MARKING SCHEME FOR FUNCTIONS AND GRAPHS

1.

1(a)	$(-2, -3)$	B1
		(1)
(b)	$(-3, -9)$	B1B1
		(2)
(c)	$(-4, 3)$	B1
		(1)
		(4 marks)

2.

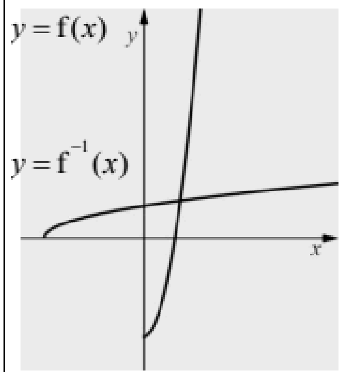
(a)	$\mathrm{f}(x)=\dfrac{2x^2-32}{(3x-5)(x+4)}+\dfrac{8}{3x-5}$	B1
	$=\dfrac{2x^2-32}{(3x-5)(x+4)}+\dfrac{8}{3x-5}=\dfrac{2x^2-32+8(x+4)}{(3x-5)(x+4)}$ or e.g. $=\dfrac{2(x-4)(x+4)}{(3x-5)(x+4)}+\dfrac{8}{3x-5}=\dfrac{2(x-4)+8}{(3x-5)}$	M1
	$=\dfrac{2x\cancel{(x+4)}}{(3x-5)\cancel{(x+4)}}=\dfrac{2x}{3x-5}$ *	A1*
		(3)
(b)	$\mathrm{f}'(x)=\dfrac{2(3x-5)-3\times 2x}{(3x-5)^2}$	M1A1
	$\mathrm{f}'(x)=\dfrac{-10}{(3x-5)^2}$ As $(3x-5)^2>0$ (for $x>2$) then $\mathrm{f}'(x)<0$ Hence f is a decreasing function *	A1cso*
		(3)

(c)	$g^{-1}(x) = e^{\frac{x-3}{2}}$	M1A1
	$x \geqslant 3$	B1
		(3)
(d)	$g\left(\frac{2a}{3a-5}\right) = 5 \Rightarrow 3 + 2\ln\left(\frac{2a}{3a-5}\right) = 5$	B1
	$\ln\left(\frac{2a}{3a-5}\right) = 1 \Rightarrow \frac{2a}{3a-5} = e$	M1
	$\frac{2a}{3a-5} = e \Rightarrow a = \ldots$	dM1
	$a = \frac{5e}{3e-2}$	A1
		(4)
(d) Way 2	$gf(a) = 5 \Rightarrow \frac{2a}{3a-5} = g^{-1}(5)$	B1
	$\frac{2a}{3a-5} = g^{-1}(5) \Rightarrow \frac{2a}{3a-5} = e$	M1
	$\frac{2a}{3a-5} = e \Rightarrow a = \ldots$	dM1
	$a = \frac{5e}{3e-2}$	A1
		(13 marks)

3.

(a)	$f(x) \leqslant 9$	B1
		(1)
(b)	$fg(1.5) = f\left(\frac{3}{2\times1.5+1}\right) = 9 - \left(\frac{3}{2\times1.5+1}\right)^2$	M1
	$= \frac{135}{16}$	A1
		(2)
(c)	$g(x) = \frac{3}{2x+1} \Rightarrow g^{-1}(x) = \frac{3-x}{2x}$	M1 A1
	$0 < x \leqslant 3$	B1
		(3)
		Total 6

4.

.(a)	$f \geqslant -5$	B1	**(1)**
(b)	 Curve starting on negative x-axis and passing through positive y-axis, in quadrants 1 and 2 only.	M1	
	Shape and position correct.	A1	**(2)**
(c)	$2x^2-5=x$ or $2x^2-5=\sqrt{\dfrac{x+5}{2}}$ or $x=\sqrt{\dfrac{x+5}{2}}$ or $2\left(2x^2-5\right)^2-5=x$	B1	
	Full attempt to solve $2x^2-x-5=0 \Rightarrow x=\ldots$ exact	M1	
	$x=\dfrac{1+\sqrt{41}}{4}$	A1	**(3)**
			6 marks

5.

