

Categorical Dualities in Algebra and Logic

This list was created during a discussion session chaired by Peter Jipsen
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General concepts

Adjunctions
Concrete dualities
Term-equivalences
Full dualities
Morita equivalences
Multi-sorted dualities
Natural dualities
Piggyback dualities
Strong dualities
Trivial dualities

Specific dualities

1. Algebra-coalgebra duality (includes BAO/descriptive general frames and similar)
2. Baker-Beynon duality
3. Banaschewski duality
4. Bezhanishvili-Holliday choice-free duality
5. Birkhoff duality
6. Bimbo duality
7. De Vries duality
8. Di Nola equivalence
9. Došen duality
10. Duality between syntax and semantics
11. Dualities based on possibility semantics
12. Dunn duality
13. Esakia duality
14. Gelfand duality
15. Ghilardi-Zawadowski duality
16. Girard duality of coherence spaces
17. Goldblatt duality
18. Hansoul duality
19. Hofmann-Mislove-Stralka duality
20. INF-SUP duality
21. Isbell duality for spatial frames

22. Isbell duality between presheaves and co-presheaves
23. Jónsson-Tarski duality
24. Kakutani duality
25. Koszul duality (for operads and dg categories)
26. Linear algebra duality
27. Marra-Reggio duality
28. Marra-Spada duality
29. Moshier duality
30. Mundici equivalence
31. Pierce duality
32. Pontryagin duality
33. Priestley duality
34. Priestly-type dualities
35. Stone duality for Boolean algebras
36. Stone duality for distributive lattices
37. Tarski duality
38. Thomason duality
39. Urquhart duality
40. Zariski duality

More details

[Adjunctions](#)

Concrete dualities, Porst, Tholen, 1991, <http://www.heldermann.de/R&E/RAE18/ctw07.pdf>

Algorithms for categorical equivalence, Bergman and Berman, 1996,
<http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.94.274&rep=rep1&type=pdf>

Term-equivalence, e.g., Boolean algebras and Boolean rings,

Morita equivalence for varieties, McKenzie, 1996,
<https://books.google.com/books?id=4JY4DwAAQBAJ&pg=PT128&lpg=PT128&dq=McKenzie#v=onepage&q=McKenzie&f=false>

Koszul duality, classical paper, formulated for algebras:

- A. Beilinson, V. Ginzburg, W. Soergel, Koszul duality patterns in representation theory. J. Amer. Math. Soc. 9 (1996), no. 2, 473–527.

Over operads:

- V. Ginzburg, M. Kapranov, Koszul duality for operads, Duke Math. J. 76 (1994), no. 1, 203–272; reprint arxiv/0709.1228; Erratum to: Koszul duality for operads, Duke Math. J. 80 (1995), no. 1, 293.

As a generalization of Morita equivalence:

- J. Chuang, A. Lazarev, W. Mannan, Koszul-Morita duality, <https://arxiv.org/abs/1408.6153v1>

Natural dualities, Clark and Davey, 1998, Natural dualities for the working mathematician, https://books.google.com/books/about/Natural_Dualities_for_the_Working_Algebr.html?id=vvfWMVYQS9YC

Natural duality, modality, and coalgebra, Maruyama, 2012
<https://www.sciencedirect.com/science/article/pii/S0022404911001666>

Full dualities

Multi-sorted dualities

Strong dualities

Piggyback dualities revisited, Davey, Haviar, Priestley, 2015 <https://arxiv.org/abs/1501.02512>

Trivial dualities, e.g. the duality between a category and its opposite category

List of specific dualities with some details

1. **Algebra-coalgebra duality**, extends a base-duality such as BA-Stone, for example
 - a. Modal algebras - neighbourhood frames
 - b. BAOs - descriptive general frames
 - c. Complete atomic BAOs - Kripke frames (generalized to discrete frames - AV-BAOs)
 - d. **Thomason-style duality** for complete atomic BAOs with arbitrary (not just complete) morphisms. Again, generalizes to discrete frames.
 - e. Ordered versions of the above and many more
 - f. **Kurz-Rosicky dualities** (coalgebraic logic in predicate lifting setting, in the cases when the transpose is well-behaved)
2. **Baker-Beynon duality** between finitely generated semi-simple vector lattices and Euclidean closed polyhedral cones, 1968, 1974
<https://londmathsoc.onlinelibrary.wiley.com/doi/pdf/10.1112/plms/s3-31.1.114>
https://www.encyclopediaofmath.org/index.php/Baker-Beynon_duality
3. **Banaschewski duality** between posets and Stone distributive lattices
4. **Bezhanishvili-Holliday choice-free duality** for Boolean algebras
<https://escholarship.org/uc/item/00p6t2v4>
5. **Birkhoff duality** between finite posets and finite distributive lattices

