FORM FOUR - MATHEMATICS

MARKING SCHEME

1 (a) Given K = 4.56×10^{5}, P = 5.8×10^{2} and Q = 0.256,

$$\frac{K}{\sqrt{Q}\times P} = \frac{4.56\times10^{5}}{\sqrt{0.256}\times5.8\times10^{2}} = \frac{456000}{293.4593} = 1553.8778$$

$\approx \underline{1.554}\times10^{3}$ ……….. …………………………………. (3 marks)

(b) Total given marks 92.

Marks awarded for writing 28% Marks awarded for reading 34%

Marks awarded for practical

Marks awarded for spelling (remaining)

100%$-$ 75% =25% ……………………………………(3 marks)

Therefore, Marks awarded foe spelling

$= \frac{25}{100}\times92$

= <u>23 marks</u>

2. (a). Given $\frac{1}{16^{(3-2y)}} \times 4^{4} \times 32^{(1-y)} = \frac{1}{8^{(2y)}}$

$$2^{(8y-12)}\times 2^{(8)}\times 2^{(5-5y)} = \times 2^{(-6y)}$$

$$2^{(8y-5y-12+5+8)} = 2^{(-6y)}$$

$$2^{(3y+1)} = 2^{(-6y)}$$

$$3y + 1 =- 6y$$

$$6y + 3y =- 1$$

$$9y =- 1$$

$y =- 1/9$ (3 marks)

(b) Given $6 \;-\frac{1}{4}625 \;-\; \left(\frac{16}{3}\right) \;-\; 43$

$=6^{3} \;-- \;16 \;+\; 3 \;-\; 3^{4}$

$= 216 \;-- \;16 \;+\; 3 \;-\; 81$

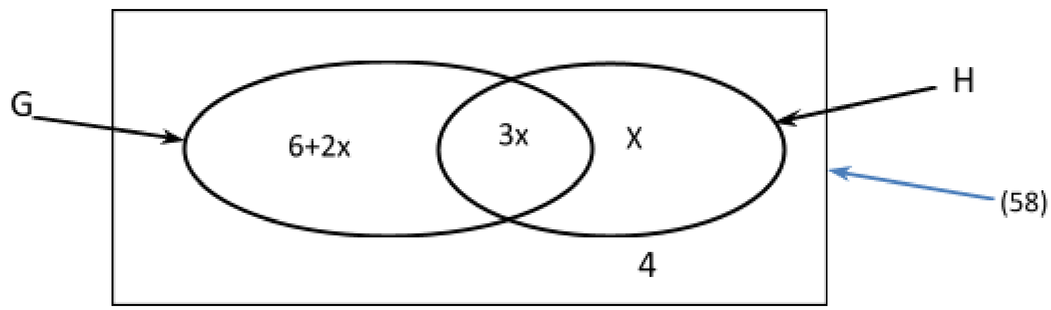
$= \left(\frac{216\times3}{5\times16\times81}\right)\left(\frac{648}{6480}\right)(0.1)$

(10)

$-\;1$ (3 marks)

3 (a) Let G be the set of those students who like geography, H be the set of students who like history and x is the set of those who like history only

By a Venn diagram



$6 \;+\; 2x \;+\; 3x \;+\; x \;+\; 4 \;=\; 58$

$6x \;+\; 10 \;=\; 58$

$6x \;=\; 48$ (3 marks)

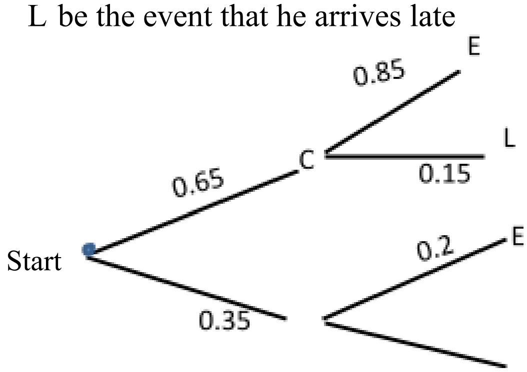
$x \;=\; 8$

<u>Therefore, the number of those who like Geography and History is 3x8= 24</u>

3(b)(i) Let C be the event that Davis goes to school by a car

C' Be the event that he does go by a car, E be the event that he arrives early

L be the event that he arrives late



(1 mark)