5.(a)	$(2, -10)$	B1 B1
		(2)
(b)	$\text{ff}(0) = \text{f}(-4) = ...$	M1
	$= 8$	A1cso
		(2)
(c)	Attempts to solve $-3(x-2)-10 = 5x+10 \Rightarrow x = ...$	M1
	$x > -\frac{7}{4}$ only	A1
		(2)
(d)	$x(\text{or } \|x\|) = \frac{16}{3}$	B1
	Attempts $3(\|x\|-2)-10=0 \Rightarrow \|x\| = k,\ k>0$	
	or $3(-x-2)-10=0 \Rightarrow x=-k$	M1
	or $3(x-2)-10=0 \Rightarrow x=k \Rightarrow x=-k$	
	$x = \left(\frac{16}{3} \text{ and}\right)\ -\frac{16}{3}$ with no other values	A1
		(3)
		(9 marks)

6.

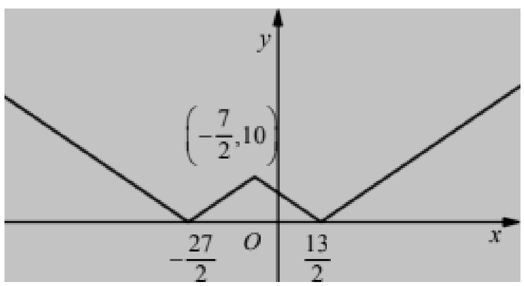
2(a)	$\text{ff}(6) = \text{f}\left(\text{"}\frac{9}{2}\text{"}\right) = \frac{\text{"}9/2\text{"}+3}{\text{"}9/2\text{"}-4} = ...;\ = 15$	M1; A1
		(2)
(b)	$\text{f}^{-1}(x) = \frac{4x+3}{x-1}$	M1A1
	$x \in \mathbb{R},\ x \neq 1$	B1
		(3)
(c)	E.g. $\left(\frac{x+3}{x-4}\right)^2 + 5 = 7$ or $\frac{a+3}{a-4} = (\pm)\sqrt{7-5}$	M1
	$\Rightarrow x^2 - 22x + 23 = 0 \Rightarrow x = ...$ or $(a+3) = (\pm)\sqrt{2}(a-4) \Rightarrow a = ...$	dM1
	$(a=) 11 + 7\sqrt{2}$ oe	A1
		(3)
		(8 marks)

Notes

7.

. (a)	$\mathrm{f}(x)=\dfrac{5x-3}{x-4}\Rightarrow \mathrm{f}'(x)=\dfrac{5(x-4)-(5x-3)}{(x-4)^2}=\dfrac{k}{(x-4)^2}$	M1 dM1
	States that $\mathrm{f}'(x)=\dfrac{-17}{(x-4)^2}\Rightarrow \mathrm{f}'(x)<0$ hence decreasing * cso	A1*
		(3)
(b)	$y=\dfrac{5x-3}{x-4}\Rightarrow xy-4y=5x-3\Rightarrow xy-5x=4y-3$	M1
	$\Rightarrow x=\dfrac{4y-3}{y-5}$ So $\mathrm{f}^{-1}(x)=\dfrac{4x-3}{x-5}$	A1
	Domain $x>5$	B1
		(3)
:) (i)	$\mathrm{ff}(x)=\dfrac{5\times\dfrac{5x-3}{x-4}-3}{\dfrac{5x-3}{x-4}-4}$	M1
	$\mathrm{ff}(x)=\dfrac{5\times(5x-3)-3(x-4)}{5x-3-4(x-4)}=\dfrac{22x-3}{x+13}$	dM1 A1
(ii)	$5<\mathrm{ff}(x)<22$	B1, B1
		(5)
		(11 marks)

8.

