14. Transformation of charge+current 4-vector field derived from Electromagnetic 8-vector field F in Cl3,0(R)

(Proof that $\rho+\mathbf{J} = \nabla^X \mathbf{F}$ transforms in $Cl_{3,0}(R)$ like 4-vectors using hermitian conjugate formula, for the Nabla operator defined as: $\nabla^{\mathbf{x}} := \partial/\partial t - (\partial/\partial x)\mathbf{g} - (\partial/\partial y)\mathbf{e} - (\partial/\partial z)\mathbf{f}$)

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Summary

This article presents a derivation of the transformation of the electromagnetic charge and current density vector $\mathcal{J} = \rho+\mathbf{J}$, where $\mathcal{J} = \nabla^{\mathbf{x}}\mathbf{F}$ [note [1]] and the electromagnetic Clifford vector field **F** (in $Cl_{3,0}(R)$) is assumed to transform under length preserving coordinate change as $\mathbf{F_2} = \exp(\mathbf{p})*\mathbf{F}*(\exp(\mathbf{p}))'$, [note[2]] . It is also assumed that the 4-vectors such as coordinates $\mathbf{q}=t+\mathbf{g}x+\mathbf{e}y+\mathbf{f}z$ and $\mathcal{J}$ must transform as $\mathcal{J}_2 = \exp(\mathbf{p})*\mathcal{J}*\exp(\mathbf{p})^T$ [note[3]].

Introduction

Clifford Algebraic electromagnetic equation has a form of a 4-dimensional (t,x,y,z) differential operator nabla [4] applied to the complex electromagnetic field 4-vector $\mathbf{F} = -S - \mathbf{E} - i\,\mathbf{B}$ [see[5]] , that must equal zero. This can be also expressed as

$$\mathbf{F} = \nabla^{|}\mathbf{a} \quad \text{(eq.1, see [4])}$$

where **a** is defined as $\mathbf{a} = \phi+\mathbf{A}$, where ϕ is electric potential and **A** is magnetic vector potential.

The Nabla operator that applies to 4-vector potential field **a** is defined in [4] as[6]:

$$\nabla^{|} = \partial/\partial t + (\partial/\partial x)\,\mathbf{g} + (\partial/\partial y)\,\mathbf{e} + (\partial/\partial z)\,\mathbf{f}$$

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[1] $\nabla^{\mathbf{x}} := \partial/\partial t - (\partial/\partial x)\mathbf{g} - (\partial/\partial y)\mathbf{e} - (\partial/\partial z)\mathbf{f}$ [this form is assumed, to prove that it leads to correct transform for $\mathcal{J} = \nabla^{\mathbf{x}}\mathbf{F}$

[2] ()' is quaternion conjugate (example **e'=-e** and (**ef**)'=**f'e'**) We restrict **p** to be a linear combination of base vectors {**g,e,f,i,j,k**} only.

[3] $()^T$ is hermitian conjugate (example $\mathbf{e}^T=\mathbf{e}$ and $(\mathbf{ef})^T=\mathbf{f}^T\mathbf{e}^T$)

[4] $\nabla = \partial/\partial t - \mathbf{g}\,\partial/\partial x - \mathbf{e}\,\partial/\partial y - \mathbf{f}\,\partial/\partial z$, $\partial/\partial t + \mathbf{g}\,\partial/\partial x + \mathbf{e}\,\partial/\partial y + \mathbf{f}\,\partial/\partial z$

[5] **F** = generalised EM field tensor = $-S - \mathbf{E} - i\,\mathbf{B} = -S - E_x\mathbf{g} - E_y\mathbf{e} - E_z\mathbf{f} - i\,(B_x\mathbf{g} + B_y\mathbf{e} + B_z\mathbf{f})$. The general full form is $\mathbf{F} = -S - \mathbf{E} - i\,\mathbf{B} + iM$, however, the imaginary scalar component M of the Clifford EM field vector **F** must be zero or constant in order to cancel the unobservable magnetic monopole charge density and magnetic monopole current density. With M=0, equation $\nabla\mathbf{F} = 0$ corresponds to the well known Maxwell equation, if we define S as $\rho + \mathbf{J} = \nabla S$.