0.8 L

$P(C \cap L) + P(C' \cap L) = 0.65 \times 0.15 + 0.35 \times 0.8$

$= 0.3775$ 2 marks

4. (a) Given L_1 passes through P(-9, 5)

L_2 : A(9, 0) and B(0, -12)

$M_2 = \frac{-12-0}{0-9} = \frac{4}{3}$ 1 mark)

Since L_1 is perpendicular to L_2, $M_1 \times M_2 = -1$

$\frac{4}{3} \times M_1 = -1$

$M_1 = -\frac{3}{4}$ 1 mark

Now, from $y = m(x - x_1) + y_1$

$y = -\frac{3}{4}(x -- 9) + 5$

$y = -\frac{3}{4}x - \frac{7}{4}$ 1 mar

(b) (i) Given vectors $\underline{a} = 4\underline{i} - 3\underline{j}$, $\underline{b} = -\underline{i} + 3\underline{j}$ and $\underline{c} = \frac{1}{3}\underline{i} - \frac{1}{6}\underline{j}$ and

$\underline{v} = 2\underline{a} - 12\underline{c} + 3\underline{b}$

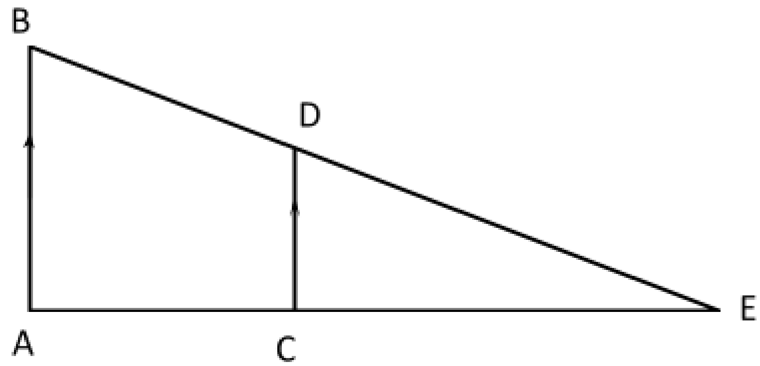
$\underline{v} = 2\begin{pmatrix} 4 \\ -3 \end{pmatrix} - 12\begin{pmatrix} \frac{1}{3} \\ \frac{-1}{6} \end{pmatrix} + 3\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

$\underline{v} = \begin{pmatrix} 8 \\ -6 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \end{pmatrix} + \begin{pmatrix} -3 \\ 6 \end{pmatrix}$

$\underline{v} = \underline{i} + 5\underline{j}$ 1 mark

(ii) $|\underline{v}| = \sqrt{1^2 + 5^2} = \sqrt{26}$ = 5.10 1 mark

(iii). the unit in the direction of vector $\frac{\underline{v}}{|\underline{v}|} = \frac{\underline{i}}{\sqrt{26}} + \frac{5\underline{j}}{\sqrt{26}}$ 1 mark



The ratio of the corresponding sides of similar triangles is equal

$\frac{\overline{AB}}{\overline{DC}} = \frac{\overline{AE}}{\overline{CE}} = \frac{\overline{BE}}{\overline{DE}}$ 1 mark

$\frac{54\ cm}{36\ cm} = \frac{CE+13.5}{CE}$ Cross multiplication $\rightarrow$ $2(\overline{CE} + 13.5\ cm) = 3\ \overline{CE}$

$= 2\overline{CE} + 27$ cm $= 3\overline{CE}$

$\overline{CE} = 27\ cm$ 1 mark

$\frac{3}{2} = \frac{BD+30}{30}$ Cross multiplication $2(\overline{BD} + 30\ cm) = 3\times30\ cm$

$(2\overline{BD} + 60\ cm) = 90\ cm$

$\overline{BD}$=15 cm 1 mark

(b) (i). Given $n = 12$, $perimeter = 217.41cm$, r =?

From $p = 2nr \sin sin \left(\frac{180}{n}\right)$ 0.5 mark

217.41 cm = $2\times12\times r \sin sin \left(\frac{180}{12}\right)$

217.41 cm = $24\times r \sin sin\ (15)$

$r = \frac{217.41}{24 \sin sin \left(15^0\right)}$

$r = 35\ cm$ 1 mark

(ii) Area of a regular polygon is given by = $\frac{1}{2}n \times r\times r \sin sin \left(\frac{360^0}{n}\right)$

$A = \frac{1}{2} \times 12 \times 35\times35 \sin sin \left(\frac{360^0}{12}\right)$

$A = 210\times35 \sin sin \left(30^0\right)$ =3,675 cm^2 1.5 marks

6. (a) The amount of money in USD \$ 4600

1 \$ = 2350 Tshs

4600\$ =? Cross multiplication 4600×2350/=

10,810,000 Tshs.