(a)	$\left(-\frac{7}{2}, -10\right)$	B1 B1
		(2)
(b)	Attempts to solve either $\frac{1}{2}(2x+7)-10 ... \frac{1}{3}x+1$	
	Or $-\frac{1}{2}(2x+7)-10 ... \frac{1}{3}x+1$	M1
	Both correct critical values $x ... -\frac{87}{8}, \frac{45}{4}$	A1
	Selects outside region for their critical values	dM1
	Correct solution $x \leqslant -\frac{87}{8}, x \geqslant \frac{45}{4}$	A1
		(4)
(c)	 Shape	B1
	Maximum	B1 ft
	One correct minimum	B1
	Fully correct	B1
		(4)
		(10 marks)

9.

(a)	$\text{fg}(5) = \text{f}(3) = \frac{5-"3"}{3\times"3"+2}, = \frac{2}{11}$	M1, A1
		(2)
(b)	$\text{f}^{-1}(x) = \frac{5-2x}{3x+1}$	M1A1
	$x \in \mathbb{R},\ x \neq -\frac{1}{3}$	B1
		(3)
(c)	$\frac{5-\frac{1}{a}}{3\times\frac{1}{a}+2} = 2(a+3)-7$	B1ft
	$\frac{5-\frac{1}{a}}{3\times\frac{1}{a}+2} = 2(a+3)-7 \Rightarrow 4a^2-a-2=0$	M1A1
	$(a=)\ \frac{1\pm\sqrt{33}}{8}$	A1
		(4)
		(9 marks)

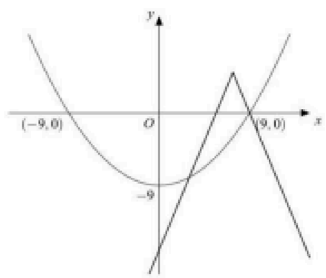
10.

(a) **(i)**	7	B1
ii)	$\frac{9}{k}$	B1
		(2)
b)	$kx-9-2=0$ and $-kx+9-2=0 \Rightarrow \frac{\dots}{k}, \frac{\dots}{k}$	M1
	CVs $\frac{7}{k}, \frac{11}{k}$	A1
	$\frac{7}{k} < x < \frac{11}{k}$	A1
		(3)
(c)	$"-2" = -2 \times "\frac{9}{k}" + 3 \Rightarrow k = \dots$	M1
	$(k =)\ 3.6$	A1
	$k > 3.6$	A1
		(3)
		(8 marks)
lt(b)	$-2 < kx-9 < 2$	
	$\frac{\dots}{k} < x < \frac{\dots}{k}$	M1
	$\frac{7}{k} < x < \frac{11}{k}$	A1A1

11.

(a)	Either $f(x) < 5$ or $f(x) \geqslant 3$	**M1**
	$3 \leqslant f(x) < 5$	**A1**
		(2)
(b)	(i) $y = 5 - \dfrac{4}{3x+2} \Rightarrow \dfrac{4}{3x+2} = 5 - y \Rightarrow \dfrac{4}{5-y} = 3x + 2 \Rightarrow x = ...$	**M1**
	$\left(f^{-1}(x)\right) = \dfrac{1}{3}\left(\dfrac{4}{5-x} - 2\right)$ oe such as $\dfrac{4}{15-3x} - \dfrac{2}{3}$ or $\dfrac{2x-6}{15-3x}$	**A1**
	(ii) Domain is $3 \leqslant x < 5$	**B1ft**
		(3)
(c)	$fg(-\pi) = f\left(\left\lvert 4\sin\left(-\dfrac{\pi}{3} + \dfrac{\pi}{6}\right)\right\rvert\right) = f\left(\left\lvert 4\sin\left(-\dfrac{\pi}{6}\right)\right\rvert\right) = f(2) = ...$	**M1**
	$= 5 - \dfrac{4}{6+2} = \dfrac{9}{2}$	**A1**
		(2)
		(7 marks)

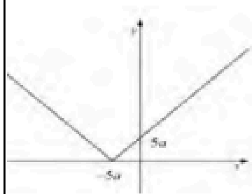
12.

