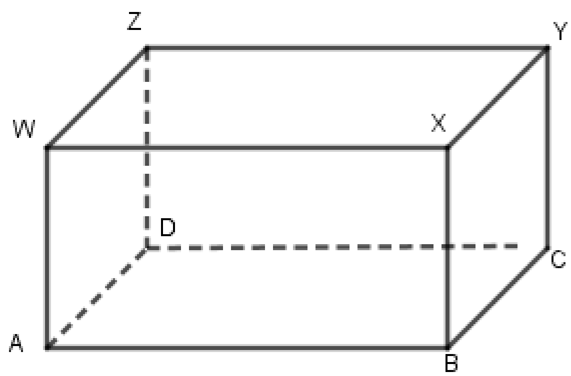
- $A\hat{O}B + A\hat{B}O + B\hat{A}O = 180^0$(Sum of degrees in triangle)
- But $B\hat{A}O = B\hat{A}O$ Since $OA = OB$ (Radii of the circle or $\Delta A\hat{O}B$ is isosceles triangle)
- Then
- $A\hat{O}B + 35^0 + 35^0 = 180^0$

- $A\hat{O}B + 70^0 = 180^0$
- $A\hat{O}B = 180^0 - 70^0$
- $A\hat{O}B = 110^0$
- To get x, $A\hat{C}B = \frac{1}{2}(A\hat{O}B)$ (Angle at the center of the circle is twice the angle at the circumference)
- $A\hat{C}B = \frac{1}{2}(110^0)$
- $A\hat{C}B = 55^0$ ………….. $03marks$

12. (a)A Rectangular box with top WXYZ and base ABCD has AB=9 cm, BC=12cm and WA=3cm

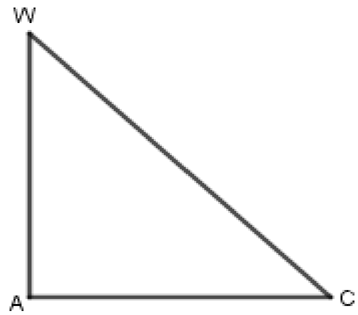
Calculate

(i) Consider a right-angled triangle ABC



- $(\overline{AB})^2 + (\overline{BC})^2 = (\overline{AC})^2$
- $(\overline{9})^2 + (\overline{12})^2 = (\overline{AC})^2$
- $AC = \sqrt{225} = 15cm$
- $AC = 15cm$ ………….. $03marks$

(ii) Consider the triangle ACW with angle WAC = 90^o



- $Tan\,\theta = \frac{Opposite}{adjacent} = \frac{3}{15}$
- $= Tan^{-1}\left(\frac{3}{15}\right) = \theta$
- $\theta = Tan^{-1}(1/5) = 11.3^{o}$

 $\therefore Angle\ between\ WC\ and\ AC\ is\ \emptyset = 11.3^{0}$ ………….. $03 marks$

(b)Two places P and Q both on the parallel of latitude 26°N differ in longitudes by 40°, find the distance between them along their parallel of latitude.

Solution

(Diagram is important)

 Distance from P to Q = $\frac{\pi R\theta cos\alpha}{180^{0}}$

 But $\theta = 40,\ R = 6370km,\ \alpha = 26,\ \pi = 3.14$

 $= \frac{40^{0}\ x\ 3.14\ x\ 6370\ xcoscos\ 26}{180^{0}}$

 $Distance\ from\ P\ to\ Q\ is\ 3995km$ ………….. $04 marks$

13. (a)If matrix A is singular, what will be the value of y given that $(3\ y\ -\ 1\ y\ +\ 1\ 1\ )$

Solution

 $(3\ y\ -\ 1\ y\ +\ 1\ 1\ )$

 For a singular matrix $|A| = 0$

 $|3\ y\ -\ 1\ y\ +\ 1\ 1\ | = 0$

 $3 - ((y + 1)(y - 1)) = 0$

 $3 - \left(y^{2} - y + y - 1\right) = 0$

 $3 - y^{2} + 1 = 0$

 $y^{2} = 4$

 $y = \mp 2$

- $The\ value\ of\ y\ is\ \mp 2$ ………….. $03marks$

(b)Solve the following simultaneous equation by inverse matrix method: $\{2x + y = 7\ 4x + 3y = 17$