Then 5510,000 spent

10, 8100, 000/=−5510, 000/= 5,300,000 remained

Then 1 Ksh= $18.50\ Tshs$

$? = 5,300,000$/= Cross multiplication

$286,486\ Kshs$ Was received by Mr. Brooklyn in Kshs 3 marks

6(b) Given $m = 240$g, b=48 cm and $l = 6cm$

m_2=? B=30 cm and $l = 20cm$

$m \propto \frac{b}{l^2}$ $\rightarrow m = \frac{kb}{l^2}$

$\rightarrow k = \frac{ml^2}{b}$ 1 mark

$k = \frac{240g \times 6^2}{48}$

$k = 180$ 1 mark

Again $m = \frac{180 \times 30}{20^2}$

$m = 13.5g$ 1 mark

7.(a)(i) Given % profit 15% , Selling price (S.P) =575,000/= Buying price (B.P)=?

From % $profit = \frac{profit\ made}{B.P} \times 100\%$

% selling price ➡ 115% =575,000/=

% buying price 100% = ? Cross multiplication

Buying price =$\frac{575,000 \times 100\%}{115\%}$ ➡ Buying price 500,000/= 1 mark

The ration of buying price the selling price BP: SP 500,000:575,000=20:23 1 mark

(ii) Profit made Selling price –buying price 575,000-500,000= 75,000/=   1 mark

(b) BRAC trading account as at the end 0f 31st Dec 2020

Opening stock		100,000	Sales	500,000
Purchases	240,000		Return inwards Tshs	50,000
Add:C.inwards…………….	40,000		Net sales	450,000
Less R.outwards	20,000			
Net purchases		260,000		
COGAS		360,000		
Less: Closing stock Tshs ……………		160,000		
COGS		200,000		
Gross profit C/D		250,000		
		450,000		450,000
				3 marks

8(a). Given the first term of an A.P A_1=a and common difference d

The sum of the first n terms and the sum of the first 2n of the same A.P are equal

Therefore, the sum of the first n terms is given by $s_n = \frac{n}{2}[(2a + (n - 1)d]$

and the sum of the first 2n terms is given by $s_{2n} = \frac{2n}{2}[(2a + (2n - 1)d]$

$s_n = s_{2n}$ $\frac{n}{2}[(2a + (n - 1)d] = s_{2n} = \frac{2n}{2}[(2a + (2n - 1)d]$

$\frac{n}{2}[(2a + (n - 1)d] = n[(2a + (2n - 1)d]$

$(2a + (n - 1)d = 2[(2a + (2n - 1)d]$

$(2a + (n - 1)d = (4a + (4n - 2)d]$

$(2a) = (nd - d) - 4nd + 2d$

$2a = d - 3nd$

$2a = d(1 - 3n)$ Hence shown   3 marks

(b). Given $G_3 = 300$ and $G_6 = 37500$

From $G_n = G_1 r^{(n-1)}$

$\frac{G_6}{G_3} = \frac{37500}{300}$

$\frac{G_1 r^5}{G_1 r^2} = \frac{375}{3}$ $1\ mark$

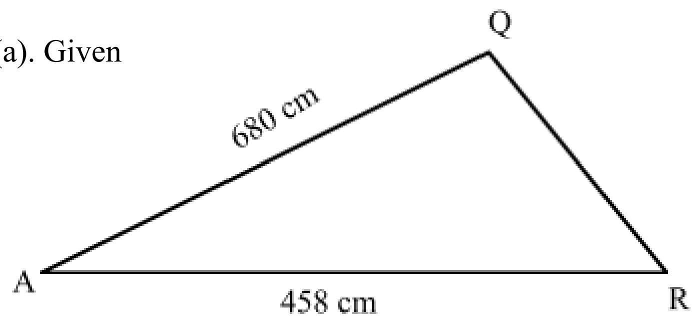
$r^3 = 125$ ⟶ $r^3 = 5^3$

$r = 5$ 1 mark

$25G_1 = 300$ $\frac{25}{25}G_1 = \frac{300}{15}$

$G_1 = 12$ 1 mark

9 (a). Given



The length $\overline{QR}$ can be calculated by cosine rule

$(\overline{QR^2}) = \left(\overline{AQ^2}\right) + \left(\overline{AR^2}\right) - 2\overline{AQ} \times \overline{AR} \cos cos\ \hat{A}$ 1 mark