(a)	(i) $\left(\frac{22}{3}, 5\right)$		B1
	(ii) $(0, -17)$		B1
			(2)
b)	$5-(3x-22)=0 \Rightarrow x=\ldots$ **and** $5+(3x-22)=0 \Rightarrow x=\ldots$		M1
	$x=9$ and $x=\frac{17}{3}$		A1
			(2)
(c)		Correct U shape symmetric about y-axis with vertex on negative y-axis.	B1
		Graphs meet $(9,0)$ with $(-9,0)$ also shown.	B1
		Intercept at $(0,-9)$ stated or labelled.	B1
			(3)
(d)	Intersect at $(9,0)$		B1
	Or when $5+3x-22=\frac{1}{9}x^2-9$		M1
	$\Rightarrow (x-3)(x-24)=0 \Rightarrow x=\ldots$		dM1
	Need smaller root $x=3 \Rightarrow y=\ldots$		dM1
	$(3,-8)$		A1
			(5)
			(12 marks)

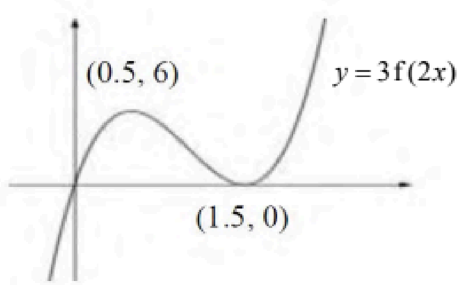
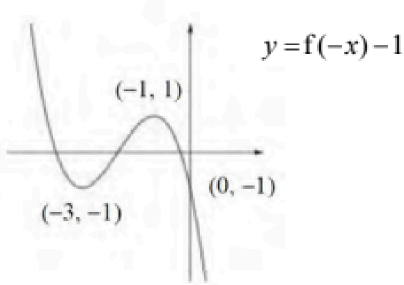
13.

(a)	$2x^2-3x-5=(2x-5)(x+1)$	B1
	$3-\frac{x-2}{x+1}+\frac{5x+26}{2x^2-3x-5}=\frac{3(x+1)(2x-5)-(x-2)(2x-5)+5x+26}{(x+1)(2x-5)}$	M1 A1
	$=\frac{(4x+1)(x+1)}{(x+1)(2x-5)}=\frac{4x+1}{2x-5}$	A1
		(4)
(b)	Correct attempt at inverse $y=\frac{4x+1}{2x-5}\Rightarrow x=\ldots$	M1
	$f^{-1}(x)=\frac{5x+1}{2x-4}$	A1
		(2)
(c)	$2<x<\frac{17}{3}$	M1 A1
		(2)
		Total 8

14.

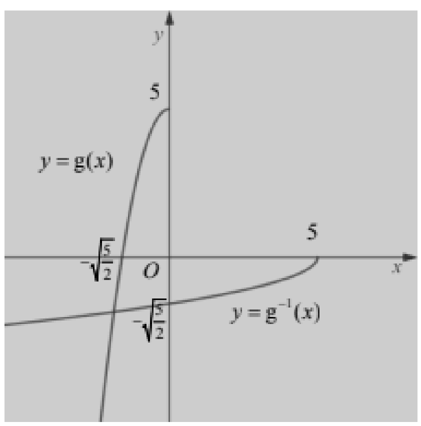
(a)	Either $x=-\frac{a}{3}$ or $y=a$	B1
	Correct coordinates $\left(-\frac{a}{3},a\right)$	B1
		(2)
(b)	 V shape in Quad 1 and 2 with the vertex on the negative x-axis	B1
	Intersects/meets axes at $(0, 5a)$ and $(-5a, 0)$	B1
		(2)
(c)	Attempts to solve a correct equation **E.g. either** $x+5a=-3x-a+a \Rightarrow x=...$ **or** $x+5a=3x+a+a \Rightarrow x=...$	M1
	$x=-\frac{5}{4}a$ **or** $x=\frac{3}{2}a$	A1
	$x=-\frac{5}{4}a$ and $x=\frac{3}{2}a$	A1
	$x=... \Rightarrow y=...$ using either equation	dM1
	$\left(-\frac{5}{4}a, \frac{15}{4}a\right)$ and $\left(\frac{3}{2}a, \frac{13}{2}a\right)$	A1
		(5)
		Total 9