Solution

- $\{2x + y = 7\ 4x + 3y = 17$

  In matrix form

- $(2\ 1\ 4\ 3)\left(\frac{x}{y}\right) = \left(\frac{7}{17}\right)$
- **Let matrix** $A = (2\ 1\ 4\ 3)$
- $|A| = |2\ 1\ 4\ 3|$
- $|A| = 6 - 4 = 2$
- $A^{-1} = \frac{1}{|A|}\left(A^T\right)$
- $A^{-1} = \frac{1}{2}(3\ -1\ -4\ 2)$
- $A^{-1} = \left(\frac{3}{2}\ -\frac{1}{2}\ -\frac{4}{2}\ \frac{2}{2}\right)$
- $A^{-1} = \left(\frac{3}{2}\ -\frac{1}{2}\ -2\ 1\right)$
- $\left(\frac{3}{2}\ -\frac{1}{2}\ -2\ 1\right)(2\ 1\ 4\ 3)\left(\frac{x}{y}\right) = \left(\frac{3}{2}\ -\frac{1}{2}\ -2\ 1\right)\left(\frac{7}{17}\right)$
- $(1\ 0\ 0\ 1)\left(\frac{x}{y}\right) = \left(\frac{\frac{3}{2} - \frac{17}{2}}{-14+17}\right)$
- $\left(\frac{x}{y}\right) = \left(\frac{2}{3}\right)$
- $\therefore \left(\frac{x}{y}\right) = \left(\frac{2}{3}\right)$ ………….. $04marks$

(c)Find the image of (3, 5) after rotation of 270° about the origin in the anti-clockwise direction.

Solution

- Given $\theta = 270^0$
  - For anticlockwise direction
  - Matrix is $(\cos cos\ \theta\ - \sin sin\ \theta\ \ \sin sin\ \theta\ \ \cos cos\ \theta\ )$
  - $(\cos cos\ 270\ \ 270\ \ \sin sin\ 270\ \ \cos cos\ 270\ )$
  - $(0\ 1\ -1\ 0)$
  - $(x\ y) = (\cos cos\ \theta\ - \sin sin\ \theta\ \ \sin sin\ \theta\ \ \cos cos\ \theta\ )(x\ y)$

    $\left(x^1\ y^1\right) = (\cos cos\ 270\ \ 270\ \ \sin sin\ 270\ \ \cos cos\ 270\ )(3\ 5)$
  - $\left(x'\ y'\right) = (\cos cos\ 270\ \ 270\ \ \sin sin\ 270\ \ \cos cos\ 270\ )(3\ 5)$

- $(x\ y) = \ (0\ 1\ \ -1\ 0)(3\ 5)$
- $(x\ y) = (5\ \ -3)$

 $\therefore The\ image\ after\ rotation\ of\ 270^0$ is $(5, -\ 3)$ .... $03 marks$

14. (a) Given the function $f(x) \ = \ \frac{1}{x-2}$, find,

i. The domain and range.

ii. $f^{-1}\left(\frac{1}{3}\right)$

Solution

$$y \ = \frac{1}{x-2}$$

i) Domain = $\{x: all\ real\ number\ numbers,\ except\ x\ =\ 2\}$
Range = $\{all\ real\ numbers,\ except\ y\ =\ 0\}$

ii) $y \ = \frac{1}{x-2}$

$$x \ = \frac{1}{y-2}$$

$$xy \ - \ 2x \ = \ 1$$

$$y \ = \frac{1+2x}{x}$$

$$f^{-1}(x) = \frac{1+2x}{x}$$

$$f^{-1}\left(\frac{1}{3}\right) = \frac{1+2\left(\frac{1}{3}\right)}{\left(\frac{1}{3}\right)}$$

$$= 1\frac{2}{3} \div \frac{1}{3}$$

$$f^{-1}\left(\frac{1}{3}\right) = 5$$

(b) Mr Akilimali has 120 shillings to spend on exercise books. At the school shop an exercise book costs 8 shillings and at stationery store an exercise book costs 12 shillings. The school shop has only 6 exercise books left and the student wants to obtain the greatest number of exercise books possible using the money he has. Find the greatest number of exercise books he can buy.