[6] Partial derivative operator applies first, then a base vector is right-multiplied to the result. The formula can be better understood if the beses g,e,f are put in the denominators ar right-divisions, that is:
$\nabla^{|} = \partial/\partial t + (\partial/\partial(x\mathbf{g})) + (\partial/\partial(y\mathbf{e})) + (\partial/\partial(z\mathbf{f}))$

Derivation of transform of $\mathcal{J}$

This assumes that $\mathcal{J}$ is a derivative of some Clifford field **F** that transforms using a formula containing quaternion conjugation). The derivation will be split in two cases: (1) Lorentz transform and (2) 3D Rotation.

1) Lorentz transform of $\mathcal{J}$

Let us take an arbitrary Lorentz transform generator vector **u** that is a linear combination of base vectors **e,f,g** . For the sake of consistency with the previously referenced articles [1] etc, it is assumed that base vector **g** represents axis x, **e** is axis y and **f** is axis z.

Without losing the generality of the proof, we can assume that a general generator vector **u** (in {**g,e,f**}) is aligned with **g** (that is there is some real U that U**g** = **u**) The reference frame can always be rotated to align **u** with **g** (we leave that without proof)

We will indicate the transformed vectors or coordinates with the subscript 2.
The time-space 4-vector **q**=t+**g**x+**e**y+**f**z transform as

$\mathbf{q_2}$ = exp(**u**) **q** exp(**u**)

$\mathbf{q_2}$ = exp(**u**) **q** exp(**u**) = exp(2**u**)t + exp(2**u**)x + **e**y + **f**z = [note[7]]

t*cosh2U + x*sinh2U + // t_2

x*cosh2U*$\mathbf{g}$ + t*sinh2U*$\mathbf{g}$ + // $x_2\mathbf{g}$

ey + // $y_2\mathbf{e}$

fz // $z_2\mathbf{f}$

(where U := **u**/**g**)

therefore:

t_2 = t*cosh2U + x*sinh2U

x_2 = x*cosh2U + t*sinh2U

y_2 = y

z_2 = z

We will transform $\mathcal{J}=\nabla^{\mathbf{x}}\mathbf{F} \leftarrow \mathcal{J}_2=\nabla^{\mathbf{x}}\mathbf{F_2}$ in reverse, starting with the formula **:**

$$\mathcal{J} = \nabla^{\mathbf{x}}\mathbf{F} = \partial\mathbf{F}/\partial t - \partial\mathbf{F}/\partial x\ \mathbf{g} - \partial\mathbf{F}/\partial y\ \mathbf{e} - \partial\mathbf{F}/\partial z\ \mathbf{f}$$

and evaluating the inner terms.

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[7] We use the following property of exponential function: exp(**g**U)*$\mathbf{g}$ = $\mathbf{g}$*exp(**g**U) , For perpendicular vectors (i.e. for anti-commuting vectors), for example **g** and **e** it follows exp(**g**U)*$\mathbf{e}$ = $\mathbf{e}$*exp(-**g**U)

$$d\mathbf{F}/dt = (d\mathbf{F}/dt_2)*(dt_2/dt) + (d\mathbf{F}/dx_2)*(dx_2/dt) + (d\mathbf{F}/dy_2)*(dy_2/dt) + (d\mathbf{F}/dz_2)*(dz_2/dt) =$$
$$(d\mathbf{F}/dt_2)*(\cosh 2U) + (d\mathbf{F}/dx_2)*(\sinh 2U)$$

$$d\mathbf{F}/dx = (d\mathbf{F}/dt_2)*(dt_2/dx) + (d\mathbf{F}/dx_2)*(dx_2/dx) + (d\mathbf{F}/dy_2)*(dy_2/dx) + (d\mathbf{F}/dz_2)*(dz_2/dx) =$$
$$(d\mathbf{F}/dt_2)*(\sinh 2U) + (d\mathbf{F}/dx_2)*(\cos 2U)$$