15.

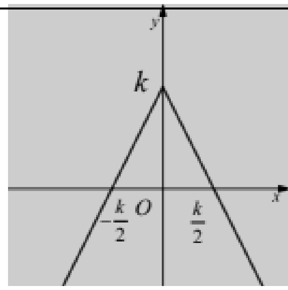
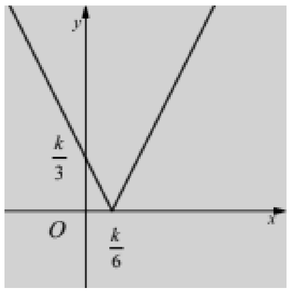
(i)	 Shape (two way stretch)	B1
	Maximum at (0.5, 6)	B1
	Minimum at (1.5, 0)	B1
		(3)
(ii)	 Shape and position	B1
	Minimum at (−3, −1) and maximum at (−1, 1)	B1
	Crosses y-axis at (0, −1)	B1
		(3)
		Total 6

16.

(a)	$\text{fg}(x) = \dfrac{4\left(5-2x^2\right)+6}{5-2x^2-5}$	M1
	$\text{fg}(x) = 3 \Rightarrow \dfrac{26-8x^2}{-2x^2} = 3 \Rightarrow x = -\sqrt{13}$	**d**M1 A1 A1
		(4)
(b)	$y = \dfrac{4x+6}{x-5} \Rightarrow yx - 5y = 4x + 6 \Rightarrow yx - 4x = 5y + 6$	M1
	$\Rightarrow x = \dfrac{5y+6}{y-4}$	A1
	$\Rightarrow \text{f}^{-1}(x) = \dfrac{5x+6}{x-4} \qquad x \in \mathbb{R},\ x \neq 4$	A1
		(3)

(c)		Shape and position for $y = \text{g}(x)$. A parabola starting on the y-axis that extends down in quadrant 2 to at least the x-axis. Condone slips of the pen e.g. if the curve heads back towards the y-axis in quadrant 3 slightly.	B1
		Shape and position for $y = \text{g}(x)$ as above that crosses the x-axis **and** $y = \text{g}^{-1}(x)$ a parabola starting on the x-axis with graphs crossing as shown in quadrant 3. Condone slips of the pen e.g. if $\text{g}^{-1}(x)$ heads back towards the x-axis in quadrant 3 slightly.	B1
		Correct intercepts. This mark is for the 4 correct intercepts as shown. Allow as values or coordinates and allow coordinates the wrong way round (e.g. (0, 5) for (5, 0)) as long as they are in the correct places. The negative intercepts must be exact as shown or equivalent. The graphs must cross the axes at these points and you can ignore any other intercepts.	B1
	Note that labels for g(x) and g^{-1}(x) are not required		
			(3)
			(10 marks)

17.

