PDM - End Behavior and One-Sided Limits   Name:______________________________

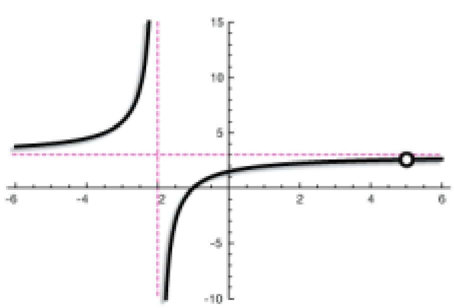
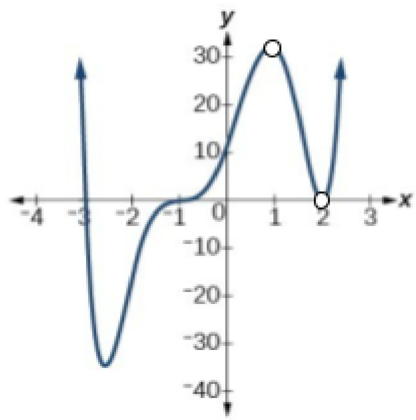
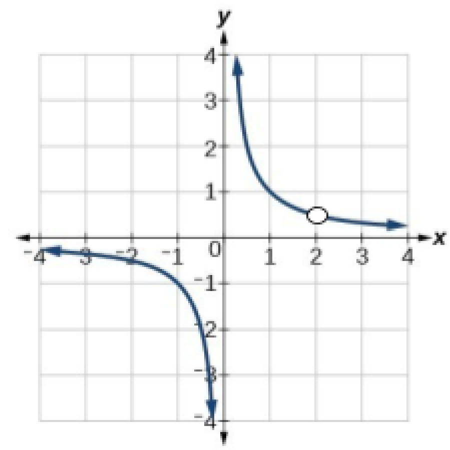
5.B Reassessment Practice

NO GRAPHING SOFTWARE allowed on this standard

Emerging:

- I can identify domain restrictions from a graph
- I can define the end behavior model for a rational function given a graph

1. For each of the graphs, find the domain restrictions and the end behavior using limit notation

		
Domain Restrictions:	Domain Restrictions:	Domain Restrictions:
End Behavior:	End Behavior:	End Behavior:

Proficient:

- I can determine the end behavior of any rational function by comparing the leading terms of the numerator and denominator
- I can describe the behavior of a function as x approaches any value using limit notation
- I can determine essential and removable discontinuities given an equation or a graph
- I can evaluate one-sided limits for any function at any point from a graph

For questions #2-4, use the following rule to answer the questions: $g(x) = \frac{a_m}{b_n} x^{m-n}$

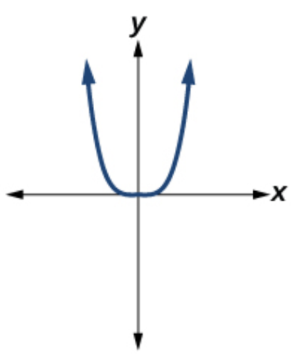
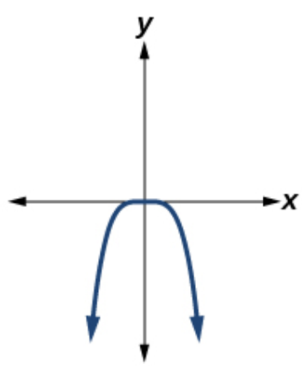
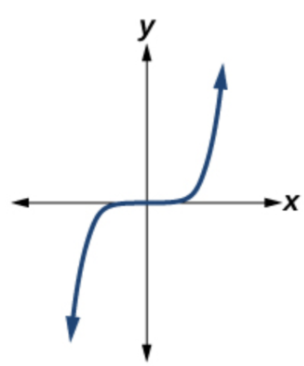
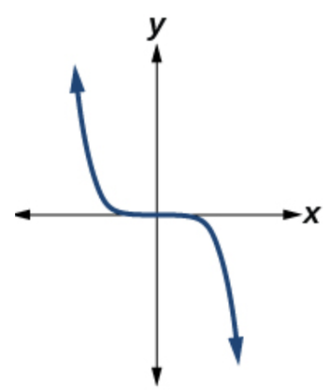
2. Fill in the blanks to summarize the different scenarios when using the rule above:

   a. When the numerator has a higher power than the denominator, the end behavior will be the same as the ________________ found when using the rule above.