Solution

Let x be the number of exercise books bought at the school shop
y be the number of exercise books at the stationary store

inequalities

- $x \geq 6$
- $8x + 12y \leq 120$
- $x \geq 0$ and $y \geq 0$

- **Objective function**

- $f(x, y) = x + y$

Table of value of $8x + 12y = 120$

x	0	15
y	10	0

THE GRAPH OF LINEAR PROGRAMMING

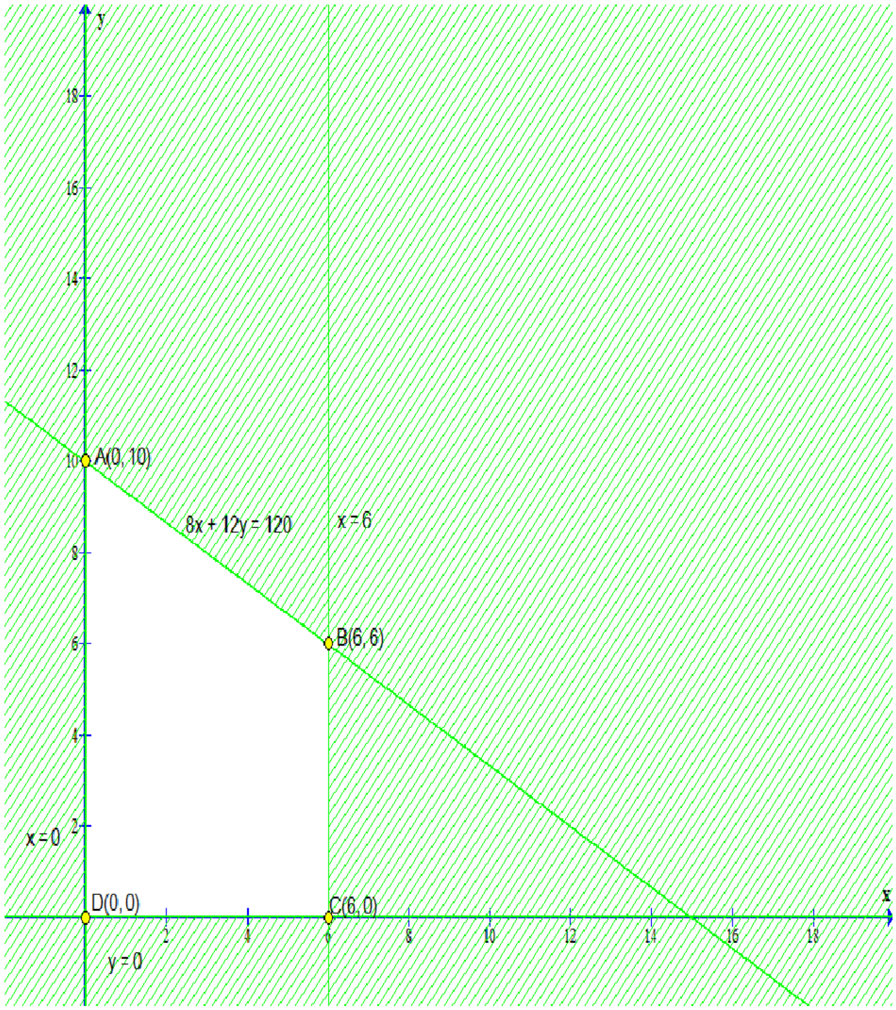


Table of solution for $f(x,\ y) = x\ +\ y$

Corner points	$f(x, y) = x\ +\ y$	Solutions
A (0, 10)	$0\ +\ 10\ =\ 10$	10
B (6, 6)	$6\ +\ 6\ =\ 12$	12
C (6, 0)	$6\ +\ 0\ =\ 6$	6
D (0, 0)	$0\ +\ 0\ =\ 0$	0

- The points $B(6,\ 6)$ offers the best solution to the problem; that is Mr.Mapunda buys 6 exercise books at the school shop and 6 at the stationery store ………….. $06 marks$