(a)(i)		Shape	M1
		Correct intercepts.	A1
(ii)		Shape	M1
		Correct intercepts.	A1
			(4)

(b)	Attempts to solve either $k+2x=-2x+\frac{k}{3}$ **or** $k-2x=2x-\frac{k}{3}$	M1
	Achieves either $x=-\frac{1}{6}k$ **or** $x=\frac{1}{3}k$ oe	A1
	Attempts to solve both $k+2x=-2x+\frac{k}{3}$ **and** $k-2x=2x-\frac{k}{3}$	M1
	Achieves either $x=-\frac{1}{6}k$ **and** $x=\frac{1}{3}k$	A1
		(4)
		(8 marks)

18.

(a)	$\dfrac{5x}{x^2+7x+12}+\dfrac{5x}{x+4}=\dfrac{5x+5x(x+3)}{(x+3)(x+4)}$	M1 A1
	$=\dfrac{5x^2+20x}{(x+3)(x+4)}=\dfrac{5x(x+4)}{(x+3)(x+4)}=\dfrac{5x}{x+3}$ *	A1*
		(3)
(b)	$y=\dfrac{5x}{(x+3)}\Rightarrow xy+3y=5x\Rightarrow 5x-xy=3y$	M1
	$\Rightarrow x=\dfrac{3y}{5-y}$ So $\mathrm{f}^{-1}(x)=\dfrac{3x}{5-x}$	A1
	Domain $0<x<5$	A1
		(3)
(c) (i)	$\mathrm{f}(x)=\dfrac{5x}{(x+3)}\Rightarrow(\mathrm{f}'(x)=)\dfrac{5(x+3)-5x}{(x+3)^2}=\dfrac{15}{(x+3)^2}$	M1 A1
(ii)	States (f is an) increasing function with a suitable reason E.g. Since $(x+3)^2$ is positive	A1
		(3)
		(9 marks)

19.

(a)	$\left(\dfrac{13}{3},5\right)$	B1 B1
		(2)
(b)(i)	$\mathrm{f}(x)\ldots 5$	B1 ft
(ii)	10	B1
		(2)
(c)	Attempts to solve either $16-2x\ldots 3x-13+5\Rightarrow$ a value or inequality in x Or $16-2x\ldots -3x+13+5\Rightarrow$ a value or inequality in x	M1
	Both correct critical values $2,\dfrac{24}{5}$	A1
	Selects inside region for their critical values	dM1
	$2<x<\dfrac{24}{5}$	A1
		(4)
(d)	$a=4, b=\dfrac{1}{3}$	B1 ft B1
		(2)
		(10 marks)

Alt(c)	$(11-2x)^2 ... (3x-13)^2 \Rightarrow 121-44x+4x^2 ... 9x^2-78x+169$ $5x^2-34x+48 ... 0 \Rightarrow$ a value or inequality in x $2, \frac{24}{5}$ Selects inside region for their critical values $2 < x < \frac{24}{5}$	M1 A1 dM1 A1

20.

.(a)	$\text{fg}\left(e^2\right) = \text{f}\left(\frac{5}{2}\ln e^2\right) = \frac{12}{\frac{5}{2}\ln e^2 + 1}, = 2$	M1, A1 (2)
(b)	$\text{f}(x) = \frac{12}{x+1}$ $\text{f}^{-1}(x) = \frac{12}{x} - 1$ $0 < x < 12$	M1 A1 B1 (3)
(c)	$\frac{12}{x+1} = \frac{12}{x} - 1 \Rightarrow 12x = 12(x+1) - x(x+1)$ $\Rightarrow x^2 + x - 12 = 0 \Rightarrow x = ...$ Must be 3TQ $x = 3$ only	M1 dM1 A1 (3) **8 marks**
Alts	Solves $\text{f}^{-1}(x) = x \Rightarrow \frac{12}{x} - 1 = x$ leading to quadratic equation, or solves $\text{f}(x) = x \Rightarrow \frac{12}{x+1} = x$ leading to quadratic equation $\Rightarrow x^2 + x - 12 = 0 \Rightarrow x = ...$ Must be 3TQ $x = 3$ only	M1 dM1 A1 **(3)**

21.