   b. When the numerator has the same power as the denominator, the end behavior approaches ______________, where ____ is the leading coefficient of the numerator and ______ is the leading coefficient of the denominator.

   c. When the numerator has a smaller power than the denominator, the end behavior approaches ______.

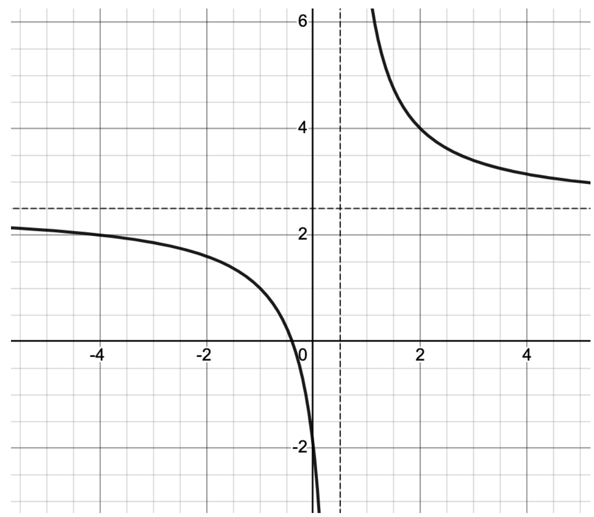
3. Considering the scenario from problem 1a, fill in the blanks in the table for the possible end behaviors for each power function:

____________ Power Function		____________ Power Function	
___________ Leading Coefficient	___________ Leading Coefficient	___________ Leading Coefficient	___________ Leading Coefficient
			
$\lim\limits_{x \to -\infty} f(x) =$ _____	$\lim\limits_{x \to -\infty} f(x) =$ _____	$\lim\limits_{x \to -\infty} f(x) =$ _____	$\lim\limits_{x \to -\infty} f(x) =$ _____
$\lim\limits_{x \to \infty} f(x) =$ _____	$\lim\limits_{x \to \infty} f(x) =$ _____	$\lim\limits_{x \to \infty} f(x) =$ _____	$\lim\limits_{x \to \infty} f(x) =$ _____

4. For each of the following, find the end behavior using limit notation:

$f(x) = \frac{-5x^7 + x - 9}{x^2 - 9}$	$g(x) = \frac{-5x^2 + x - 9}{x^2 - 9}$	$h(x) = \frac{-5x - 9}{x^2 - 9}$

5. Consider the function $f(x) = \frac{10x^2 + 9x + 2}{4x^2 - 1}$



a. Factor the numerator and denominator:

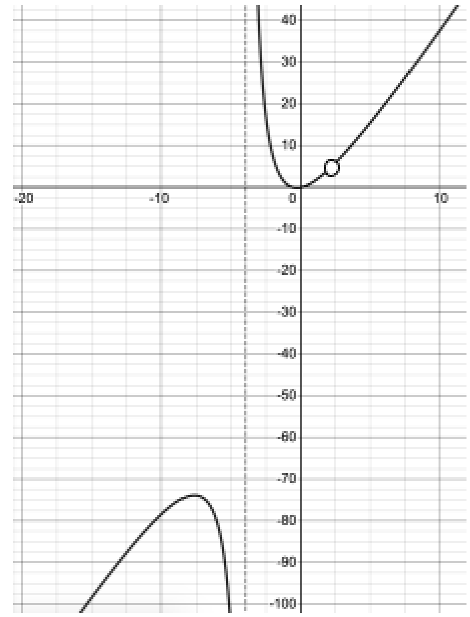
b. For each of the following x-values, determine whether it is a discontinuity or not. If so, is it considered essential or removable?