$$d\mathbf{F}/dy = (d\mathbf{F}/dt_2)*(dt_2/dy) + (d\mathbf{F}/dx_2)*(dx_2/dy) + (d\mathbf{F}/dy_2)*(dy_2/dy) + (d\mathbf{F}/dz_2)*(dz_2/dy) = (d\mathbf{F}/dy_2)$$
$$d\mathbf{F}/dz = (d\mathbf{F}/dt_2)*(dt_2/dz) + (d\mathbf{F}/dx_2)*(dx_2/dz) + (d\mathbf{F}/dy_2)*(dy_2/dz) + (d\mathbf{F}/dz_2)*(dz_2/dz) = (d\mathbf{F}/dz_2)$$

Now, we will substitute the **F** expressed in terms of the reverse transformed $\mathbf{F_2}$ into the above formulae, that is: $\mathbf{F} = \exp(-\mathbf{u})\, \mathbf{F_2} \exp(+\mathbf{u})$. This will yield:

$$\begin{aligned}
\mathcal{J} = \nabla^{\mathbf{x}}\mathbf{F} = \quad & \exp(-\mathbf{u})*((d\mathbf{F_2}/dt_2)*\cosh 2U + (d\mathbf{F_2}/dx_2)*\sinh 2U)*\exp(+\mathbf{u}) + \\
& -\exp(-\mathbf{u})*((d\mathbf{F_2}/dt_2)*\sinh 2U + (d\mathbf{F_2}/dx_2)*\cosh 2U)*\exp(+\mathbf{u})*\mathbf{g} + \\
& -\exp(-\mathbf{u})*(d\mathbf{F_2}/dy_2)*\exp(+\mathbf{u})*\mathbf{e} + \\
& -\exp(-\mathbf{u})*(d\mathbf{F_2}/dz_2)*\exp(+\mathbf{u})*\mathbf{f} \\
= \quad & \exp(-\mathbf{u})*((dF_2/dt_2)*(\cosh 2U - \sinh 2U*\mathbf{g})) + \\
& - \exp(-\mathbf{u})*((dF_2/dx_2)*(\cosh 2U - \sinh 2U*\mathbf{g})*\mathbf{g})*\exp(+\mathbf{u}) + \\
& - \exp(-\mathbf{u})*(dF_2/dy_2)*\mathbf{e}*\exp(-\mathbf{u}) + \\
& - \exp(-\mathbf{u})*(dF_2/dz_2)*\mathbf{f}*\exp(-\mathbf{u}) \\
= \quad & \exp(-\mathbf{u})*((dF_2/dt_2) * \exp(-2\mathbf{u}))*\exp(+\mathbf{u}) + \\
& -\exp(-\mathbf{u})*((dF_2/dx_2)*\mathbf{g}*\exp(-2\mathbf{u}))*\exp(+\mathbf{u}) + \\
& -\exp(-\mathbf{u})*(dF_2/dy_2)*\mathbf{e}*\exp(-\mathbf{u}) + \\
& -\exp(-\mathbf{u})*(dF_2/dz_2)*\mathbf{f}*\exp(-\mathbf{u}) \\
= \quad & \exp(-\mathbf{u})*((dF_2/dt_2) +(dF_2/dx_2)\mathbf{g} +(dF_2/dy_2)\mathbf{e} +(dF_2/dz_2)\mathbf{f})*\exp(-\mathbf{u}) = \\
= \quad & \exp(-\mathbf{u})*(\nabla^{\mathbf{x}}{}_2\mathbf{F_2})*\exp(-\mathbf{u}) \\
= \quad & \exp(-\mathbf{u})*(\mathcal{J}_2)*\exp(-\mathbf{u})
\end{aligned}$$

therefore, finally:

$$\boxed{\mathcal{J}_2 = \exp(\mathbf{u})*(\mathcal{J})*\exp(\mathbf{u})}$$

Therefore, Lorentz transform of $\mathcal{J}$ follows 4-vector transform and is consistent with the hermitian conjugate formula.