.(a)	$(2.5, 3)$ oe	B1 B1
		(2)
(b)	Attempts one solution usually $4x-10+3=3x-2 \Rightarrow x=5$	M1 A1
	Attempts both solutions $-4x+10+3=3x-2 \Rightarrow x=\frac{15}{7}$	dM1 A1
		(4)
(c)	Attempts to solve $y=kx+2$ with $x=2.5, y=3$ or states that $k<4$	M1
	$k ... \frac{2}{5}$	A1
	States $\frac{2}{5}<k<4$	A1
		(3)
		9 marks

22.

(a)	$\mathrm{ff}(6)=\mathrm{f}(13)=-1$	M1 A1
		(2)
b)	Attempts $21+2(2-x)=5x \Rightarrow x=...$ or $21-2(x-2)=5x \Rightarrow x=...$	M1
	$x=\frac{25}{7}$ only	A1
		(2)
(c)	Either $k<21$ or $k \geqslant 17$	M1
	$17 \leqslant k<21$	A1
		(2)
(d)	$a=\frac{1}{7}$ $b=4$	B1 B1
		(2)
		(8 marks)

23.

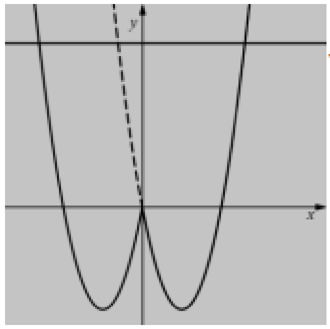
(a)	$(5,10)$	B1 B1
		(2)
(b)	Attempts to solve e.g. $-2(x-5)+10...6x \Rightarrow x..."\frac{5}{2}"$	M1
	$x<\frac{5}{2}$ o.e.	A1
		(2)
(c)	$(7,30)$	B1ft, B1ft
		(2)
		(6 marks)

24.

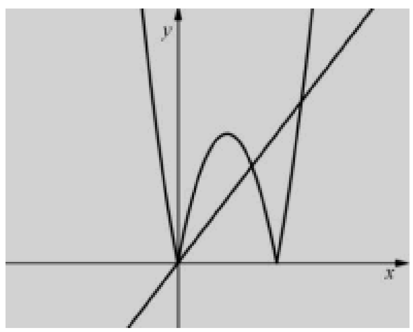
(a)	$\text{gf}(2)=\text{g}\ (3)=3^2+5$	M1
	$=14$	A1
		(2)
(b)	Correct attempt at inverse $y=6-\frac{21}{2x+3} \Rightarrow 2x+3=\frac{21}{6-y} \Rightarrow x=\left(\frac{21}{6-y}-3\right)\div 2$	M1
	$\text{f}^{-1}(x)=\frac{3x+3}{2(6-x)}$ or $\text{f}^{-1}(x)=\frac{21}{2(6-x)}-\frac{3}{2}$	A1
	$-1 \leqslant x<6$	B1
		(3)
(c)	$\text{gg}(x)=126 \Rightarrow \left(x^2+5\right)^2+5=126 \Rightarrow x^2+5=\sqrt{121}$	M1
	$\Rightarrow x^2="6" \Rightarrow x=(\pm)\sqrt{6}$	dM1
	$\Rightarrow x=\pm\sqrt{6}$	A1
		(3)
		Total 8

25.