$(\overline{QR^2}) = 680^2 + 458^2 - 2 \times 680 \times 458 \cos cos\ (32^0)$

$(\overline{QR^2}) = 462,400 + 209,764 - 924,800 \times 0.8480$ 1 mark

$(\overline{QR^2})$ =672,164-528,232 =143,931.8

$\overline{AQ}$=379.38 m 1 mark

9(b) given $\tan tan\ \beta = 1\frac{7}{8}$ $\tan tan\ \beta = \frac{15}{8}$ but $\tan tan\ \beta = \frac{opposite}{adjacent}$

By Pythagoras theorem $c^2 = a^2 + b^2$

$15^2 + 8^2 = c^2$

$225 + 64 = c^2$

$289 = c^2$

$\sqrt{c^2} = \sqrt{289}$

C =17 1 mark

$Sin\beta = \frac{15}{17}$ and $cos\beta = \frac{8}{17}$

Required $\frac{2Sin\beta + Cos\beta}{Sin\beta - 3} = \frac{2\left(\frac{15}{17}\right)+\frac{8}{17}}{\left(\frac{15}{17}\right)-3} = \frac{\left(\frac{30}{17}\right)+\frac{8}{17}}{\left(\frac{15}{17}\right)-3} = \frac{\frac{38}{17}}{-\frac{36}{17}} = \frac{38}{17} \times -\frac{17}{36} = -\frac{38}{36}$

Therefore $\frac{2Sin\beta + Cos\beta}{Sin\beta - 3} = -\frac{19}{18}$ 2 marks

10(a). Abigail is five years older than her Grace; Four years to come the product of their age will be 150.

Let x be Grace's years old

$Grace \quad Abigael$

(x+4)× ((x+5) +4)=150   1 mark

(x+4)(x+9)=150

$x^2 + 4x + 9x + 36 = 150$

$x^2 + 4x + 9x + 36 - 150 = 0$

$x^2 + 13x - 114 = 0$ $a = 1,\ b = 13\ and\ c = -114$

General quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ 1 mark

$x = \frac{-13 \pm \sqrt{13^2 - 4\times1\times-114}}{2\times1}$

$x = \frac{-13 \pm \sqrt{169+456}}{2} = \frac{-13 \pm \sqrt{625}}{2} = \frac{-13 \pm 25}{2} = \frac{-13+25}{2} = \frac{12}{2}$

$x = 6$

Therefore, Grace is 6 years old and Abigail is 6 is 6+5=11 years old. 1 mark

(b). given $2x^2 - x - 15 = 0$

$a = 1,\ b = -1\ and\ c = -15$ 1 mark

$2x^2 - x = 15$ Divide by 2 through out

$\frac{2}{2}x^2 - \frac{x}{2} = \frac{15}{2} \rightarrow \quad x^2 - \frac{x}{2} = \frac{15}{2}$ ---------------- (i)

We add $\left(\frac{b}{2a}\right)^2$ on both sides to equation (i) i.e. $\left(\frac{-1}{4}\right)^2$

$x^2 - \frac{x}{2} + \left(\frac{-1}{4}\right)^2 = \frac{15}{2} + \left(\frac{-1}{4}\right)^2$

$\left(x - \frac{1}{4}\right)^2 = \frac{15}{2} + \frac{1}{16} \quad \rightarrow \left(x - \frac{1}{4}\right)^2 = \frac{121}{16}$ Under root on both sides 1 mark

$\sqrt{\left(x - \frac{1}{4}\right)^2} = \sqrt{\frac{121}{16}}$

$\left(x - \frac{1}{4}\right) = \pm\frac{11}{4}$

$x = \frac{11}{4} + \frac{1}{4} = \frac{12}{4} = 3$ Or $x = -\frac{11}{4} + \frac{1}{4} = \frac{-10}{4} = \frac{-5}{2}$ 1 mark