2) Rotation transform of $\mathcal{J}$. [note[8]]

We take as previously, a generator vector **v** aligned with the base vector **g** (x axis), except this time **v** has imaginary coefficient. **v** = *i* V **g** where V is a real number and *i* is imaginary unit (also *i*=**efg**)

$\mathbf{q_2} = \exp(\mathbf{v})\ \mathbf{q}\ \exp(-\mathbf{v})$

$\mathbf{q_2} = \exp(\mathbf{v})\ \mathbf{q}\ \exp(-\mathbf{v}) = t + \mathbf{g}x + \exp(2\mathbf{v})\mathbf{e}y + \exp(2\mathbf{v})\mathbf{f}z =$ [note[9]]

$t +$
$\mathbf{g}x +$
$\mathbf{e}y*\cos(2V) - \mathbf{f}y*\sin(2V) +$
$\mathbf{f}z*\cos(2V) + \mathbf{e}z*\sin(2V)$
$=$
$t +$ // t_2
$\mathbf{g}x +$ // $x_2\mathbf{g}$
$\mathbf{e}y*\cos(2V) + \mathbf{e}z*\sin(2V) +$ // $y_2\mathbf{e}$
$\mathbf{f}z*\cos(2V) - \mathbf{f}y*\sin(2V)$ // $z_2\mathbf{f}$

therefore:

$t_2 = t$
$x_2 = x$
$y_2 = y*\cos(2V) + z*\sin(2V)$
$z_2 = z*\cos(2V) - y*\sin(2V)$

We will transform now (in reverse) the $\mathcal{J} = \nabla^{l}\mathbf{F}$ **:**

$\mathcal{J} = \nabla^{x}\mathbf{F} = d\mathbf{F}/dt - (d\mathbf{F}/dx)\mathbf{g} - (d\mathbf{F}/dy)\mathbf{e} - (d\mathbf{F}/dz)\mathbf{f}$

$d\mathbf{F}/dt = (d\mathbf{F}/dt_2)*(dt_2/dt) + (d\mathbf{F}/dx_2)*(dx_2/dt) + (d\mathbf{F}/dy_2)*(dy_2/dt) + (d\mathbf{F}/dz_2)*(dz_2/dt) = (d\mathbf{F}/dt_2)$
$d\mathbf{F}/dx = (d\mathbf{F}/dt_2)*(dt_2/dx) + (d\mathbf{F}/dx_2)*(dx_2/dx) + (d\mathbf{F}/dy_2)*(dy_2/dx) + (d\mathbf{F}/dz_2)*(dz_2/dx) = (d\mathbf{F}/dx_2)$
$d\mathbf{F}/dy = (d\mathbf{F}/dt_2)*(dt_2/dy) + (d\mathbf{F}/dx_2)*(dx_2/dy) + (d\mathbf{F}/dy_2)*(dy_2/dy) + (d\mathbf{F}/dz_2)*(dz_2/dy) =$
$(d\mathbf{F}/dy_2)*(\cos 2V) - (d\mathbf{F}/dz_2)*(\sin 2V)$
$d\mathbf{F}/dz = (d\mathbf{F}/dt_2)*(dt_2/dz) + (d\mathbf{F}/dx_2)*(dx_2/dz) + (d\mathbf{F}/dy_2)*(dy_2/dz) + (d\mathbf{F}/dz_2)*(dz_2/dz) =$
$(d\mathbf{F}/dz_2)*(\cos 2V) + (d\mathbf{F}/dy_2)*(\sin 2V)$

Now, we will substitute the **F** expressed in terms of the inverse-transformed $\mathbf{F_2}$ into the above formulae: $\mathbf{F} = \exp(-\mathbf{v})\ \mathbf{F_2}\ \exp(\mathbf{v})$. This will yield:

$\mathcal{J} = \nabla^{x}\mathbf{F} = \exp(-\mathbf{v})*(\ (d\mathbf{F_2}/dt_2))*\exp(\mathbf{v}) +$

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[8] This chapter is almost identical as in ref [13], because rotation transform is the same for EM field Clifford 8-vector **F** as for 4-vectors such as **a** and $\mathcal{J}$.

