Core Problems:

1.1.4 How big is a million?

Investigating a Proportional Relationship

Can you imagine a tower built out of one million pennies? How tall would it be? In this lesson, you will work with your team to predict just how tall such a tower would be. To make sense of this question, you will measure some pennies and investigate the relationship between the height of a tower of pennies and the number of pennies in that tower. This is an example of a special relationship called a **proportional relationship**, which you will learn more about later in this course.

1-23. How tall would a tower of a million pennies be? Would it be taller than your school building? Would it be taller than Mount Everest? (Mount Everest is approximately 8848 meters or 29,029 feet high.)

Discuss these questions with your team and make a prediction. Record your prediction on the table provided by your teacher.

1-24. When you are working with your team to answer the "Penny Tower" questions, as well as other problems in this course, it is important to work effectively with other people. Effective math conversations are a valuable part of the learning process throughout this course. Choose a member of your team to read the "Collaborative Learning Expectations" below out loud.

COLLABORATIVE LEARNING EXPECTATIONS

Working with other students allows you to:

- Develop new ways of thinking about mathematics,
- Learn to communicate about math, and
- Understand ideas better by having to explain your thinking to others.

The following expectations will help you get the most out of working together.

T Together, work to answer questions.
E Explain and give reasons.
A Ask questions and share ideas.
M Members of your team are your first resource.
S Smarter together than apart.

1-25. TINY TOWERS

To begin to investigate this question, start by collecting data. Parts (a) and (b) will lead you through the data-collection process.

a. How many pennies does it take to build a tower that is one centimeter tall? Use the tools provided by your teacher to answer this question.

b. Work with your team to complete the missing information in the table below. Be prepared to explain your reasoning to the class.

Height of Tower (cm)	# of Pennies
1	
2	
3	
4	
5	
10	
20	

30	
100	

1-26. THE HUNDRED-PENNY TOWER

"I have an idea!"Carol said. "If I know how tall a tower of one hundred pennies would be, maybe that can help me figure out how tall a tower of one million pennies would be."

a. Discuss this idea with your team. How could Carol's idea work?

b. Use the table below. Work with your team to figure out how tall a tower of one hundred pennies would be. Can you find more than one way to figure this out? Be sure that each member of your team is prepared to explain your team's reasoning to the class.

Height of Tower (cm)	# of Pennies
1	
2	
3	
4	
5	
	100

1-27. THE MILLION-PENNY TOWER

Now it is time to answer the big question: How tall would a tower of one million pennies be?

Your task: Work with your team to calculate the height of a tower of one million pennies as accurately as you can. Can you find the height more than one way? Be prepared to explain your ideas to the class.

Discussion Points

Where would one million pennies belong in our table?

How can the height of the hundred-penny tower help us?

What is the relationship between the number of pennies and the height of the tower?

Further Guidance

1-28. Carol said, *"To use my hundred-penny height to find the height of one million pennies, I have to know how many hundreds are in one million."*

a. How many groups of 100 pennies are in 1000 pennies?

b. How many 1000s are in 1,000,000? How do you know?

c. Now work with your team to figure out how many towers of 100 pennies it would take to build a tower of 1,000,000 pennies. Be prepared to explain your reasoning to the class.

d. Use this result along with the height of a hundred-penny tower that you found in problem 1-27 to find the height of a tower of one million pennies.

1-29. Anouk was working with Carol. *"I have another way,"* she said. *"I can see in the table that the number of pennies is always seven times the number of centimeters. How can we reverse that to find the number of centimeters if we know the number of pennies?"*

a. Discuss Anouk's question with your team. How can you find the height of the tower if you know the number of pennies in it?

b. Find the height of a tower of 1,000,000 pennies using this method. Does it agree with the result you got in problem 1-28?

Further Guidance section ends here.

1-30. Anouk wants to compare the million-penny tower to the height of Mount Everest. She read on the Internet that Mount Everest is approximately 8848 meters tall. Her calculation for the height of the million-penny tower is in centimeters.

a. How can she change the units so that she can compare them? Discuss this with your team.

b. Compare your own calculation for the million-penny tower to the height of Mount Everest.

c. Look at the predictions made by your class at the beginning of this lesson and decide which team came closest to predicting the actual height.

1-31. How accurate is your result? Is there any reason to believe that there may be some amount of error in your calculation? How much do you think this error would matter? Be prepared to share your ideas with the class.

1-32. Additional Challenge: How many pennies would be in a tower that is 10 miles high?

1-33. Additional Challenge: The Taipei-101 is the second-tallest building in the world. There is a staircase up to the 91st floor with an average of 22 steps from one floor to the next.

a. How many steps would you have to climb to get from the 1st floor to the 91st floor?

b. When you are standing on the 91st floor, you are 1285 feet above the ground. How many stories are below you? About how many feet tall is each story of the building?

c. About how high is each step?

1-34. LEARNING LOG

Throughout this course, you will be asked to reflect about your understanding of mathematical concepts in a Math Blog. Your Learning Log Math Blog will contain explanations and examples to help you remember what you have learned throughout the course. It is important to write each post of your Math Blog in your own words so that later you can use your Math Blog as a resource to refresh your memory. Remember to label each entry with a title and **a date** so that it can be referred to later.

For your first post, consider what you know about proportional relationships. The relationship between the number of pennies in a stack and the height of that stack is an example of a proportional relationship. Talk with your team about how you can describe this relationship.

Then record your ideas in your Math Blog, using numbers, words, and tables to help show your thinking. Title this entry **"Beginning to Think About Proportional Relationships"** and label it with today's date.