

black art of manipulating numbers

TRIG

fund #1 - triangle angles add up to 180

fund #2 - right triangles are called "special"

$\sin \theta = \text{opposite/hypotenuse}$

$\cos \theta = \text{adjacent/hypotenuse}$

$\tan \theta = \text{opposite/adjacent}$

assume circle with radius 1 (unit circle)

angle at center, then $\sin \text{ angle} = y$, $\cos \text{ angle} = x$

$\sin 90 = 1 = \pi/2$

$\cos 90 = 0$

$\sin 0 = 0$

$\cos 0 = 1$

$\sin 180 = 0$

$\cos 180 = -1$

application: Projector throw

ratio of distance to image width

ex: $T = 2:1$ from 2 meters image width is 1

$\tan \theta/2 = 1/2t$

from throw distance

$\theta = 2 \cdot \arctan(1/2 \cdot \text{throw})$

VECTORS

vector == offset

vector notation

$\rightarrow \quad \rightarrow \quad \rightarrow$

$V = (10,5) = 10i + 5j = V_x i + V_y j + V_z k$

magnitude = $\sqrt{x^2 + y^2}$

Vector arithmetic is component-wise addition is commutative

To visualize = draw vectors running tip to tail

Vector subtraction is not commutative

To visualize = put both tails at zero and draw vector between tips

Vector-scalar math

can't add

multiply is scaling, i.e. $(10,5) \times 2 = (10,5) + (10,5)$

Vector multiplication is mathematically undefined

Vector libraries perform component-wise multiplication

Vector multiplication is possible through dot and cross PRODUCT

DOT PRODUCT

Dot = $(x,y) \cdot (x_0,y_0) = \text{mag}(v_0) \cdot \text{mag}(v_1) \cdot \cos \theta$ (θ is angle between) = $(x_0x_1) + (y_0y_1) + (z_0z_1)$

dot product is "shadow" of a vector is a scalar

normalizing a vector = make magnitude 1

$\text{sqrt}(v_x^2 + v_y^2 + v_z^2) / (v_x^2 + v_y^2 + v_z^2)^{0.5}$;

reflection

vector reflected off normal n

take tangent and normal

tangent

CROSS PRODUCT

cross = $v_0 \times v_1 = \text{mag}(v_0) \cdot \text{mag}(v_1) \cdot \sin \theta$

dot product is shadow (scalar), cross product is vector of rotation, that is what is the vector you rotate v_0 around to get v_1

getting poly strips from a vector v_0 defined by p_1 and p_2

need to get perpendicular vectors v_1 and v_2

$p_2 - p_1$ gives us direction vector (p_0)

we need an up vector $up = (0,0,1)$

$p_0 \times up$ gives us p_1

multiply $p_1 \cdot \text{normalized}$ * desired width(w)/2 = offset vector

$p_1 + \text{offset}$ and $p_1 - \text{offset}$ gives us v_1

$p_2 + \text{offset}$ and $p_2 - \text{offset}$ gives us v_2

optimization tip for finding contour direction

put neighboring vectors tail to tail

take dots

check cross only if dot meets thresh

finding "hills" in 3d

cross dot up

MATRICES and XFORMS

Matrices == denote frame of reference for xform

Matrix == xform applied to axes

Matrix*Vector number of columns in matrix must match number of elements in vector

Matrix*Vector = Matrix

Generally use 4x4

identity matrix:

[1 0 0] = x axis

[0 1 0] = y axis

[0 0 1] = z axis

using identity matrix

example camera look

camera o, target t

direction vector $dv = (t-o).norm$

world up vector $wup = (0,0,1)$

right vector $rv = (d \times wup).norm$

object up vector $ov = (d \times r)$

look matrix =
$$\begin{bmatrix} rv & 0 & 0 & 0 \\ 0 & ov & 0 & 0 \\ 0 & 0 & dv & 0 \\ ox & oy & oz & 1 \end{bmatrix}$$

Wow...all this shit finally makes sense, holy crap. Thanks, Memo!