[9] We use the following property of exponential function: $\exp(\mathbf{g}U)*\mathbf{g} = \mathbf{g}*\exp(\mathbf{g}U)$, For perpendicular vectors (i.e. for anti-commuting vectors), for example **g** and **e** it follows $\exp(\mathbf{g}U)*\mathbf{e} = \mathbf{e}*\exp(-\mathbf{g}U)$

$-\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial x_2))*\exp(\mathbf{v})*\mathbf{g}\ +$
$-\exp(-\mathbf{v})*((\partial\mathbf{F_2}/\partial y_2)*(\cos 2V) - (\partial\mathbf{F_2}/\partial z_2)*(\sin 2V))*\exp(\mathbf{v})*\mathbf{e}\ +$
$-\exp(-\mathbf{v})*((\partial\mathbf{F_2}/\partial z_2)*(\cos 2V) + (\partial\mathbf{F_2}/\partial y_2)*(\sin 2V))*\exp(\mathbf{v})*\mathbf{f}$
$=$
$\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial t_2))*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*((\partial\mathbf{F_2}/\partial x_2))*\exp(\mathbf{v})*\mathbf{g}\ +$
$-\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial y_2)*(\cos 2V)*\mathbf{e} - (\partial\mathbf{F_2}/\partial z_2)*(\sin 2V)*\mathbf{e}\ +$
$\quad (\partial F_2/\partial z_2)*(\cos 2V)*\mathbf{f}\ +\ (\partial\mathbf{F_2}/\partial y_2)*(\sin 2V)*\mathbf{f}\ )*\exp(-\mathbf{v})$
$=$
$\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial t_2))*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial x_2))*\mathbf{g}*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial y_2)*(\cos 2V)\mathbf{e}\ +\ (\partial\mathbf{F_2}/\partial y_2)*(\sin 2V)*\mathbf{ggf}\ +$
$\quad (\partial\mathbf{F_2}/\partial z_2)*(\cos 2V)\mathbf{f}\ -\ (\partial\mathbf{F_2}/\partial z_2)*(\sin 2V)*\mathbf{gge}\ )*\exp(-\mathbf{v})$
$=$
$\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial t_2))*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial x_2))*\mathbf{g}*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*(\ (\cos 2V)*(\partial\mathbf{F_2}/\partial y_2)\mathbf{e}\ +\ (\sin 2V)*(\partial\mathbf{F_2}/\partial y_2)*\mathbf{g(gf)}\ +$ //note[10]
$\quad (\cos 2V)*(\partial\mathbf{F_2}/\partial z_2)\mathbf{f}\ -\ (\sin 2V)*(\partial\mathbf{F_2}/\partial z_2)*\mathbf{g(ge)}\ )*\exp(-\mathbf{v})$
$=$
$\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial t_2))*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial x_2))*\mathbf{g}*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*(\ (\cos 2V)*(\partial\mathbf{F_2}/\partial y_2)*\mathbf{e}\ -\ (\sin 2V)*(\partial\mathbf{F_2}/\partial y_2)*\mathbf{g(e}i\mathbf{)}\ +$
$\quad (\cos 2V)*(\partial\mathbf{F_2}/\partial z_2)*\mathbf{f}\ -\ (\sin 2V)*(\partial\mathbf{F_2}/\partial z_2)*\mathbf{g(}i\mathbf{f)}\ )*\exp(-\mathbf{v})$
$=$
$\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial t_2))*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial x_2))*\mathbf{g}*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial y_2)*(\cos 2V\ -\ \mathbf{g}i*\sin 2V)*\mathbf{e}\ +$
$\quad (\partial\mathbf{F_2}/\partial z_2)*(\cos 2V\ -\ \mathbf{g}i*\sin 2V)*\mathbf{f}\ )*\exp(-\mathbf{v})$
$=$
$\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial t_2))*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial x_2))*\mathbf{g}*\exp(\mathbf{v})\ +$
$-\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial y_2)\ \exp(-2\mathbf{v})\ *\mathbf{e}*\ +\ (\partial\mathbf{F_2}/\partial z_2)*\exp(-2\mathbf{v})*\mathbf{f}\ )*\exp(-\mathbf{v})$
$=$
$\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial t_2))*\exp(\mathbf{v})\ +$
$\exp(-\mathbf{v})*(-\ (\partial\mathbf{F_2}/\partial x_2))*\mathbf{g}*\exp(\mathbf{v})\ +$
$\exp(-\mathbf{v})*(-\ (\partial\mathbf{F_2}/\partial y_2)*\mathbf{e}*\exp(+2\mathbf{v}) - (\partial\mathbf{F_2}/\partial z_2)*\mathbf{f}*\exp(+2\mathbf{v})\ )*\exp(-\mathbf{v}) =$
$\exp(-\mathbf{v})*(\ (\partial\mathbf{F_2}/\partial t_2)) - (\partial\mathbf{F_2}/\partial x_2))*\mathbf{g} - (\partial\mathbf{F_2}/\partial y_2)*\mathbf{e} - (\partial\mathbf{F_2}/\partial z_2)*\mathbf{f}\ )*\exp(+\mathbf{v})$
$= \exp(-\mathbf{v})*(\ \nabla^{\mathbf{x}}{}_{\mathbf{2}}\mathbf{F_2}\ )*\exp(\mathbf{v})$
$=\ \exp(-\mathbf{v})*(\ \mathcal{J}_{\mathbf{2}}\ )*\exp(+\mathbf{v})$
Finally