x-value	x = 1/2	x = -1/2	x = -2/5
Is it a discontinuity?			
If yes, essential or removable?			

c. Use limit notation to describe the behavior of f(x) as $x \to 1/2$ from the left and right

d. Use limit notation to describe the end behavior of f(x)

6. Consider the function $f(x) = \dfrac{5x^3 - 17x^2 - 12x}{x^2 - 16}$



a. Factor the numerator and denominator:

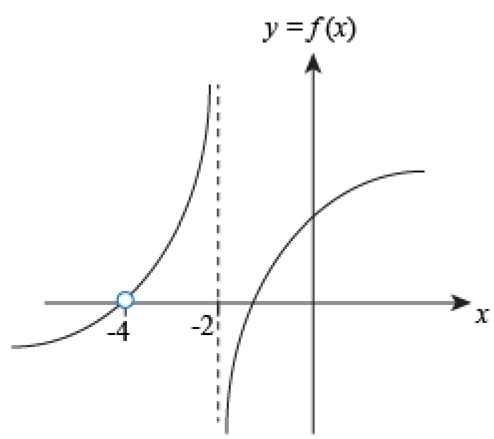
b. For each of the following x-values, determine whether it is a discontinuity or not. If so, is it considered essential or removable?

x-value	x = 4	x = -4	x = -3/5	x = 0
Is it a discontinuity?				
If yes, essential or removable?				

c. Use limit notation to describe the behavior of f(x) as $x \to -4$ from the left and right

d. Use limit notation to describe the end behavior of f(x)

7. Consider the graph of f(x) shown:



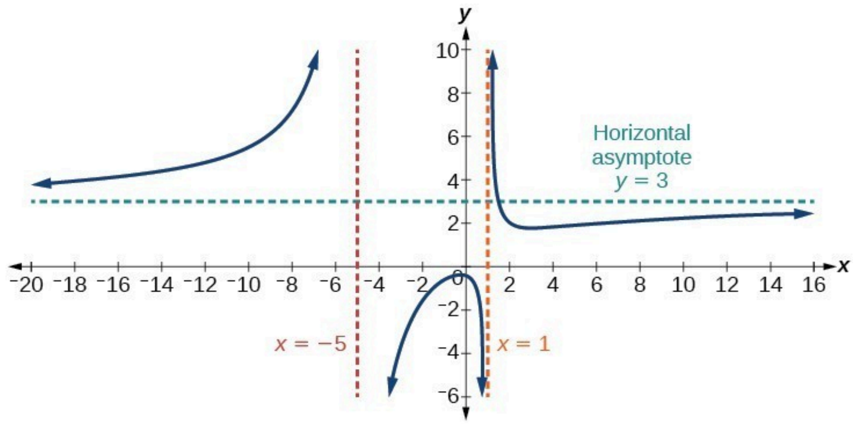
a. State the domain restrictions:

b. Does x = -4 represent a removable or essential discontinuity?

c. Does x = -2 represent a removable or essential discontinuity?

d. Use limit notation to describe the behavior of f(x) as $x \to -2$ from the left and right

8. Consider the graph of f(x) shown:
    a. State the domain restrictions:



    b. Does x = -5 represent a removable or essential discontinuity?

    c. Does x = 1 represent a removable or essential discontinuity?

    d. Use limit notation to describe the behavior of f(x) as $x \to -5$ from the left and right

    e. Use limit notation to describe the behavior of f(x) as $x \to 1$ from the left and right

    f. Use limit notation to describe the end behavior of f(x).

