Advanced Math
Date:
Period:

Name:

3.2 & 3.3 Homework

1. Consider $g(x) = (x-3)(x+6)(x-4) = x^3 - x^2 - 30x + 72$. Give estimates for each of these values.

a. $g(-6) = 0$

b. $g(-6.001) \approx 0$

c. $g(0) = 72$

d. $g(1{,}000{,}000{,}000) \approx$ large positive number

e. $g(-1{,}000{,}000{,}000) \approx$ large negative number

2. a. Find a cubic polynomial function with a graph that crosses the x-axis at (2, 0), (7, 0), and (-4, 0). The graph also contains the point (4, -144).

$g(x) = A(x-2)(x-7)(x+4)$

$-144 = A(4-2)(4-7)(4+4)$

$-144 = A(2)(-3)(8)$

$-144 = A(-48)$ $\quad A = 3$

$g(x) = 3(x-2)(x-7)(x+4)$

b. Travis graphed the function he wrote for part a. Travis said his function falls, rises, and falls again when looking at it from left to right. Explain why you know Travis made an error with his function.

g(x) is a cubic function with a positive leading coefficient. Therefore g(x) should be "line like with positive slope". This means g(x) should rise, fall, then rise. Travis has a neg. L.C.

3. a. Find a cubic polynomial function with a graph that crosses the x-axis at (-2, 0), (3, 0), and (6, 0). The graph also contains the point (1, -15).

$f(x) = A(x+2)(x-3)(x-6)$

$-15 = A(1+2)(1-3)(1-6)$

$-15 = A(3)(-2)(-5)$

$-15 = A(30)$

$-\frac{1}{2} = A$

$f(x) = -\frac{1}{2}(x+2)(x-3)(x-6)$

b. Dalton graphed the function he wrote for part a. Dalton said his function rises and then falls when looking at it from left to right. Explain why you know Dalton made an error with his function.

Dalton would have an even degree function to rise and fall only. However, the function has 3 real zeros so it is an odd degree function.

For questions 4-7: You may write the equation of your line in point-slope or slope intercept form.

4. Let $f(x) = 2x^3 + 4x - 3$. Find the equation of the secant line through the points (1, f(1)) and (2, f(2)).

$f(1) = 2(1)^3 + 4(1) - 3 = 3 \rightarrow (1, 3)$

$f(2) = 2(2)^3 + 4(2) - 3 = 21 \rightarrow (2, 21)$

$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{\text{Rise}}{\text{Run}} = \frac{21-3}{2-1} = \frac{18}{1} = 18$

$y - 3 = 18(x - 1)$

OR

$y - 21 = 18(x - 2)$

OR

$y = 18x - 15$

only one of these is needed as they are all equivalent.

5. Let $f(x) = x^3 - 2x + 1$. Find the equation of the secant line through the points (2, f(2)) and (2.01, f(2.01)).

$f(2) = 2^3 - 2(2) + 1 = 5 \rightarrow (2, 5)$

$f(2.01) = (2.01)^3 - 2(2.01) + 1 = 5.100601$

$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{5.100601 - 5}{2.01 - 2} = \frac{.100601}{.01} = 10.0601$

$\boxed{y - 5 = 10.0601(x - 2)}$

other formats are possible

6. Let $f(x) = 2x^4 - 5x^2 + 3$. Find the equation of the secant line through the points (1, f(1)) and (3, f(3)).

$f(1) = 2(1)^4 - 5(1)^2 + 3 = 0$

$f(3) = 2(3)^4 - 5(3)^2 + 3 = 120$

$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{120 - 0}{3 - 1} = \frac{120}{2} = 60$

$y - 0 = 60(x - 1)$

OR

$y - 120 = 60(x - 3)$

OR

$y = 60x - 60$

only one format is needed

7. Let $f(x) = x^3$. Find the equation of the secant line through the points (2, f(2)) and a point of very close to (2, f(2)).

AMV: I used $x = 2.0001$ as the point close to $x = 2$

$f(2) = 2^3 = 8$

$f(2.0001) = (2.0001)^3 = 8.00120006$

$\text{slope} = \frac{8.00120006 - 8}{2.0001 - 2} = \frac{.00120006}{.0001} = 12.00060001$

slope is almost 12

so Really close to: $y - 8 = 12(x - 2)$

8. (Level III) Why must all odd degree polynomial functions have at least one real zero? Explain.

Answers may vary but would likely include one of the two following main ideas:

Polynomial functions are continuous, meaning there are no breaks in them (i.e. no holes, vertical asymptotes, etc.) We know that a negative number to an odd power is a negative and a positive number to an odd power is positive. This means that odd degree polynomials are guaranteed to have both negative and positive outputs. Since the graphs of polynomial functions are continuous, the only way to get from a negative to positive output is to go through an output of 0.

OR

We know the graphs of odd degree polynomials are like a line with positive slope when the leading coefficient is positive and like a line with negative slope when the leading coefficient is negative. Graphs of polynomial functions are continuous and any "line" (that is not horizontal) must eventually go through an output of 0 (i.e. cross the x-axis).

9. (Level III) Why don't all even degree polynomial functions have at least one real zero? Explain.

Answers may vary but would likely include one of the two following main ideas:

We know that a negative number to an even power is a positive and a positive number to an even power is also positive. If we multiply this by a positive leading coefficient, we get a positive output. This means all of the outputs could be greater than 0 (i.e. positive) so there is never a time when the output is 0.   Similarly, we know that a negative number to an even power is a positive and a positive number to an even power is also positive. If we multiply this by a negative leading coefficient, we get a negative output. This means all of the outputs could be less than 0 (i.e. negative) so there is never a time when the output is 0.

OR

We know the graphs of even degree polynomials are like a parabola that opens up when the leading coefficient is positive and like a parabola that opens down when the leading coefficient is negative. This means the entire graph of the even degree polynomial could be above the x-axis or it could all be below the x-axis.