[10] **gf** = -**fg** = -**eefg** = -**e***i* and **ge** = **geff** = **efgf** = ***i*f**

$$\boxed{\mathcal{J}_2 = \exp(+\mathbf{v})*(\mathcal{J})*\exp(-\mathbf{v})}$$

Conclusion

In a general case, we can combine the **u** and **v** cases in one formula that employs the Hermitian conjugate $(\ )^T$ [note[11]]

$$\boxed{\mathcal{J}_2 = \exp(\mathbf{p})*(\mathcal{J})*(\exp(\mathbf{p}))^T}$$

References:

1)
"Representing 3D vectors, 4-vectors, rotation and Lorentz transform using Clifford Algebra $Cl_{3,0}(R)$.", Stan Bleszynski, 4-March-2018 https://goo.gl/wuB43b

2) reserved

3)
"Zero Length Bases in Clifford Algebra $Cl_{3,0}(\{1,h\})$", Stan Bleszynski 16-April-2018 (Updated 15/06/2018) link: https://goo.gl/kTh5jb

4)
"Electromagnetics in Clifford Algebra"
Stan Bleszynski, David Coffey, 13-Jan-2019,
https://docs.google.com/document/d/1ki-N2au3AVE5Mo7KR6CVY6TZDwwkxcnJYg0mJ--K58M/edit

5) "Hamilton Equation in Clifford $CL_{3,0}(R)$ ", Stan Bleszynski
17/10/2019]
https://docs.google.com/document/d/1_jWOF3cku0K8FwpMcgCUtkz69uJy0_emsQdblJdF9lg/edit?usp=sharing

6) "Standard model physics from an algebra?" by C. Furey
PhD thesis, Waterloo, Ontario, Canada, 2015
https://arxiv.org/pdf/1611.09182.pdf

[11] exp(**p**) := exp(**u**)*exp(**v**) , and exp(**u**)*exp(**v**))' = exp(**v'**)*exp(**u'**)

13) Proof that EM Clifford 8-vector field F transforms in $Cl_{3,0}(R)$ differently, using quaternion conjugate formula, unlike Clifford 4-vectors transforming using hermitian conjugate formula.
https://docs.google.com/document/d/1e37GgeSA9JOr9nNNRrosEGmPmlhhJX1dBQzuvS8B_04/edit?usp=sharing