6. **Bimbo duality** for gaggles
7. **De Vries duality** for compact Hausdorff spaces
8. **Di Nola-Lettieri equivalence** between lattice ordered abelian groups and perfect MV-algebras.
9. **Došen duality** between modal algebras and neighbourhood frames (see above: algebra-coalgebra duality)
10. **Duality between syntax and semantics**, Awodey and Forssell, 2018, First Order Logical Duality <https://arxiv.org/pdf/1008.3145.pdf>
 - a. A duality between the resulting categories of syntax and semantics, expressed primarily in the form of a contravariant adjunction, is established by homming into a common dualizing object, now Sets, regarded once as a boolean category, and once as a groupoid equipped with an intrinsic topology.
11. Dualities based on **possibility semantics**: duality between complete modal algebras with completely additive operators and (a reflective subcategory of) full possibility frames; choice-free duality between modal algebras and (a reflective subcategory of) possibility frames. <https://escholarship.org/uc/item/0tm6b30q>
12. **Dunn duality** between distributoids and Dunn spaces
13. **Esakia duality** between Heyting algebras and Esakia spaces
14. **Gale duality** between polarized arrangements of linear spaces by a vector and covector
15. **Gelfand duality** between commutative C-star algebras and (locally) compact topological spaces
16. **Ghilardi-Zawadowski** (pre)sheaf duality for finitely presented modal and Heyting algebras
17. **Girard duality** of coherence spaces
18. **Goldblatt duality** between ortholattices and orthospaces
19. **Hansoul duality**
20. **Hofmann-Mislove-Stralka duality** between semilattices and algebraic lattices

21. **INF-SUP duality** between complete join-semilattices and complete meet-semilattices
22. **Isbell duality** between sober spaces and spatial frames
23. **Isbell duality** between presheaves and co-presheaves,
 - a. Works well to extend finite dualities, for example
 - i. Finite BAs-Finite Sets to BAs-Stone and CABA-Set
 - ii. Similarly for finite DLs-finite posets
 - iii. Related to profinite completion - ind completion
 - b. <https://ncatlab.org/nlab/show/Isbell+duality>
24. **Jónsson-Tarski duality** between Boolean algebras with operators and relational structures
25. **Kakutani duality** between unital real vector lattices and compact Hausdorff spaces
26. **Koszul duality** between quadratic operads as generalization of dg duality between symmetric and exterior algebras of a vector space (a linear algebra duality)
27. **Linear algebra duality** between finite dimensional vector spaces and their dual vector spaces
28. **Marra-Reggio duality** between σ -MV-algebras and compact Hausdorff spaces
29. **Marra-Spada duality** between semisimple MV-algebras and Tychonoff spaces and \mathbb{Z} -maps
30. **Moshier duality** between complete semilattices and polarities
31. **Mundici equivalence** between MV-algebras and abelian ℓ -groups with a strong order unit
32. **Pierce duality** between commutative rings and sheaves of indecomposable rings over Stone spaces
33. **Pontryagin duality** between abelian groups and compact abelian groups
34. **Priestley duality** between distributive lattices and Priestley spaces
35. **Priestly-type dualities** for more general classes of partially ordered structures than distributive lattices, e.g. free frames on posets [introduced by [Caramello](#)]

- 36. **Stone duality** between Boolean algebra and Stone spaces (see above: algebra-coalgebra duality)
- 37. **Stone duality** between distributive lattices and spectral spaces (= coherent spaces)
- 38. **Tarski duality** between complete and atomic Boolean algebras and sets (see above: algebra-coalgebra duality)
- 39. **Urquhart duality** between lattices and Urquhart spaces
- 40. **Zariski duality** between affine schemes and commutative rings with unity

The category of Chu spaces is $\text{Chu}(\text{Set}, 2)$. A nice point emphasized by Pratt is that for many instances of concrete dualities where the underlying set of the schizophrenic object is 2, the category and its opposite embed as dual subcategories in the self-dual category of Chu spaces. This applies in particular to Stone duality, the duality between ordinals and intervals [emphasized by Joyal in connection with his category of disks], the self-duality of sup-lattices, and many others that you can think of.

Posted by: Todd Trimble on January 19, 2007

2018 A Unified View of Piecewise Linear Neural Network Verification

Rudy Bunel, Ilker Turkaslan, Philip H.S. Torr, Pushmeet Kohli, M. Pawan Kumar

<https://arxiv.org/abs/1711.00455>

Category theory is a formal game combining the structure of monoids and partial orders
-- Axel Poigné, Basic Category Theory, Handbook of Logic in Computer Science, 1992

"A mathematician is a person who can find analogies between theorems; a better mathematician is one who can see analogies between proofs and the best mathematician can notice analogies between theories. One can imagine that the ultimate mathematician is one who can see analogies between analogies." Stefan Banach

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