Problems of Knowledge

Why are modern axioms anything but self-evident?

Axioms in classical times were simple and clear --- one of the required characteristics was that it should be so obvious that any rational person would automatically accept them as correct. Nice idea. However, such "clear and obvious" axioms turned out not to be either clear or obvious --- they tended to be somewhat ambiguous and lacked the necessary precision required by deductive logic. So, as time went by the axioms became more precise, more technical, more symbolic and less and less "obvious". On the positive side, axioms are now rigorously defined with little room for ambiguous interpretation or sloppy notation. The down side is that axioms can now only really be understood by 'experts', trained in the language and syntax of the subject. So, the mathematical community has accepted the pay-off of having precision and unambiguous axioms instead of the self-evident building blocks of the ancients.

If two conflicting sets of axioms exist, each leading to consistent results, which is the "right" one?

This becomes something of a philosophical question, and depends on ones stance on the question of the nature of mathematics. If you believe mathematics is a fundamental part of Nature, and that mathematical truths reflect and explain physical truths (i.e. you are a Platonist) then this question causes you some serious problems. For Platonist having several 'correct' geometries, for example, is something of an embarrassment. Nature, or the Universe, for the Platonist, must be described by mathematics, so how can we have more than one competing (yet apparently equally valid) candidates for what geometry to choose? Surely one must be the right one? But which? This is an unsolved issue, but some Platonists would say that our choice will either become clear when new results about the nature of the Universe shows one geometry to be the right one, or, results will require a fundamental rewriting of the axioms which will avoid multiple geometries, yet still produce a mathematical framework that can be used to derive all the results we find so useful, e.g. Pythagoras' Theorem.

If you are a formalist, you can smugly sit back and watch the Platonist squirm as you simple pronounce each correct set of axioms just represents a different "game", but none of them are what actually describes the Universe, because the Universe is not described mathematically.

Can mathematics ever be finished?

No; at least not if Gödel is correct. And Gödel will be correct if our Axioms of arithmetic are to hold true since Gödel proved his famous Incompleteness Theorem from nothing more complex than simple arithmetic. It is unimaginable to create a mathematics without arithmetic, so most modern mathematicians accept what seem the inevitable, and conclude mathematics will never be finished (or to put it in more precise form, there will always be true results in mathematics which we will be unable to prove true). This was initially (and to many still is) a major flaw in the subject. But all is not lost. What we do know, and have proved, is true (assuming the axiom sets chosen). What is more, some would argue Nature herself is full of holes --- areas we cannot get a precise answer about (quantum mechanics and Heisenberg's Uncertainty Principle come to mind) so it is not so surprising that mathematics too has its limitations.

If mathematics deals in knowledge that is 100% certain, how then can statistics be used (or mis-used) to describe data in various biased forms?

Statistics used correctly generates measures that describe data. The results obtained are probabilistic in nature, so will contain a degree of uncertainty, but are nevertheless accurate representations of the data. What is misleading is not the mathematics, but rather the way unscrupulous or ignorant people choose to display the results. Putting aside the issues of incorrectly applying statistics (through things like inadequate sampling or invalid hypothesis testing, which can be classed simply as errors) the more interesting cases of mis-leading statistics are due to such devices as inappropriate scaling, selective choice of values and invalid and biased interpretation of the results. Through such means the general public is often fooled into believing that which is not substantiated by the statistical analysis. It is not the statistics which is to blame, but rather those who misuse it, and those who are inadequately educated to understand what limitations statistical analysis involves.