Frequency distribution table

C.I	x	f	d	fd	c.f	U.R.L
$32 - 39$	35.5	4	-24	-96	4	39.5
$40 - 47$	43.5	10	-16	-160	14	47.5
$48 - 55$	51.5	8	-8	-64	22	55.5
$56 - 63$	59.5	5	0	0	27	63.5
$64 - 71$	67.5	7	8	56	34	71.5
$72 - 79$	75.5	3	16	48	37	79.5
$80 - 87$	83.5	2	24	48	39	87.5
$88 - 95$	91.5	1	32	32	40	95.5
1mark	0.5 mark	N=40 1 mark	0.5 mak	$\sum fd$=-136 0.5 mark	0.5 mark	

Given assumed mead A=59.5

From $\overline{X} = A + \frac{\sum fd}{\sum f}$ (1 mark)

$\overline{X} = 59.5 + \frac{-136}{40}$ (1 mark)

$\overline{X} = 56.1$ (1 mark)

Therefore , the estimated median is 53.4 (3 marks)

12.(a) (i). Given two towns P (32^0S, 12^0E) and Q (32^0S, 28^0W)

$\theta = 40^{\circ}$ $\alpha = 32^{\circ}$, R=6370 km and $\pi = 3.14$ (0.5 mark)

Length PQ= $\frac{\pi R\theta COS\alpha}{180^0}$ (0.5 mark)

$= \frac{3.14 \times 6370 \times 40 COS32^0}{180^0}$ km (0.5 mark

Length PQ =3769.44 km ≈ 3,769 km (1 mark)

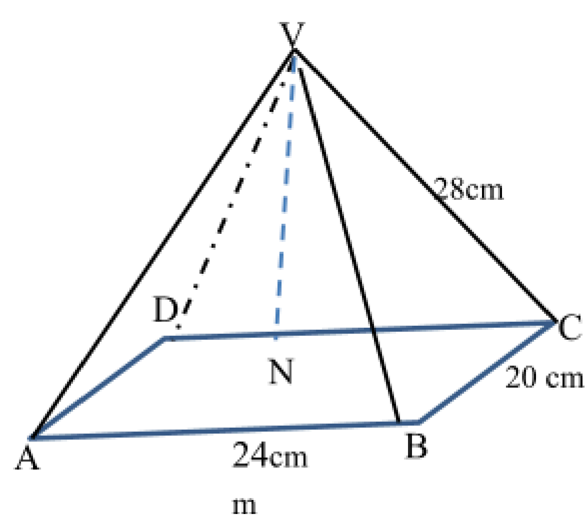
(ii) Given speed =75 km/ hour (1 mark)

75 km = 1 hour (0.5 mark)

3,769 km=? Cross multiplication

It will take 50 hours and 15 minutes (1 mark)

12(b). Given pyramid



Consider ΔABC

$(\overline{AB^2}) + (\overline{BC^2}) = (\overline{AC^2})$ (1 mark)

$(\overline{AC^2}) = 24^2 + 20^2$

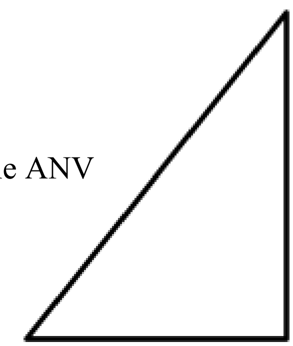
$(\overline{AC^2}) = 976cm$

$\sqrt{\overline{AC^2}} = \sqrt{976}$

$\overline{AC} = 31.25\ cm$

$\overline{AN}$=15.62cm (0.5 mark)

Again consider the triangle ANV



$(\overline{AN^2}) + \left(\overline{VN^2}\right) = \left(\overline{AV^2}\right)$

$(\overline{VN^2}) = 28^2 - 15.62^2$

$\overline{VN} = 23.24\ cm$ (1 mark)

(ii). Angle VAN tan $tan\ \emptyset\ = (\frac{23.24}{15.62})$=1.4875 (1 mark)

$\emptyset\ = (1.487)$

$\emptyset\ = 56^{\circ}$ (1 .5 marks)

13. (a) given matrix B=$(3\ \ -1\ \ -2\ 5\ )$

$|B|$=$3\times5\ -(-\ 2\times\ -\ 1)$

$|B| = 13$

B^{-1}=$\frac{1}{13}(5\ 1\ 2\ 3\ )$ $B^{-1} = \left(\frac{5}{13}\ \frac{1}{13}\ \frac{2}{13}\ \frac{3}{13}\ \right)$ $(\ 2\ marks)$