(a)

 $y = 48$ Graphical interpretation of $f(\lvert x \rvert) = 48$	
One of $8, -8$	B1
Attempts to solve an appropriate equation E.g. $2x^2 - 10x = 48 \Rightarrow x^2 - 5x - 24 = 0 \Rightarrow (x \pm 8)(x \pm 3) = 0 \Rightarrow x = ...$	M1
$x = 8, -8$ with no additional values	A1
	(3)

(b)

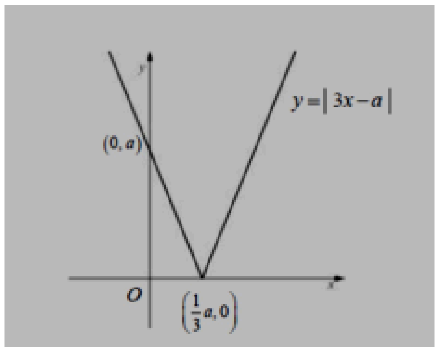
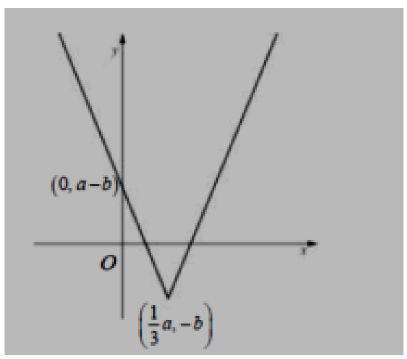
 $y = \frac{5}{2}x$ Graphical interpretation of $\lvert f(x) \rvert ... \frac{5}{2}x$	
Attempts to solve $2x^2 - 10x = \frac{5}{2}x \Rightarrow 4x^2 - 25x = 0 \Rightarrow x = \frac{25}{4}$ OR attempts to solve (o.e.) $10x - 2x^2 = \frac{5}{2}x \Rightarrow 4x^2 - 15x = 0 \Rightarrow x = \frac{15}{4}$	M1
Attempts to solve $2x^2 - 10x = \frac{5}{2}x \Rightarrow 4x^2 - 25x = 0 \Rightarrow x = \frac{25}{4}$ AND attempts to solve (o.e.) $10x - 2x^2 = \frac{5}{2}x \Rightarrow 4x^2 - 15x = 0 \Rightarrow x = \frac{15}{4}$	dM1

	Achieves both critical values $x = \frac{15}{4}$, $x = \frac{25}{4}$	A1
	Correct set of values $x \leqslant \frac{15}{4}$ or $x \geqslant \frac{25}{4}$	A1
		(4)
		Total 7

26.

(a)	$y = \frac{4x+3}{x-2} \Rightarrow yx - 2y = 4x + 3$ $\Rightarrow yx - 4x = 2y + 3$	$y = 4 + \frac{11}{x-2} \Rightarrow y - 4 = \frac{11}{x-2}$ $x - 2 = \frac{11}{y-4}$	M1
	$\Rightarrow x = \frac{2y+3}{y-4} \Rightarrow f^{-1}(x) = \frac{2x+3}{x-4}$	$\Rightarrow x = 2 + \frac{11}{y-4} \Rightarrow f^{-1}(x) = 2 + \frac{11}{x-4}$	A1
	$x \neq 4$		B1
			(3)
(b)	$\mathrm{ff}(x) = \dfrac{4 \times \frac{4x+3}{x-2} + 3}{\frac{4x+3}{x-2} - 2}$ or $\mathrm{ff}(x) = 4 + \dfrac{11}{4 + \frac{11}{x-2} - 2}$		M1
	$= \frac{4 \times (4x+3) + 3(x-2)}{4x + 3 - 2(x-2)} = \frac{19x+6}{2x+7}$		dM1 A1
			(3)
(c)	Either $x = 1$ or $y = 38$		M1
	$(1, 38)$		A1
			(2)
			(8 marks)

27.

7 (a)(i)		Shape Vertex $\left(\frac{1}{3}a, 0\right)$ y intercept $(0, a)$	B1 B1 B1
(a)(ii)		Shape Vertex $\left(\frac{1}{3}a, -b\right)$ y intercept $(0, a-b)$	B1ft B1ft B1ft
(b)	Attempts to solve $-3x + a - b = 5x \Rightarrow x = ...$ $\Rightarrow x = \frac{a-b}{8}$ ONLY		M1 A1 **(2)** **(8 marks)**