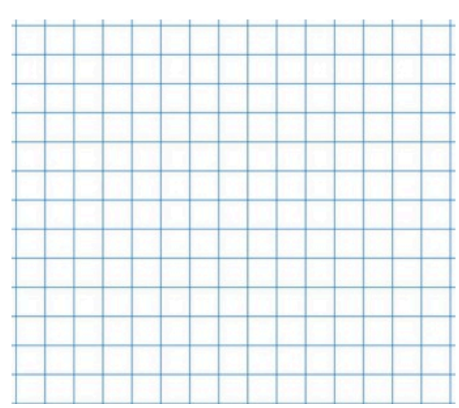
Advanced: Draw the graph of a single function below that has all of the following properties:

$$\lim_{x \to -\infty} f(x) = 0$$

$$\lim_{x \to \infty} f(x) = 0$$

$$\lim_{x \to 3^{+}} f(x) = -3$$

$$\lim_{x \to 3^{-}} f(x) = 5$$



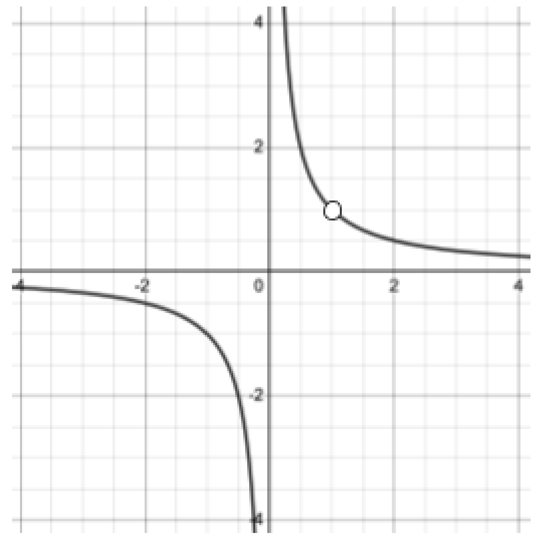
PDM - Standard 5B Practice Quiz **Name:**____

End Behavior and One Sided Limits

You may NOT use graphing software on this test

Emerging:

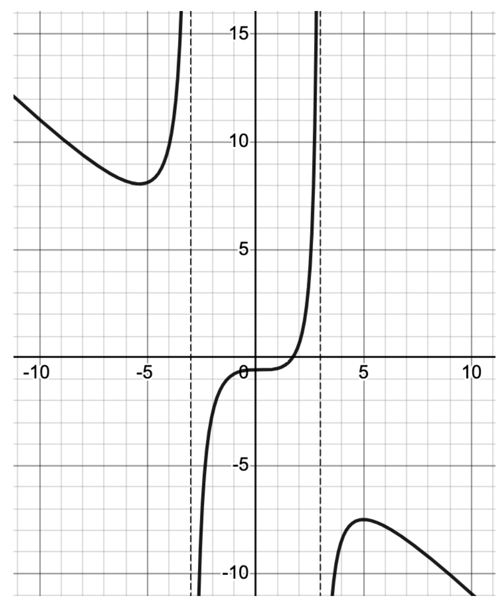
1. Consider the graph of *j(x)* below
   a. State the domain restrictions.



   b. Describe the End Behavior of $j(x)$.

Proficient:

1. Consider the function *k* defined by $k(x) = \dfrac{-x^3 + 5}{x^2 - 9}$ shown.



   a. Using Limit notation, describe the behavior of *k(x)* as $x \to -3$ (There should be 2 limits!)

   b. Using Limit notation, describe the behavior of *k(x)* as $x \to 3$ (There should be 2 limits!)

2. Use limit notation to describe the End Behavior Model for the following Rational Functions

$f(x) = \dfrac{3x - 5}{-x^2 - 2x}$	$g(x) = \dfrac{3x^7 - x^4 + 2}{-2x^3 + 5x}$	$h(x) = \dfrac{5x^3 - 2x + 5}{-2x^3 - 2x}$

3. Consider the function *f(x)* defined by $f(x) = \dfrac{3x^3 + 2x^2 - 5x}{x^2 - 1}$

   a. Factor the function completely:

   b. Which x-values, if any, represent a removable discontinuity?

   c. Which x-values, if any, represent an essential discontinuity?

Advanced:

4. Draw the graph of a single function below that has all of the following properties.

$$\lim_{x \to -\infty} f(x) = -\infty$$

$$\lim_{x \to \infty} f(x) = -\infty$$

$$\lim_{x \to 1^+} f(x) = -2$$

$$\lim_{x \to 1^-} f(x) = 1$$