$\left(\frac{5}{13}\ \frac{1}{13}\ \frac{2}{13}\ \frac{3}{13}\ \right)(3\ \ -1\ \ -2\ 5\ )\left(\frac{x}{y}\right) = \left(\frac{5}{13}\ \frac{1}{13}\ \frac{2}{13}\ \frac{3}{13}\ \right)\left(\frac{16}{11}\right)$

$(1\ 0\ 0\ 1)\left(\frac{x}{y}\right) = \left(\frac{\frac{5\times16}{13}+\frac{1\times11}{13}}{\frac{2\times16}{13}+\frac{3\times33}{13}}\right) = \left(\frac{\frac{91}{13}}{\frac{51}{13}}\right)$ (1 mark)

$\left(\frac{x}{y}\right) = \left(\frac{7}{5}\right)$ (1 mark)

(b). $Given\ mtrix\ (x - 2\ 4\ 2\ x)$

$|x - 2\ 4\ 2\ x| = 0$ its determinant is equal to zero (1 mark)

$(x)(x - 2) - 8 = 0$

$x^2 - 2x - 8 = 0$

$x = 4\ or\ x = - 2$ (1 mark)

c). $M_{y=-x}P(x,y)$. $M_xP(x,y)$, Given point $P(-1, 6)$

For $M_xP(x,y)$ transomartion matrix T= $(1\ 0\ 0\ -1)$

$p'\left(\frac{x}{y}\right)' = (1\ 0\ 0\ -1)\left(\frac{-1}{6}\right) = \left(\frac{-1}{-6}\right)$ (2 marks)

For $M_{y=-x}P(x,y)$ transomartion matrix T= $(0\ -1\ -1\ 0)$

$p''\left(\frac{x}{y}\right)'' = (0\ -1\ -1\ 0)\left(\frac{-1}{-6}\right) = \left(\frac{6}{1}\right)$

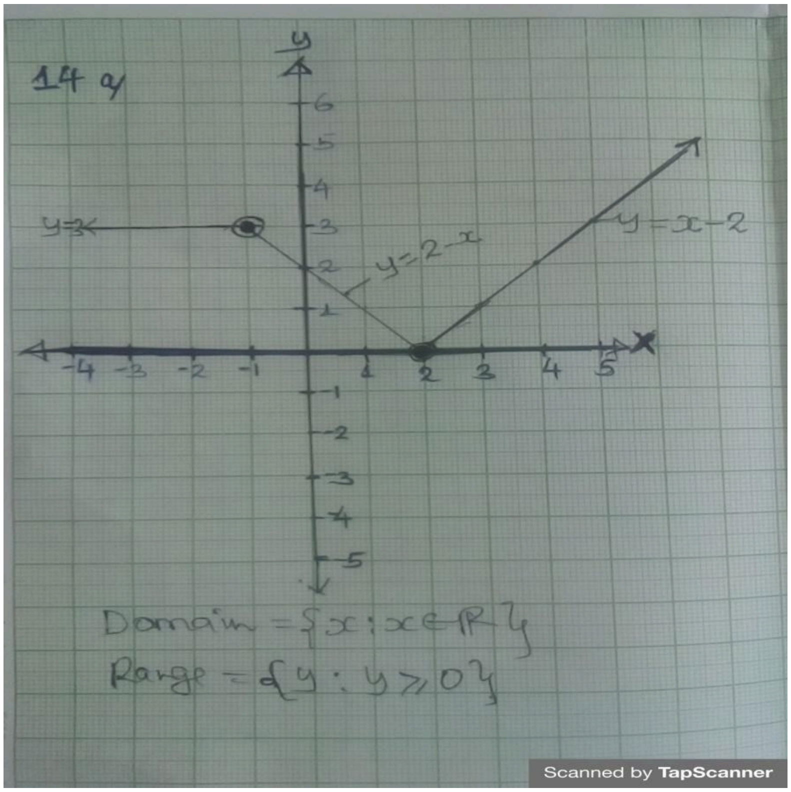
$p''\left(\frac{x}{y}\right)'' = \left(\frac{6}{1}\right)$ (2 marks)

14 (a). $y = 2 - x$

x	-1	0	1	2
y	3	2	1	0

$y = x - 2$ (0.5 mak)

x	2	3	4	5	6
y	0	1	2	3	4 (0.5mark)



Graph (3 marks)

(domain and range of f 0.5 mark each)

14 (b). Let x be the number of bags of wheat and y be the number of bags of rice.

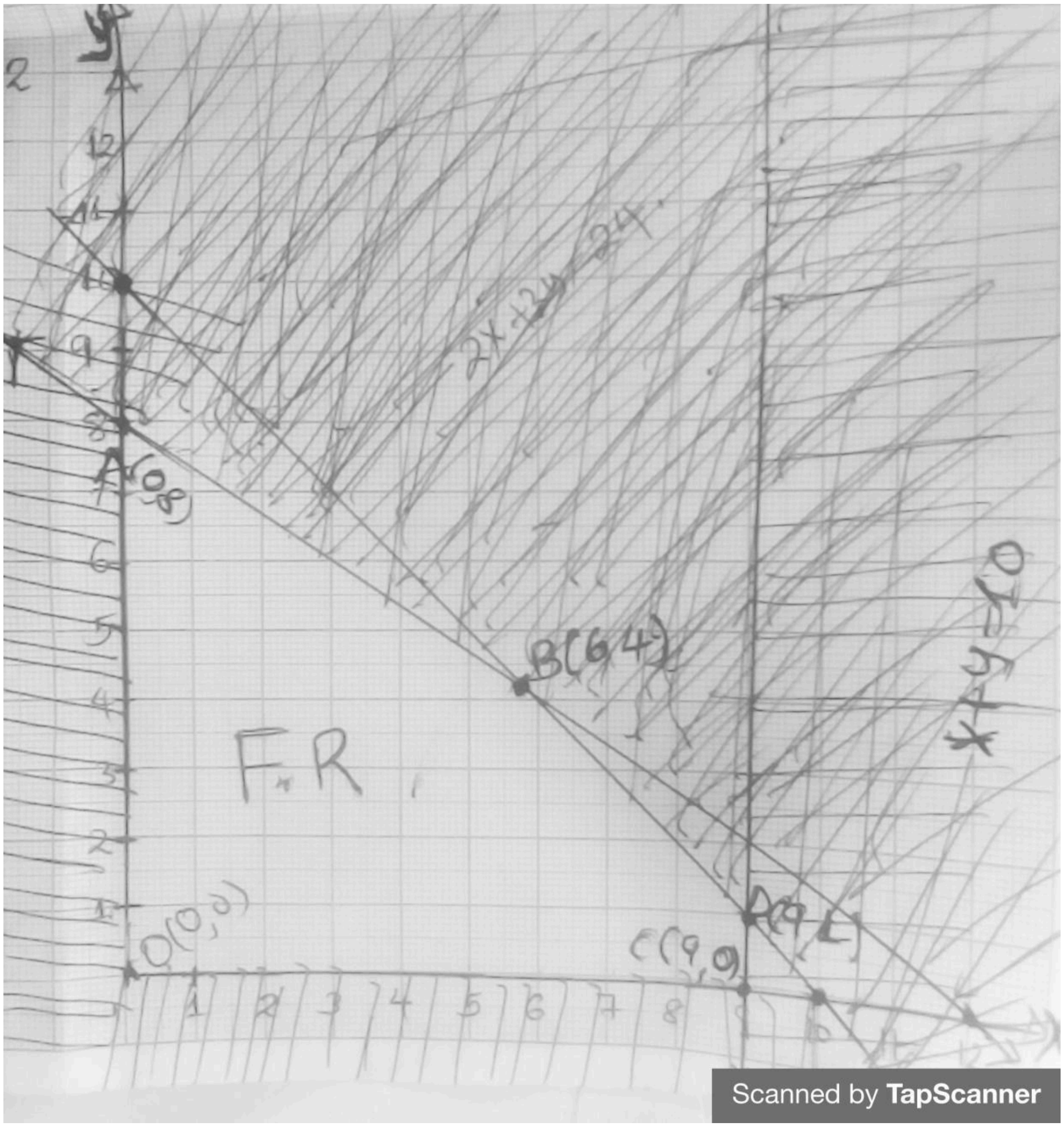
Max$f(x,\ y)$=$8,000x\ +\ 10,000y$ (1 mark)

S.t $20,000x\ +\ 30,000y\leq 240,000$ (12, 8)

$x\ +\ y\leq 10$ (10, 10)

$x\leq 9$ (1 mark)

$x\geq 0,\ y\geq 0$



(graph plus corner points 3 marks)

God Bless Us Teachers And Students